The Valuation of Residential Rental Options

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Abstract

This paper develops a formula for pricing a residential option with respect to a tenant's so called outside option in which two new parameters are introduced; the tenant's transaction cost of moving and moving threshold. This is then used to simulate the price given a number of "reasonable" parameter values. We believe that the pricing formula developed in this paper provides a potentially useful way of conceptualizing the way households might actually think at the time of deciding to buy an option. Simulation results show, as could be expected, that the value of an option increases with higher transaction cost of moving, and decreases with higher moving threshold. These findings imply households to reflect over what could be a reasonable level of its moving threshold and transaction cost of moving. With a low moving threshold (i.e. a liquidity constrained household) and/or a high transaction cost of moving, the rental option may have a high value for this household. This value might therefore be higher than the option value calculated the standard way. Naturally, the opposite situation occurs for a household with a high moving threshold and/or a low transaction cost of moving.

Keywords: rent regulation, rent control, rent insurance, rental option, real option

1 Introduction

1.1 Background

Rent control (or rent regulation) systems exist in several housing markets worldwide.[1] Typically, protection given by rent control covers rent increases related to increases in demand caused by an increasing general attractiveness of an area (Lind 1999a). Although they have different features, a common characteristic of such rent control policies is that they lack any mechanism that will result in equality between the controlled long run rent levels and the market rent levels. Both economists and landlords usually argue that this discrepancy caused by rent control policies create many drawbacks. For instance, there is a widespread agreement that rent control systems discourage new construction, cause abandonment, retard maintenance, reduce mobility, generate mismatch between housing units and tenants, create black markets, exacerbate discrimination in rental housing, encourage the conversion of rental to owner-occupied housing, and generally short-circuit the market mechanism for housing (Arnott 1995). But some types of rent control systems may give rise to more severe negative effects than others do. For instance, Lind (2001) identifies five different types of rent regulation systems (A-E). While two of these systems (A and B) only protects sitting tenants, the three other systems (C-E) also cover the rent in contracts with new tenants (see appendix A for further explanation of the various types of categories). Given this classification, it is most likely that the most severe criticism of rent regulation system can be leveled against rent systems that belong to any of the last three categories.

As a specific example, consider the Swedish rent regulation system. According to Lind (2001), the current system of rent regulation in Sweden belongs the type E, i.e. a system that protects a sitting tenant against rents higher than the market rent, and that also aims at keeping rents in new contracts below the market level. Ellingsen and Englund (2003) point out that this system has caused rent levels to be far below clearing levels in most of Stockholm and central parts of other major cities in Sweden, and this in turn has caused negative effects like illegal key money, a flourishing market for second-hand contracts, rapid conversion of rental apartment buildings into housing cooperatives, tenants locked into sub-optimal housing arrangements etc.

In spite of the fact that different types of rent control policies create more or less severe drawbacks, it may still not be desirable to carry through a far-reaching reform that will result in total absence of specific regulations concerning rental contracts and rent protection mechanisms. One reason not to introduce a rental market that only relies on general contract law, is that sitting tenants may have high transaction costs of moving, but also weak bargaining power. Therefore, in order to mitigate the negative effects of regulated rents, as well as to avoid undesirable effects that may occur if there exist no specific rules for the rental market at all besides general contract law, it is of interest to find other ways of protecting at least sitting tenants against major increases in market rents.

[1] For an updated and exhaustive overview and analysis of rent regulation systems with main focus on Sweden, see “Swedish Economic Policy Review”, volume 10, 2003. I will treat rent control and rent regulation as synonyms in this article.

Furthermore, Ellingsen and Englund (2003) argue that equity and efficiency goals that can be obtained through traditional rent control can be obtained more efficiently either through voluntary contracting or through some other cheaper intervention. Indeed, it is of particular interest nowadays to discuss the implementation of "market solutions" based on individual and voluntary contracting when risk reducing devices on the market are both more developed and well known, compared to the time period when rent control policies were introduced. Therefore, it might be of interest to consider a regulatory reform that aims at finding some kind of market solution that may serve as a substitute for a regulatory system.[2]

Lind (1999a) and Lind (1999b) propose the use of more sophisticated lease agreements and in particular some type of "rental option" contracts, where tenants pay an insurance premium to obtain a protection against high rent increases. Such a "market based" risk mitigating instrument can indeed be an attractive alternative to traditional rent regulation policies. The proposed option policy may also increase the possibility to find acceptance for letting rents in vacant apartments be set so they reflect supply and demand. There also exist other arguments in favour of market solution. For instance, with market solutions, adaptation to individual desires can be obtained in a better way. Furthermore, the political risk might be reduced, i.e. the risk that the protection given by rent regulation suddenly disappears due to a change in the political majority. Song (2004) further develops how a rental option can be constructed. It is the pricing of this kind of rental option that is the main subject of this paper.

1.2 Some basic properties of the proposed rental option policy

The aim of the proposed rental option policy is to provide sitting tenants with protection against sharp increases in market rent, for instance due to increases in demand. The insurance policy may be viewed as a call option. That is, a tenant who owns this insurance has a right to reside in the current apartment after the next rent review paying a rent that is the lower of the markeet rent and the so called strike price (or strike rent). In other words, the owner of this option will only exercise it if the market rent exceeds the strike price at the time of the rent review (i.e. the maturity date). On the other hand, the tenant will not exercise this option, if the market rent ends up below the strike price, since he or she can pay the lower market rent for the following rental period.[3] For this right, the tenant must pay an option premium.

