Dynamic Based Control Strategy for Haptic Devices

Khan Suleman
Machine design dept. Royal Institute of Technology-KTH.

Kjell Andersson
Machine design dept. Royal Institute of Technology-KTH.

Jan Wikander
Machine design dept. Royal Institute of Technology-KTH.

ABSTRACT

Transparency is a key performance measure for haptic devices. In this paper, we investigate a control strategy to increase the transparency of a haptic device. This control strategy is based on careful analysis of the dynamics of the haptic device, computed torque feed forward control and current feedback based force control. The inverse dynamic equation of motion for the device is derived using the Lagrangian formalism and the dominating terms are identified for some representative motion trajectories. The user contact dynamic model is identified using experiments on the device with different users. A PI controller using motor current measurements is used to follow the reference force from the virtual environment. Experimental results illustrate the effectiveness of the control strategy.

KEYWORDS: Haptic devices, transparency, computed torque control, inverse dynamics modeling, simulation and control design.

INDEX TERMS: K.1 [Introduction]: K.2 [control strategy]: K.3 [Experimental results]:

1 INTRODUCTION

A haptic device provides a communication link between an operator and a virtual or remote world. The device reflects forces/torques to the operator based on interaction and manipulation within the virtual or remote world. The intended application of the device presented in this paper is in a medical/dental simulator [1]. In this scenario a haptic device is used to achieve manipulation capabilities and force/torque feedback in six degrees of freedom (6-DoF) during simulation of surgical or dental procedures in hard tissue such as bone structures [1]. Such procedures involve removing bone by drilling or milling, including processing of channels and cavities, hence requiring 5-6 degrees of freedom. Use of a haptic device in the above described application leads to three important design requirements on the control design of the device [2, 3]:

- Stiffness
- Transparency and stability
- Feedback of force and torque in 6-DoF.

The requirement on transparency means that motion in free space should feel free while motion in contact with a virtual or remote object should result in feedback forces and torques as close as possible to those appearing in the remote or virtual world. In free space motion, transparency is affected by the dynamics (moving inertia, friction) of the device.

Keeping the device inertia as low as possible as well as compensating for it in control design will increase the transparency. However, dynamics based control imposes hard requirements on the real time implementation of the controller, thus requires a complete dynamics and control analysis for the device. The focus of this paper is the dynamics analysis and corresponding compensation for a 6-DoF parallel kinematic device used in an impedance control scheme.

1.1 Review

To derive the inverse dynamics equation for a parallel haptic device several methods have been reported in the literature, including Newton Euler formulation [4, 5, 6], Lagrangian formulation [7, 8], principle of virtual work [9, 10] and generalized momentum approach [11]. The Newton Euler method is based on a free body diagram of bodies and involves calculation of all forces and moments at each joint including all constraints. The Lagrange formalism is based on the total energy of the system and provides resulting equations of motion in a form well structured for control design. S. Lee et al. [8] used the Lagrange formalism to derive the equation of motion for a 6 DoF parallel haptic device. However, both of the above classical formulations usually require high computation due to their complexity. To reduce the computational cost of the Lagrange dynamics approach, some researchers have used a dynamic formulation based on the principles of virtual work and generalized momentum. Tsai [9] formulates the inverse dynamics of a Stewart–Gough manipulator by means of the principle of virtual work. Lopes et. al. [11] used a new formalism based on generalized momentum to develop equations of motion for a Stewart platform and 6-DoF Robotic Control Impedance Device (RCID) parallel manipulators. This method is based on the kinetic component of the generalized momentum acting on each rigid body of the manipulator. However, for the real time application of control, the above discussed methods end up with inverse dynamic models that still result in too high computational load to be effectively used [12, 13].

In our research, the Lagrangian formalism is used to derive and analyze the equations of motion. This analysis of the full dynamics is used to compare contributions from the different dynamic effects on the required motor torques for some representative motion profiles. Based on this, simplified equations of motion are derived to form a well structured and computationally efficient inverse dynamics model to be used for control design.

As given in the literature, two control structures; impedance and admittance control are widely used for haptic devices [14, 15, 16]. To implement impedance control, an open-loop control structure is traditionally used [17]. It is then assumed that the dynamics of the device e.g. time dependent disturbances, friction, inertial and gravitational forces are negligible. However, it is also reported in literature that the quality of haptic feedback (forces/torques) in open-loop is susceptible to environment model errors and time dependent disturbances [18]. Therefore, for high transparency of a...
device, it is well motivated to compensate for the dynamics and frictional forces of the device with in closed loop.

1.2 Approach
To address the above described problems, a 6-DoF parallel haptic device has been optimally designed for force/torque capabilities and kinematic isotropy by the authors as reported in [2, 3]. The parallel mechanism (as opposed to a serial mechanism) is selected based on its general merit of a high stiffness-to-inertia ratio. But still it is necessary to consider dynamics of the device in control design to achieve the required transparency. Thus the main focus of this work is to evaluate the dynamic behavior of the developed haptic device and to develop a control strategy that is in accordance with the design targets of the device itself – high transparency and high stiffness. The remaining part of the paper is organized in sections. Section 2 presents the dynamics model and simulation of the developed 6-DoF haptic device; section 3 identifies the user model; section 4 investigates a control strategy; and finally results and discussions are presented in section 5 and 6 respectively.

2 DEVICE DYNAMICS

2.1 Structure of the developed 6-dof haptic device

The developed haptic device is based on a parallel mechanism in the form of a modified version of the JP Merlet Kinematic structure [19, 3]. The structure consists of a fixed base, a moving platform, and six identical legs connecting the platform to the base see figure 1. Each leg consists of an active actuator fixed to the base, a spherical joint, a constant length proximal link, and a universal joint. The joint attachment point pairs are symmetrically separated 120° and lie on a circle, both on the base and platform. The platform attachment points are rotated 60° clockwise from the base attachment points. The 6-PSU (active prismatic, spherical and universal) joint configuration is used to get 6-DoF motion at TCP. Six independent DC motors connected to cable transmissions are used to provide a linear motion of each actuator.

The first term on the left hand side of equation 1 represents inertial forces, the second term accounts for the coriolis and centrifugal forces, and the last term is the gravitational force. On the right hand side f is the generalized Cartesian force at the end-effector, \( \mathcal{F} \) is the reflected force from contacts with virtual objects (i.e. delivered by the device actuators) and \( \mathcal{F} \) accounts for the frictional forces. In this paper the effect of frictional force is for simplicity neglected in dynamic simulation, but is considered in the control design of the device. The resultant generalized forces acting on the end effector can be transformed to a generalized torque \( \tau \) in joint space through the Jacobian matrix \( J \) as \( \tau = J^{-1}(q) f \). The inertial, coriolis and centrifugal forces are derived from the kinetic energy equations and the gravitational force is calculated from the potential energy, for each rigid body. In the following sections the Lagrange formalism is used to derive the equation (1) of motion for the developed 6-DoF haptic device.

2.2 Dynamic modeling using Lagrange formalism

The closed form dynamic equation of motion for a general parallel haptic device in Cartesian space can be described as.

\[
M(q)\ddot{q} + V(q, \dot{q}) + G(q) = f - f_e - f_f, \tag{1}
\]

where \( q \) is the joint coordinate vector, \( M(q) \) is the mass matrix, \( V(q, \dot{q}) \) is the potential energy, \( G(q) \) is the gravity vector, \( f \) is the generalized input force, \( f_e \) is the reflected force from contacts with virtual objects, and \( f_f \) is the frictional forces. The left hand side \( M(q)\ddot{q} + V(q, \dot{q}) + G(q) \) represents the actuator velocity in joint space through the Jacobian matrix \( J \). The inertial, coriolis and centrifugal forces are represented in section 2.2.1 Platform Kinetic energy. For simplicity, the platform is considered a circular plate of radius \( R_p \) and mass \( m_p \) acting at the geometrical center of the platform. Thus the platform kinetic energy is