The proposed insurance policy will comprise a rule that says that the landlord is obliged to offer the tenants a rental option. The proposed option policy should thus be regarded as a substitute for rent regulation, and if as – in Sweden – we start from a situation with rent regulation, it can be seen as a "trade" where rents are at least partly deregulated at the same time as the landlords are obliged to offer rent options. The idea of only partly deregulating the rental market is further embodied by keeping some particular regulations concerning rental contracts. Above all, the rental contracts must be in writing, tenants shall possess the right of not being evicted under ordinary circumstances, and they shall still have a right to cancel a contract with at most three month's notice. For a more detailed discussion concerning the framework of the proposed option policy, see Song (2004). Here, I state the main points of the proposed option policy:

[2] Another way for households to reduce rent risks is to enter ownership. But not all households have enough economic mean to enter ownership (for instance enough liquidity to meet down payment requirements, see Atterhög and Song, 2003). Yet another way is to enter long-term lease agreements.

[3] We always assume that a tenant has a right to legal protection of tenancy, in the sense that he or she always has a right to reside in the rented apartment as long as he or she "behaves well".

- When leasing to a new tenant, rents are based on prevailing market rent levels.
- The landlord is the insurer and the landlord is obliged to offer his tenants insurance against major rent increases.
- Tenants who have not purchased any insurance contract, are supposed to pay rents equal to market rents whenever rents are being reviewed.
- For an insured tenant, the new rent will be set to the market rent or the strike rent, whichever is lower.
- The landlord is obliged to offer tenants a minimum amount of different alternatives of insurance periods, e.g. 5 or 10 years.
- The maximum premium a tenant has to pay for the rental option shall not exceed a pre-specified level, for instance 5% of the periodic rental payments during the time before the rent review.
- A tenant shall have a right to split up the cost of the option premium at least during a time period between the purchase of the option (which typically will coincide with the time a new agreement is signed) and the time of the rent review. In practice, the partitioned costs of the option should be added to the periodic rental payments.

The insurance policy proposed in this paper should be understood as combining long-term leases with a rental option, and that these two types of contracts together define the length of the insured period. The insured period simply equals to the sum of the lengths of the two long-term lease periods that occur before and after the rent review (assuming single rent review insurance contracts).

To clarify this, consider following example. A tenant signs a new rental contract today at market rent. The rental contract states that the rent will be reviewed five years from today. Until that day, the rent will be adjusted with respect to yearly changes in some index, e.g. the consumer price index.[4] At the same time, he or she also buys a rental option with a maturity date that is equal to the date for the next rent review. Then, at the time of the rent review, the rent will either be set to the strike rent or the market rent, whichever is lower. The reviewed rent will after the review again be adjusted according to the changes in the underlying index for yet another five years. Thus the sitting tenant will enjoy a protection against strong rent increases for a period of ten years.

1.3 The pricing problem considered in this paper

Several authors have dealt with the problem of developing appropriate pricing models for different kinds of commercial leases with embedded options. In particular, so-called upward-only adjusting leases have received much attention among scholars (see for instance Baum *et al*. 1998; Ward and French 1997; Booth and Walsh 2001a, 2001b; Ambrose *et al.* 2002; Clapham 2004). But also turnover (or overage or percentage) leases and leases with renewal options have been analyzed in the literature (see e.g. Buetow and Albert 1998; Hendershott 2002; Hendershott and Ward 2003; Clapham 2003). Several of these articles apply risk-

[4] Indeed, with an indexed rent tied to the CPI, the yearly changes in the indexed rent should be sufficiently foreseeable, since price stability is a comprehensive goal for Swedish central bank as well as for many other central banks in developed countries.

neutral valuation approach in a continuous time setting, which has its origin in the seminal papers of Black and Scholes (1973) and Merton (1973). But it is not known by the author if there exist any articles that explicitly deals with the problem of pricing residential lease or rental options. One natural explanation of this might be that there exist no residential rental contracts with explicitly defined options like features.

But note that there typically exist (at least in Sweden) one major difference between commercial and residential rental contracts; residential rental contracts are not time limited as opposed to commercial leases. As long as a tenant "behaves well", he or she has a right to reside in the apartment for the time being. Therefore, it can be argued that residential leases have embedded "multiple" renewal options. Buetow and Albert (1998) discuss the value of a commercial renewal option for the special case a lessee has an option to renew a time-limited commercial lease, where the strike (or renewal) rent is defined as the market rent at the time the lease expires. Buetow and Albert argue that such an option will still have a value to the lessee, even if the option by definition will be at-the-money, or have an intrinsic value of zero at expiration. Without modeling this value, they stipulate that the value to the lessee is simply the present value of the combined cost of relocating and the locational goodwill, and that this amount is the most a lessee will pay for this option. Thus, we may establish that an option to renew at market gives the owner of the option some value.[5] And of course, if the strike price of the renewal option is defined in such a way that the option also gives the lessee an insurance against sharp rent increases, then this option will be even more valuable.

Thus, while the commercial renewal option not only gives protection against strong rent increases (given that the strike price is not defined as the market rent at the time the lease expires), but also assures that the lessee can stay in the rented unit for another lease period, the sole object of the proposed residential rental option is to provide insurance against strong rent increases; there is typically no need for a residential tenant to acquire a renewal option. Nevertheless, option pricing formulas that are related to renewal options may be useful in valuing the proposed residential rental option, since they share some common important features. Therefore, some "standard" renewal option pricing formulas will be presented below in section two.

The rental option pricing formulas presented below are based on an implicit assumption that a tenant will stay in the apartment he or she currently lives in after the first rent review, no matter if he or she owns a rental option. In other words, the model does not explicitly consider the fact that a tenant may choose to exercise his or her right to terminate a current rental agreement. For instance, if the market rent at the time for the rent review ends up above an uninsured tenant's reservation price for the current apartment, this tenant may choose to terminate the rental contract in order to move to a cheaper apartment. This right, henceforth called the "outside option", may indeed in one way or another affect the value of a rental option. Therefore, it is of interest to see if the inclusion of this outside option in a pricing model yields different option prices compared to the "standard" case where this outside option is not taken into account.