\[
K_p = \frac{1}{2} \dot{q}^T (M(q)_p) \dot{q}, \tag{2}
\]

where \( \dot{q} = \begin{bmatrix} p_x & p_y & p_z & \dot{\alpha} & \dot{\beta} & \dot{\gamma} \end{bmatrix}^T \) is a vector of general linear and angular velocities and \( M(q)_p \) is the mass matrix of the platform.

\[
M(q)_p = \begin{bmatrix}
m_p & 0 & 0 & 0 & 0 & 0 \\
0 & m_p & 0 & 0 & 0 & 0 \\
0 & 0 & m_p & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & I_p \\
0 & 0 & 0 & 0 & 0 & 0 \\
I_p & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

where \( I_p \) is the platform inertia tensor at the centroid, and \( ^bR_p \) is the rotation matrix of the platform coordinates with respect to the base frame.

2.2.2 Kinetic energy of the actuator Link \( l \)

The actuator link \( l \) of the developed mechanism provides translational motion only in Z-direction. As a result, the velocity of the actuator link has a component only along \( z \)-direction in the joint coordinate system. This component is calculated from the position vector of leg \( l \) as

\[
l_{ui} = (0, 0, \dot{z}_{ui}).
\]

Let’s assume that the mass of the actuator link is a point mass \( (m_l) \) acting at the geometrical center of the link then the kinetic energy of the link is

\[
K_{al} = \frac{1}{2} l_{ui} ^T m_l l_{ui}. \tag{3}
\]

It is needed to transform all the energy equations to Cartesian space in order to derive the equation of motion in the same reference frame. Let \( l_1, \ldots, l_n \) represent the actuator velocity in joint coordinates and \( \dot{q} \) represent motion in Cartesian space. Then the Jacobian matrix \( J(l) \) (derived in [3]) relates the motion in joint space \( l \) to the motion in Cartesian space as

\[
l = J\dot{q}, \tag{4}
\]

Finally, the actuator link kinetic energy is represented in the Cartesian space by equation (5).

\[
K_{al} = \frac{1}{2} J^T \dot{q}^T m_l J\dot{q}. \tag{5}
\]
2.2.3 Kinetic energy of the proximal link \( \text{ci} \)

In the developed device, the proximal link \( \text{ci} \) can have both translational and rotational motions, but the universal joint constrains the rotation around x-direction such that the link can rotate only around y and z-directions \((\theta_y, \theta_z)\) respectively in its local coordinate system. The local coordinates of the proximal link is fixed as; \(X_{\text{pli}}\) axis is kept along the proximal link (parallel to vector \(a_i\)) and points from the proximal joint towards platform, \(Y_{\text{pli}}\) axis is kept perpendicular to \(X_{\text{pli}}\) axis and it is always parallel to the base plane, while \(Z_{\text{pli}}\) is fixed through right-hand rule and its projection along axis \(Z_{\text{pli}}\) is always positive. We assume that the center of mass of each proximal link \(\text{ci}\) is at a constant distance \(P_{\text{cm}}\) from the platform connecting point \(P_i\) as shown in figure 2. The position vector \(a_{\text{Pcm}}\) of the center of mass, relative to the fixed frame \{B\} can be derived from a closed loop vector equation as

\[
\text{b}_{P_{\text{cm}}} = \text{b}_{P_i} + \text{b}_{P_{\text{cm}}} = \text{d} + p_i - \frac{P_{\text{cm}}}{C} \cdot a_i, \quad (6a)
\]

\[
\text{b}_{P_{\text{cm}}} = \text{b}_{P_i} + \text{b}_{P_{\text{cm}}} = \text{d} + p_i - \frac{P_{\text{cm}}}{C} (\text{b} + p_i - h_i - l_{ai}), \quad (6b)
\]

Where \(\text{b}_{P_{\text{cm}}}\) is a vector that represents the co-ordinates of center of mass of proximal link \(\text{ci}\) expressed in base frame \{B\}. Taking the derivative of \((6b)\) with respect to time in the base frame results in linear velocity of center of mass as

\[
\text{b}_{P_{\text{cm}}} = (1 - \frac{P_{\text{cm}}}{C}) \cdot \text{w}_x \times p_i + \frac{P_{\text{cm}}}{C} (\text{b}_i - l_{ai}). \quad (6c)
\]

Where

\[
\text{w}_x \times p_i = \begin{bmatrix} z & x & y \\ 0 & \theta_z & \theta_y \\ s_{p_i/z} & p_i/y & p_i/z \end{bmatrix}
\]

and \(i = J\dot{q}\),

\[
\text{R}_{\text{pli}} = \left[ \begin{array}{ccc} x_{\text{pli}} & y_{\text{pli}} & z_{\text{pli}} \\ 1 & 1 & 1 \end{array} \right] \quad \text{and} \quad \text{R}_{\text{pli}} = \text{R}_{\text{pli}} \text{R}_{\text{pli}}^{-1}.
\]

Finally, the rotation matrix for proximal link \(\text{ci}\) is given by equation \((6d)\)

\[
R_{\text{pli}} = \left[ \begin{array}{ccc} x_{\text{pli}} & y_{\text{pli}} & z_{\text{pli}} \\ 1 & 1 & 1 \end{array} \right].
\]

2.2.4 Potential energy of platform, actuator and proximal link \(i\)

The potential energy of platform, actuator and proximal link is given by equation \((10a)\), \((10b)\) and \((10c)\) respectively.

\[
U_p = m_g \text{g} d_z. \quad (10a)
\]

Where \(m_g = [0 \ 0 \ 0]^T\) and \(d_z\) is the Z-component of the position vector of the center of mass of platform expressed in the base coordinates.

\[
U_{\text{ai}} = \sum_{i=1}^{m} m_{\text{ai}} \text{g} r_{\text{ai}}/z. \quad (10b)
\]

Where \(m_{\text{ai}}\) is the mass of actuator \(i\) and \(r_{\text{ai}}/z\) is the Z-component of the position vector of the center of mass of actuator link \(l_i\) expressed in the base coordinates.

\[
U_{\text{ci}} = \sum_{i=1}^{m} m_{\text{ai}} \text{g} r_{\text{ci}}/z. \quad (10c)
\]

Where \(m_{\text{ci}}\) is the mass of proximal link \(\text{ci}\) and \(r_{\text{ci}}/z\) is the Z-component of the position vector of the center of mass of the proximal link \(i\) expressed in the base coordinates.

\[
r_{\text{ci}}/z = \left[ 1 - \frac{P_{\text{ai}}}{C} Z_y + \left( 1 - \frac{P_{\text{ai}}}{C} \right) P_{\text{ai}} X_y + \frac{P_{\text{ai}}}{C} l_i \right].
\]

2.3 Equation of motion

Based on the derived kinetic and potential energies of all moving components, the equation of motion \((1)\) is derived using
Lagrange formalism according to equation (11). The resultant equation of motion is given in equation (12a) and (12b).

\[
\frac{d}{dt} \left( \frac{\partial \mathbf{K}}{\partial \dot{q}_i} \right) - \frac{\partial \mathbf{K}}{\partial q_i} + \frac{\partial \mathbf{U}}{\partial q_i} = f_i \quad i = 1, \ldots, 6, (11)
\]

\[
M(q)\ddot{q} + M(q)\dot{q} + \dot{q}^T M(q)\dot{q} + G(q) = f - f_e - f_f, \quad (12a)
\]

\[
M(q)\ddot{q} + V(q, \dot{q})\dot{q} + G(q) = f - f_e - f_f, \quad (12b)
\]