There may exist several ways to incorporate this outside option into a pricing model. In this paper, I choose to model the impact of the outside option by introducing two parameters: a tenant's moving threshold (or reservation price) and the same tenant's total transaction cost of moving to another housing unit. In this paper, total transaction cost should be interpreted in a

[5] See also Geltner and Miller (2001, pp. 824) for a discussion of this problem.

broad sense; it includes both costs for new housing and transaction costs (see further below). By introducing these two parameters, I try to model how a tenant actually may think when he or she will decide upon buying a rental option offered by the landlord rental option (the idea behind the inclusion of these two parameters is discussed in detail in section 3.2 below).

Hence, the purpose of this paper is to derive a pricing model for a rental option in which a tenant's right to terminate his or her current contract in order to move to another housing unit is explicitly taken into account when a tenant's moving threshold and total transaction costs enter the formula as parameters.

Since a vast majority of commercial renewal option pricing formulas are based on standard continuous time financial calculus, it is appropriate to develop the pricing model for the residential option in the same manner.

The rest of the paper is organized as follows. The next section presents some basic facts regarding the assumed stochastic process governing the market rent, some characteristics of the proposed rental option, and a brief survey of some option pricing formulas from the literature on lease renewal options. In section 3, a pricing model is developed where a tenant's moving threshold and relocation costs are included as parameters in the pricing model. In Section 4, numerical results are presented and discussed. Finally, section 5 concludes this study and proposes guidelines for future studies and policy implications.

2 Rental option pricing – the standard model

2.1 The rental process assumption

In order to derive a pricing formula for the rental option, we need to define the stochastic process that governs the market rent. The market rent may be modeled as an exogenous process, or may be endogenously determined through some type of equilibrium condition (Clapham and Gunnelin 2003). Here, the spot rent process R_t (the value of the rent level at time *t*) is assumed to follow an exogenous geometric Brownian motion (GBM), also known as lognormal random walk, with constant expected drift rate μ and constant volatility σ. Several papers model R_t this way (e.g. Clapham 2004; Clapham 2003; Booth and Walsh 2001a and 2001b; Grenadier 1996). Thus I assume that the following stochastic process describes the evolution of the free market rent:[6]

$$dR_t = \mu R_t dt + \sigma R_t dW_t \tag{1}$$

where *W* is a Wiener process under the objective (or real-world) probability measure.[7] The uncertainty in future rent level is thus represented by *dW* (amplified with σ), where *dW* is a normally distributed random variable with zero mean and variance *dt*. Indeed, when *R* follows (1), the change in the logarithm of *R* between time *t* and *T* (where $T > t$), *d*ln*R*, is given by:

$$d\ln R_t = \left(\mu - \tfrac{1}{2}\sigma^2\right)dt + \sigma dW_t. \tag{2}$$

From now on though, we always presuppose that $t = 0$. In other words, we are only interested in the case in which a tenant signs a contract in the beginning of a new rental period and hence the value of the option at time $t = 0$.

The logarithm of *R* is said to follow a generalized Wiener process (see Booth and Walsh 2001b). Since *d*ln*R* is normally distributed, we have that

$$\ln R_T \in N\left[\ln R0 + \left(\mu - \frac{1}{2}\sigma^2\right)T, \sigma\sqrt{T}\right]. \tag{3}$$

Because $\ln R_T$ is normally distributed, R_T, i.e. the rent level at some future time *T*, is lognormally distributed. The expected value of R_T, $E[R_T]$, is simply given by

$$E[R_T] = R_0 e^{\mu T}. \tag{4}$$

In order to clarify the basic results concerning the future rent level when it is assumed that rents is governed by GBM, a simple numerical example is given now. Assume that the market rent level today R_0 is 1000 SEK per square meter and year. Assume furthermore that the expected drift μ and volatility σ is 3% and 7,5% per annum respectively. We will first

[6] Grenadier (1995) model the market rent endogenously, determined of current demand and supply.
[7] The increment of the standard Wiener process is therefore *dW*.

calculate a 90% confidence interval for R_T and then the expected value for R_T, both in 5 years time (T = 5).

A 90% confidence interval for R_T is given by (see appendix B)

$$\left[R_0 e^{(\mu-\sigma^2/2)T-1.64\sigma\sqrt{T}}, R_0 e^{(\mu-\sigma^2/2)T+1.64\sigma\sqrt{T}}\right] \tag{5}$$

Thus, there is a 90% probability that the rent level in five years time will lie between 870 and 1510. From (4), we obtain that the expected value $E[R_T]$ equals 1160 SEK per square meter and year.

2.2 Risk neutral valuation of the expected option pay-off

While the assumed "real-world" or actual rent process (1) above is governed by the objective probability measure or the *P*-measure, what really matters in risk neutral valuation is the stochastic process governed by the risk neutral, or *Q*-probability, measure (Björk 2004). Here, we denote the risk neutral (or the risk-adjusted) rental process by

$$dR_t = \alpha R_t dt + \sigma R_t dV_t \tag{6}$$

where α denotes the expected risk neutral drift.

Much like discount rates, finding the risk neutral drift of stochastic process can be based on intuitive reasoning, derived by calibrating pricing results to market data, or by using a formal model such as the CAPM (Clapham 2004). Indeed, it is commonly applied in e.g. the real options literature, to infer risk neutral drift from general equilibrium arguments (Gunnelin and Clapham 2003). This makes it possible to motivate the use of risk neutral pricing, even if such an approach is traditionally motivated by arbitrage arguments, which is not always a realistic assumption for real estate (Clapham 2003).

Several papers, for instance Grenadier (1995), Booth and Walsh (2001a and 2001b), and Hendershott and Ward (2003) discuss how the risk neutral drift α can be estimated using the continuous time CAPM. In short, the expected risk neutral drift can be calculated by reducing the expected actual growth rate of rents with the required risk premium on an investment in an asset whose return has the stochastic properties of rents. Indeed, given the continuous time CAPM, it holds that the risk premium can be expressed as $\lambda\sigma = \beta(r^M - r)$, where $r^M - r$ is the market risk premium, β is the beta of some process (with stochastic properties of residential rents) in relation to the market process, and where λ is "the market price of risk". In other words, the expected risk neutral growth can be expressed as $\alpha = \mu - \lambda\sigma$.

In this case, the geometric Brownian motion expresses the dynamics of *R* under the risk neutral measure *Q*:

$$dR_t = (\mu - \lambda\sigma)R_t dt + \sigma R_t dV_t. \tag{7}$$

Henceforth, the risk neutral drift will simply be denoted by α.[8]

When the market rent follows GBM, the rent level at a future time T in a risk neutral world is given by

$$R_T = R_0 e^{(\alpha - \sigma^2/2)T + \sigma W_T} \tag{8}$$

and the expected value is given by

$$E^Q[R_T] = R_0 e^{\alpha T} \tag{9}$$

(see Björk 2004).

The holder of a rental option has the right to enter into a new T year lease period at a certain new rent level, R^{new} (per square meter per period) by a certain future time T (day of maturity). R^{new} will either be set to the market rent at maturity, R_T, or the strike price, K, whichever is lower: $R^{new} = min(K, R_T)$. Naturally, the tenant will only exercise the option if it is in-the-money, e.g. when $R_T > K$. The payoff function at the maturity date of the option is therefore given by

$$\max(R_T - K, 0). \tag{10}$$

Given the Q-dynamics of R, the price of an rental option F_t, with strike price K, is given by the risk neutral valuation formula

$$F = e^{-rT} E^Q[\max(R_T - K, 0)]. \tag{11}$$

In other words, the value of the option, given today's date (and given today's rent level) is calculated by taking the risk neutral expectation of the payoff function (10) at date of maturity T and then discounting the expected payoff at a continuously compounded risk-free rate r for T years, where r is assumed to be a constant. For the particular pay-off structure considered in this paper, a closed-form analytical pricing formulas can be derived by explicitly evaluating this expectation (see below).

Since rents are quoted as per square meter per year and usually paid monthly or quarterly in advance during a several years' lease period, the payoff function (10) should be premultiplied with the space of the apartment of interest, S, and an appropriate annuity factor, A, respectively, in order to calculate the total option value.[9] In this case the payoff of the option at maturity is given by

$$SA[\max(R_T - K, 0)]. \tag{12}$$

[8] Note that in the original Black and Scholes (1973) option pricing formula for a European call option, the risk-neutral drift of a stock price is simply given by the risk-free interest rate r. By assuming that the risk neutral drift is given by r, as for instance Ward and French (1997) do, the only parameter that has to be estimated in order to use the pricing formula is the volatility, which clearly simplifies the use of the option pricing formula.

[9] I assume that the annuity amount $R_T - K$ will remain constant during the new T year lease period (or that any changes in the difference is of negligible size).

But since we are mainly concerned to compute and compare prices per square meter, S will be set to one henceforth.

From general option pricing theory, we know that the pricing function F_t of the option must satisfy a fundamental pricing equation (see Black-Scholes 1973; Merton 1973) given by the partial differential equation (PDE)

$$\frac{\partial F}{\partial t} + \alpha R \frac{\partial F}{\partial R_t} + \frac{1}{2}\sigma^2 R^2 \frac{\partial^2 F}{\partial R^2} = rF \tag{13}$$

subject to the boundary condition given by the payoff structure (10) above (t-subscript dropped here to simplify notation). Indeed, another way of deriving the pricing formulas below is by solving the fundamental pricing equation subject to the boundary condition determined by the particular payoff structure considered in this paper. That is, the closed-form pricing formulas that will be presented below can be derived from either risk neutral pricing or by solving the partial differential equations subject to a particular boundary problem (the so called PDE approach). These two approaches are equivalent because when F is a solution to the boundary value problem given by the contractual terms of a derivative (e.g. the payoff structure (10) above), F has a stochastic representation formula given by the risk neutral valuation formula (11) above. The representation (11) is called the Feynman-Kač stochastic representation formula (see Björk 2004).

The risk neutral valuation approach may be the most intuitively appealing method of finding a pricing formula of a derivate, and I will use it whenever a pricing formula is derived (see appendix C).

2.3 Review of some pricing formulas

It is possible to construct many different types of options. For instance, simple call options may differ with respect to how the strike price is determined. Three examples of determining the strike price:

1. Fixed strike price: K is contractually set at a fixed rate at time $t = 0$.
2. Indexed strike price: K is set at maturity and will equal the current rent adjusted by the cumulative change in some index X_t from $t = 0$ till T. Given that the index is normalized to 1 at $t = 0$, the strike price becomes $K = R_0 X_T$.
3. Strike price as a function of the market rent: K is set maturity and will equal a percentage p of market rent at time T. The strike price at maturity is $K = pR_T$.

Indeed, a discussion of these three alternatives can be found in real estate articles that study the problem of valuing commercial leases with renewal options (see e.g. Buetow and Albert 1998; Booth and Walsh 2001a and 2001b; Clapham 2003).