Equation (12b) gives the forces in the Cartesian coordinate system. These forces can be transformed to the leg (joint) coordinate system using the relations,

\[
\tau = \mathbf{J}^{-T}(q) f, \quad \mathbf{i} = \mathbf{J}\dot{q} \quad \text{and} \quad \dot{\mathbf{i}} = \mathbf{J}\ddot{q} + \mathbf{J}\dot{q}.
\]

Finally, the equation of motion that shows the resultant forces on actuators in the joint coordinate system is

\[
\mathbf{M}(l)\ddot{\mathbf{i}} + \mathbf{V}(l, \dot{\mathbf{i}}) + \mathbf{G}(l) = \tau.
\]

2.4 Simulation of the dynamic model

The model is implemented in Maple® 13 and Matlab® 9 using the input parameters as specified in table 1.

**TABLE I: PARAMETERS USED IN DYNAMIC MODEL SIMULATION**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of platform ($R_p$)</td>
<td>55 mm</td>
</tr>
<tr>
<td>Length of proximal link ($l_{pli}$)</td>
<td>124 mm</td>
</tr>
<tr>
<td>Actuator length ($l_a$)</td>
<td>130 mm</td>
</tr>
<tr>
<td>Gravitational acceleration</td>
<td>9800 mm/s²</td>
</tr>
<tr>
<td>Mass of platform ($m_p$)</td>
<td>0.1 kg</td>
</tr>
<tr>
<td>Mass of proximal link ($m_{pli}$)</td>
<td>0.035 kg</td>
</tr>
<tr>
<td>Mass of actuators+motor inertia ($m_a$)</td>
<td>0.135 +0.385 kg</td>
</tr>
<tr>
<td>Force and torque on TCP</td>
<td>0 (free space)</td>
</tr>
</tbody>
</table>

For the dynamic model simulation, a 5th order trigonometric trajectory in Cartesian space is developed for each degree of freedom of the platform, to get a smooth motion without jerk. The motion starts and ends with zero velocity and acceleration. The position and velocity trajectories for each degree of freedom, in Cartesian space, are shown in figure 3 and 4. The trajectories of actuator motion (from simulation) are shown in figure 5.

Similarly, the trajectories of the corresponding actuator torques are shown in figure 6. Due to the space limitation only the actuator having maximum torque requirements is shown. The simulations were repeated with different input velocities and accelerations but with the same position trajectories to study the
effects of different components of the dynamic equation. It was concluded that the major component of the total torque on each motor is due to the gravitational and inertial components. The effect of coriolis and centrifugal forces is very small as compared to the other components as shown in figure 6. The same model was also verified in Adams View software using the same input. From a dynamics compensation point of view, gravitational and inertial components are the more important and should be considered in the control design.

3 USER DYNAMICS

To model and identify the contact impedance between user and the tool connected to the haptic device, a simplified one degree of freedom second order system (mass, spring and damper) is used. The equation of motion for contact impedance between user and tool is given by

\[ M_h \ddot{x}_h + d_h \dot{x}_h + k_h x_h = F_{\text{active}} - F_a \] (14)

Where \( M_h \) is the mass of user hand, \( F_{\text{active}} \) is the user intention al force, \( F_a \) is the actuator force and \( x_h \) is the position of tool. The contact impedance can be represented in the Laplace domain as

\[ Z_h(s) = \frac{M_h s^2 + d_h s + k_h}{s} \] (14a)

The parameters of the above equation are associated with major uncertainty due to a number of influence factors. These factors are for example type of grasp, variation in the hand mass, grasping force and physiological condition. Therefore the mechanical parameters in equation (14a) were identified by measurements for five users. In these experiments the users held the tool firmly but with \( F_{\text{active}} = 0 \) and a reference current signal with varying frequency and amplitude was applied to the actuator to represent \( F_a \). The resultant velocity was recorded with a sampling rate of 1 kHz from both the model and the real system. A Matlab identification toolbox was used to estimate the model parameter values as \( M_h = 0.05, k_h = 14 d_h = 1.5 \).