With a fixed strike price K, the corresponding call option pricing formula is almost identical with the well-known Black-Scholes formula for the price at a general time $t < T$ of a European call option on a non-dividend-paying stock. For our purposes the closed-form pricing formula for a rent option with predetermined strike price K is given by

$$F = e^{-rT} A(R_0 e^{\alpha T} N(d_1) - KN(d_2)) \tag{14}$$

where N denotes the cumulative standard normal distribution $N[0, 1]$ for the value d. Here d_1 and d_2 are given by

$$d_1 = \frac{\ln(R_0 / K) + (\alpha + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(R_0 / K) + (\alpha - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

As stated above, the expected value of R at time T under the risk neutral measure Q is

$$E^Q(R_T) = R_0 e^{\alpha T}$$

and from this result an eventually more intuitively appealing pricing formula can be obtained by some algebraic manipulation:[10]

$$F = e^{-rT} A[E^Q(R_T) N(d_1) - KN(d_2)] \tag{15}$$

where d_1 and d_2 are given by

$$d_1 = \frac{\ln(E^Q(R_T) / K) + (\sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$\mathrm{d}_2 = \frac{\ln(E^Q(R_T) / K) - (\sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

In the second example (with indexed strike price), the strike rent is uncertain when the option contract is written (at time t). Instead, the strike rent will depend on the evolution of the index X from the time the option contract is written until the time of maturity T. Assume that the risk neutral process of this index evolves according to

$$dX_t = \pi X_t dt + \sigma_x X_t dW_t^X \tag{16}$$

where (6) and (16) constitutes correlated geometric Brownian motions and where the correlation coefficient between the two driving Wiener processes is ρ. Fisher (1978) has proposed a closed-form solution for pricing call options when the strike price is uncertain. Buetow and Albert (1998) and Clapham (2003) have studied the problem of pricing lease options when the strike rent is determined by $K = R_0 X_T$. While Buetow and Albert use a

[10] This formula is used in Hull (2003), chapter 28, example 28.1, which in fact is a real option example considering a lease option.

numerical method to price lease options with indexed strike price, Clapham proposes an analytical pricing formula similar to the one studied by Fisher (1978) for pricing call option with uncertain strike prices. The value of an indexed option is (at any time up to maturity) given by (Clapham, 2003)

$$F_t = A(R_0 e^{(\alpha-r)(T-t)} N(d_1) - R_0 X_0 e^{(\pi-r)(T-t)} N(d_2)). \tag{17}$$

$$d_1 = \frac{\ln(R_0 / R_0 X_t) + (\alpha - \pi + \hat{\sigma}^2 / 2)(T-t)}{\hat{\sigma}^2 \sqrt{T-t}}$$

$$\mathrm{d}_2 = \frac{\ln(R_0 / R_0 X_t) + (\alpha - \pi - \hat{\sigma}^2 / 2)(T-t)}{\hat{\sigma}^2 \sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}$$

$$\hat{\sigma} = \sqrt{\sigma^2 + \sigma_X^2 - 2\rho\sigma\sigma_X}.$$

For a constant exercise rent (i.e. replace R_0X_t with a constant K), $\sigma_X = 0$ and $\pi = 0$, the pricing formula given by (17) reduces to the pricing formula for a constant strike rent given by (14) or (15).

In case 3 (strike rent is defined as a fraction of market price), the option will always be in-the-money, i.e. the *inequality* $R_T > K$ will always hold since $pR_T < R_T$ for all $0 < p < 1$. Buetow and Albert (1998) report that the value of the call option should be priced according to

$$F = e^{-rT}(1-p)AE[R_T] = (1-p)AR_0 e^{(\alpha-r)T+\sigma^2 T/2} \tag{18}$$

Clapham argues correctly that the formula above is incorrect since it erroneously contains a variance term in the exponent. Furthermore, Clapham points out that there is no information regarding under which probability measure the expectation is taken, implying that we do not know whether the drift μ is the objective (real-world) or the risk neutral drift. For our purposes, it is the risk neutral or Q-measure that should be the appropriate one. Thus according to Clapham the option should be priced according to the formula

$$F = e^{-rT}(1-p)AE^Q[R_T] = (1-p)AR_0 e^{(\alpha-r)T} \tag{19}$$

which indeed is the correct formula.[11]

[11] Contact the author for a derivation.

3 Rental option value with outside option

As discussed above in section 1.2, this paper aims at developing a pricing model for the proposed rental option when the so called outside option is included in the model by incorporating the moving threshold and total transaction cost parameters (section 3.2 below discusses these two parameters in more detail). For simplicity, both parameters are assumed to be exogenously given constants.

For tractability, the pricing formula will be developed in a two-period setting where the rent for each period is supposed to be paid in advance (i.e. the rent for the first period is paid at time $t = 0$, while the rent for the second period is paid at time T_2). I will also restrict the analysis to the case when the strike price K is supposed to be known at time $t = 0$.

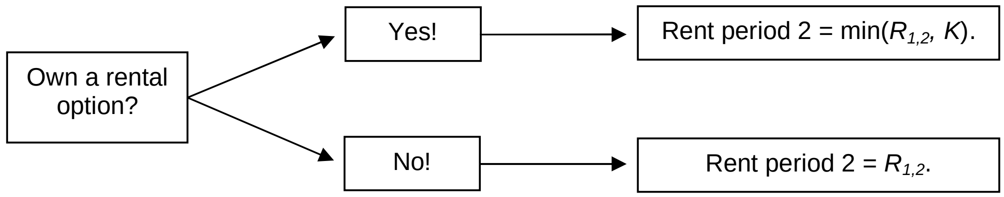
Though, for convenience, I will first show how a “standard” option pricing formula (formula 14 or 15 for fixed strike price K) can be derived for a two-period rental model (see section 3.1). Then, a second two-period model will be developed which comprises the outside option (see section 3.2). Below, the standard option price at origin (i.e. at time $t = 0$) is denoted $F(s)$, while $F(o)$ denotes the option price where the outside option is taken into consideration. As we will see below, given the same option contract (i.e. same contractual terms regarding the strike price and exercise date) $F(o)$ may either be larger or smaller than $F(s)$, mainly depending on the size of the costs that occur when the outside option is exercised.

3.1 Standard option pricing model revisited

We have seen above that the rental option can be considered to be an instrument that provides insurance (or hedge) against high future rents.[12] Furthermore, we have above implicitly considered a model with two rental period characterized as follows: at time $t = 0$, a tenant signs a new rental contract. The rent for the first rental period is the market rent level given by $R_{0,1}$. At time T_1, the rent will be reviewed. The reviewed rent for the second rental period (time T_1 to T_2) will be set to the market rent for the second period, $R_{1,2}$, if the tenant has not bought a rental option at time $t = 0$. However, if the tenants has bought a rental option, then the rent for the second period will either be set to the strike price K or to the market rent $R_{1,2}$, whichever is lower, i.e. the reviewed rent will equal $\min(R_{1,2}, K)$. In this standard model, the tenant is supposed to stay in the same apartment (at least) until time T_2 whether he or she has acquired an option or not. Figure 1 below shows possible outcomes for this model.

Figure 1. Possible outcomes for the standard two-period model in which a tenant is supposed to stay in the same apartment in both rental periods, no matter if the tenant owns a rental option or not.

[12] The option can also be regarded as an instrument for speculation. Indeed, a tenant may wish to bet that the rent will increase sharply until the next rent review. A speculating tenant has an opportunity to make a good deal, i.e. that the realized present value of the payoff from the option will exceed the option premium paid. In theory, this profit may be infinite. On the other hand, the speculator’s loss is limited to the amount paid for the option premium, based on the payoff structure given by $\max(R_T - K, 0)$.



Based on this two-period model, it can be shown (see below) that the expected value of the rental option will be given by the general risk neutral valuation formula (11) above. In particular, if the dynamics of the market rent follows a GBM given by equation (6) above, a "standard" closed-form analytical pricing formula for the rental option can be derived (see formula 14 and 15).

In order to derive an explicit option pricing formula based on this two-period model, consider now a tenant who chooses to not buy an option. That is, the tenant signs a new rental contract at time $t = 0$, and the tenant's intention is to stay in the apartment until time T_2. At time T_1, the rent will be reviewed. While $R_{0,1}$ is paid at time $t = 0$ and therefore known when the rental contract is written, $R_{1,2}$ is a random variable which is realized at time T_1. Thus, the expected present value of rental payments for this tenant is given by

$$R_{0,1} + e^{-rT_1} E^Q[R_{1,2}]. \tag{20}$$

A tenant who wants to hedge against the rent risk can buy a rental option at time $t = 0$, with strike price K and time of maturity T_1. This option gives the insured tenant the right to pay a rent for the second period that is the lower of K and $R_{1,2}$. In this case, the rent for the second period is given by the random variable $\min(R_{1,2}, K)$ and hence, the expected present value of total costs now becomes

$$F(s) + R_{0,1} + e^{-rT_1} E^Q[\min(R_{1,2}, K)]. \tag{21}$$

The maximum price a tenant is willing to pay for the option could be derived from indifference pricing. Thus by equating (20) with (21) and solving for $F(s)$, we get the following expression for the standard option price:

$$F\ (s) = e^{-rT_1} E^Q[R_{1,2} - \min(R_{1,2}, K)]. \tag{22}$$

Since $R_{1,2} - \min(R_{1,2}, K) = \max(R_{1,2} - K, 0)$, $F(s)$ can be expressed as the familiar risk neutral valuation formula

$$F\ (s) = e^{-rT_1} E^Q[\max(R_{1,2} - K, 0)]. \tag{23}$$

Note that formula (23) fully resembles the risk neutral valuation formula (11). Thus, assuming that the risk neutral process of R_t is given by the GBM

$$dR_t = \alpha R_t dt + \sigma R_t dV_t,$$

the standard option price $F(s)$ can be calculated using the standard option pricing formula (14) or (15) above.

3.2 Model for the option value with outside option

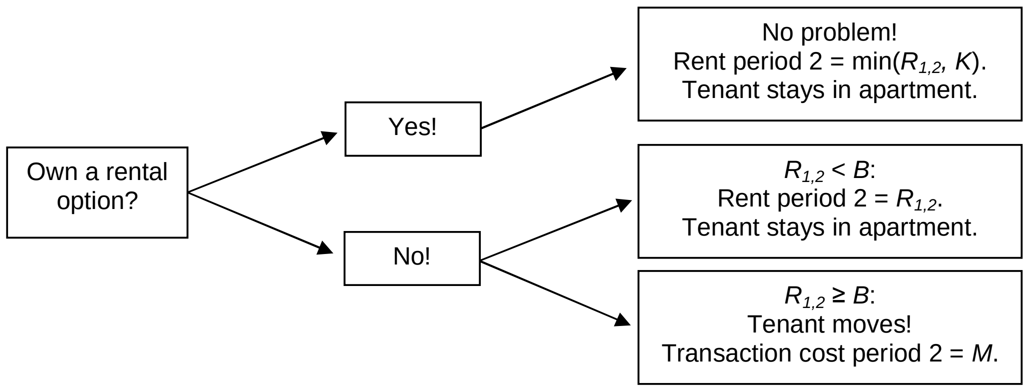
As mentioned above, the two-period model above does not take into account the possibility that a tenant may use his or her right to terminate the current rental agreement, what we in this paper call the outside option. To model the impact of the outside option, we may stipulate that there exist roughly two kinds of tenants: those that are liquidity constrained, and those that are not. Households that are not liquidity constrained are supposed to not face any risk of being forced to move due to a sharp increase in rents. Thus richer households that do not speculate only give up a possibility to make a good gain if they do not buy an option. But liquidity constrained and uninsured households do not only give up potential gains, these households also run the risk of having to move to a cheaper apartment if rents reach a certain level, henceforth called the *moving threshold.* Furthermore, if the rent level at time of rent review exceeds the moving threshold such that the uninsured tenants must move, the household will also face *transactions costs* due to the move. This means that liquidity constrained households may be extra eager to acquire a rental option, especially if the moving threshold is low and the tenant's transaction costs are high.