4 DESIGN OF CONTROL STRATEGY

In designing a control strategy for the haptic device, to achieve high transparency, we consider the device dynamics (derived in section 2, 3) in a closed loop. We apply a computed torque based high transparency, we consider the device dynamics (derived in section 2) in a closed loop. We apply a computed torque based control structure is given in figure 7. The device was connected to a virtual environment, with the user trying to move the virtual tool in free space and in contact with a virtual object (wall) having the stiffness 50 N/mm. The haptic collision detection and force feedback program is implemented on the same computer. The position measurement resolution in each actuator leg is 0.01mm and the update rate of the controller is 1kHz.

Then a compensation for the dynamic influence \( F \) of the device, is added to the control signal as a feed-forward term. The aim of this feed-forward term is to increase the transparency of the device, i.e. the user should not feel the inertia and friction of the device itself, only of the tool. For this purpose, the dynamic model derived in section 2 is used to calculate the required actuation forces \( F \). As the inverse dynamics of the 6-dof haptic device is non-linear and complex, a simplified model was considered for the control design. It is clear from the simulation that for the motion profiles considered the effect due to coriolis and centrifugal forces is less than 2% of the total torque, thus we did not consider those in control design. Also it was observed that the torque contribution from the proximal link is very small. Thus we added the mass of the proximal link to the actuator mass and tuned it to get approximately the same dynamic results as with full dynamics. The simplified equation used for control design is

\[ F = M(q)\ddot{q} + f_s(q) + G(q) \] (15)

Where \( f_s(q) \) represents velocity dependent compensation terms (friction in motor and joints, back-Emf in the motor), and \( F \) is the compensation.

5 EXPERIMENTAL SETUP AND RESULTS

The developed 6-dof haptic device is connected to a personal computer using a dSpace 1103 board as shown in figure 8. The proposed control structure is implemented in Simulink on that PC and the target controller code is executed on the dSpace board with 1kHz sampling rate. The haptic collision detection and force torque feedback program is implemented on the same computer. The position measurement resolution in each actuator leg is 0.01mm and the update rate of the controller is 1kHz. In order to investigate the performance of the control structure, experiments were performed with different device control strategies: open loop; computed torque with dynamic compensation; and computed torque plus force feedback (Figure 7). The device was connected to a virtual environment, with the user trying to move the virtual tool in free space and in contact with a virtual object (wall) having the stiffness 50 N/mm. The reference force from the virtual object contact and the forces produced by the device (calculated from motor currents) were recorded for the different control strategies; open loop; computed torque; and computed torque plus force feedback with results as given in figures 9, 10 and 11.
To handle these problems, first a parallel mechanism was selected for low inertia, and its structure optimized for workspace size and actuator force requirements. Second, and as presented in this paper, a device internal control strategy was designed and experimentally verified. The influence of the dynamics of the device is compensated, considering an analysis based simplification of the inverse dynamic model and current based force feedback control, thus improving the transparency of the system. From the experimental results of the proposed control strategy, it is concluded that device performance is improved. Even though the device structure is optimized for low inertia, it is evident from Table 1 that the motor inertia is substantial. In current work, motor performance, motor inertia, transmission ratio, friction and sensor resolution are being reconsidered and optimized to further improve device performance.

6 CONCLUSION

The application context, surgery in bone and skull, leads to two conflicting design requirements on the haptic device: stiffness and transparency. High stiffness and zero backlash has an influence on transparency in the sense that they typically increase inertia and friction. To handle these problems, first a parallel mechanism was selected for low inertia, and its structure optimized for workspace

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