Hence, I will below develop a pricing formula for the rental option when the effect of the outside option is taken into account by introducing two new parameters; the moving threshold parameter denoted B, and the transaction costs parameter, denoted M. For simplicity, I assume that a tenant has a right to terminate the rental contract at the time of the rent review, T_1, i.e. when the tenant has knowledge about the size of the rent for the second rental period, $R_{1,2}$.

The definition of the moving threshold B is straightforward; If the market rent for the second period ends up below an uninsured and liquidity constrained tenant's moving threshold, i.e. if $R_{1,2} < B$, he or she will stay in the apartment in the second rental period for sure. On the other hand, if $R_{1,2} \geq B$ at the time of the rent review, this tenant will move to another housing unit due to affordability problems. Therefore the tenant will face a transaction cost of size M. The definition of the transaction cost parameter in this model is much more subtle and complicated than the definition of the moving threshold. In short, M represents all costs that a tenant that moves will have to pay for the second rental period. In other words, the size of M is the sum of the rent for the new apartment, direct or tangible transaction costs and indirect or intangible transaction costs (see separate discussion below). In the analysis below, both B and M are for tractability assumed to be exogenously given parameters, known at time $t = 0$.

Figure 2 below depicts the possible outcomes for this two-period model.

Figure 2. Possible outcomes for the two-period model where an uninsured will exercise his or her outside option if the market rent level exceeds the tenants moving threshold at the time of the rent review.



The components of the transaction cost parameter

It is a difficult task to define the transaction cost in such a way that its numerical value can be interpreted in a intuitive way in the model. Usually transaction cost refers to costs that can be related to a move, e.g. search costs, direct costs of moving, new social investments and a multiple of other factors (see e.g. Lind 1994). In order to simplify the model, M will here be defined as the sum of the quality adjusted cost/rent of the new housing unit plus the "ordinary" transaction costs. Furthermore, this total transaction cost M is supposed to be evenly distributed during the second rental period. This allows us to compare for instance the rent for the second period for the current apartment with the total transaction costs in a straightforward way in case the tenant moves.

In his analysis of default options embedded in percentage retail leases, Sing (2003) argues that relocation costs for a retail tenant may include tangible items like costs related to reinstating the existing store, fitting-up the new store, search cost, mover costs and commission fee, as well as intangible items like loss of goodwill associated with the existing store location and disruption of business. For the purpose of modeling the default option, Sing (2003) assumes that the relocation cost is a one-time lump sum fixed expense, incurring at the time the default option is exercised.

The same types of costs arise for a household that relocates. The tangible costs are roughly the same as for the retail lease. For the household the intangible items concern various types of utility losses that occur during the (initial) period after the move, e.g. because it takes time to find new friends and learn about the characteristics of the local neighborhood (best shops, best parks etc).

These transactions costs, especially the intangible costs, can be expected to differ much between different types of households. The cost can be very low for a young person with minimal stuff to move and a social network not related to the local neighborhood. For a family with children that goes to a local day-care center or a local school, and a strong local network, the cost can be much higher. Measuring some kind of average cost is therefore rather meaningless and all figures presented below should be seen as examples - and the prices for the option the price that a family with this transaction costs would be willing to pay.

The model

Now consider the pricing of a rental option when a tenant faces a moving threshold. As in the first model, the tenant signs a new rental contract at time $t = 0$, and the tenant's intention is to stay in the apartment until time T_2. At time T_1, the rent will be reviewed.

Again, the tenant chooses between two main strategies: buy or not buy an option with strike price $K < B$. If the tenant buys an option at time $t = 0$, he or she will for sure stay in the same apartment until time T_2. That is, if $R_{1,2}$ ends up above K, the insured tenant will exercise his or her option. But now, the situation becomes a little bit more complicated if the tenant chooses to not buy an option, compared with model one. If the market rent for the second period at time of the rent review, $R_{1,2}$ ends up below the moving threshold B, then the tenant will stay in the apartment until time T_2 for sure. But if $R_{1,2}$ ends up above the moving threshold, the tenant will move at time T_1, and as a consequence, the tenant will face a transaction cost of size M (which as explained above, also includes the cost for the new housing unit).

How will this affect the size of the option premium a liquidity constrained tenant may want to pay at time $t = 0$? If M is high, then the tenant may be willing to pay an extra option premium in addition to the standard option price $F(s)$. But if the tenant expects that $R_{1,2}$ ends up above his or her transaction cost M, the tenant may be willing to pay an amount that is below the standard option price $F(s)$. Thus, the option value $F(o)$ may either be greater or lower than $F(s)$, depending on the relationship between M and some expected value of $R_{1,2}$ (we will below derive this result).

Consider first the strategy when the tenant does not buy an option. Then the expected present value of costs for the two periods is given by

$$R_{0,1} + e^{-rT_1} E^Q[R_{1,2}\,\mathbf{1}(R_{1,2} < B) + M\,\mathbf{1}(R_{1,2} \geq B)] \tag{24}$$

where $\mathbf{1}(R_{1,2} < B)$ is the indicator function assigning 1 to the case $R_{1,2} < B$ and 0 otherwise. Similarly, $\mathbf{1}(R_{1,2} \geq B)$ is the indicator function assigning 1 to the case $R_{1,2} \geq B$ and 0 otherwise.[13] In other words, if the market rent for the second rental period, $R_{1,2}$, is below the moving threshold B at the time of the rent review, the tenant will stay in the apartment he or she currently lives in. But if $R_{1,2}$ exceeds B at the time of the rent review, the tenant will move, paying a transaction cost of size M (which also includes the cost for the new housing unit, see above).[14]

As mentioned above, let $F(o)$ denote the price a tenant who considers the level of a moving threshold in the pricing of the option is willing to pay. Again, assume that the tenant at time $t = 0$ buys a rental option with strike price K and maturity date T_1, which implies that the rent for the second period is given by the random variable $\min(R_{1,2}, K)$. A tenant who buys an option will face the expected present value of costs given by

$$F(o) + R_{0,1} + e^{-rT_1} E^Q[\min(R_{1,2}, K)]. \tag{25}$$

[13] More formally, the indicator function of a subset A of a set B is the function with domain B, whose value is 1 at each point in A and 0 at each point that is in B but not in A. 1_A is a random variable whose expected value is equal to the probability of A.

[14] Naturally, we must assume that M > B, otherwise the tenant will move at any time.

Equating (24) and (25) and solving for $F(o)$ yields

$$F(o) = e^{-rT_1} E^Q[R_{1,2}\mathbf{1}(R_{1,2} < B) + M\mathbf{1}(R_{1,2} \geq B) - \min(R_{1,2}, K)]. \tag{26}$$

Again, note that $\min(R_{1,2}, K) = R_{1,2} - \max(R_{1,2} - K, 0)$. Therefore (26) can be expressed as

$$F(o) = e^{-rT_1} E^Q[R_{1,2}\mathbf{1}(R_{1,2} < B) + M\mathbf{1}(R_{1,2} \geq B) - R_{1,2} + \max(R_{1,2} - K, 0)].$$

We can now observe that the last term on the right hand side is nothing else but the payoff function of the rental option at time T_1, and therefore its expected present value is given by standard option price $F(s)$. Now define the extra premium a liquidity constrained household may be willing to pay as

$$\pi = F(o) - F(s). \tag{27}$$

Notice also that $R_{1,2}\mathbf{1}(R_{1,2} < B) - R_{1,2} = -R_{1,2}\mathbf{1}(R_{1,2} \geq B)$. Hence we may write

$$\pi = e^{-rT_1} E^Q[(M - R_{1,2})\mathbf{1}(R_{1,2} \geq B)]. \tag{28}$$

By definition, the expectation of a random variable X restricted to the event A, i.e. $E[X\mathbf{1}_A]$, is given by $E[X\mathbf{1}_A] = E[X \mid A]P(A)$.[15] Applying this definition on (28), we may now state the following result.

Proposition 1:
Given the two-period model above, the extra option premium a household may be willing to pay for a rental option, when a known constant moving threshold B and a constant transaction cost M are take into consideration, is given by

$$\pi = F(o) - F(s) = e^{-rT_1}(M - E^Q[R_{1,2} \mid R_{1,2} \geq B])Q(R_{1,2} \geq B). \tag{29}$$

Because $e^{-rT} > 0$ and $Q(R_{1,2} \geq B) > 0$, we find that the sign of π is defined by the difference $M - E^Q[R_{1,2} \mid R_{1,2} \geq B)$. We also see that the total price of the rental option with strike price K, moving threshold B and transaction cost M, is simply

$$F(o) = F(s) + \pi. \tag{30}$$

There is a natural economic interpretation of the formula (29): the expected present value of the premium π equals the present value of the difference between the transaction cost of moving and the expected market rent for the second period, *given* that $R_{1,2}$ exceeds the moving threshold at time of renewal, multiplied with the probability that $R_{1,2} \geq B$.

This result tells us that the size of $F(o)$ is the result of two offsetting effects. Consider for instance an increase of the drift α. This will result in a higher expected value of $R_{1,2}$ (see formula (4) above). Then first, with a higher expected value of $R_{1,2}$, the expected payoff of the

[15] Note that if the expectation is taken under the risk neutral measure Q, P(A) is exchanged with Q(A).

standard option, $R_{1,2} - K$,[16] will increase, so $F(s)$ should increase as well. Second, a higher expected value of $R_{1,2}$, given M, decreases the value of the extra premium according to formula (29).

To illustrate the discussion above, consider following two cases. First, if the outcome at the time of the rent review is given by $M > R_{1,2} > B > K$, then the corresponding *total* payoff of owning a rental option is given by $(R_{1,2} - K) + (M - R_{1,2}) = M - K$. That is, an insured tenant will pay K for the second period, instead of moving and paying M. Because $M > R_{1,2}$, the total payoff in this case is larger than the payoff given by the standard payoff $(R_{1,2} - K)$, i.e. $(M - K) > (R_{1,2} - K)$. In this case, a tenant "would" have been willing to pay an extra premium π for the rental option.

On the other hand, the extra premium π will be negative if M is expected to end up below $R_{1,2}$, implying that $F(o) < F(s)$. Consider the outcome $R_{1,2} > M > B > K$. The total payoff = $(R_{1,2} - K) - (R_{1,2} - M) = (M - K)$, but now $M < R_{1,2}$. Hence the he total payoff in this case is less than the standard payoff $(R_{1,2} > K)$, i.e. $(M - K) < (R_{1,2} - K)$. In this case, a tenant "would" not have been willing to pay an extra premium π for the rental option.

The extra premium will be zero if $Q(R_{1,2} \geq B) = 0$, or if the expected difference $M - R_{1,2}$ will equal zero (given that $R_{1,2} \geq B$). We can rule out the case that $Q(R_{1,2} \geq B) = 0$, since this will only happen if the moving threshold tends to infinity, which for most households is very unlikely. Thus the premium will become zero only if we expect M to be equal to $R_{1,2}$. In this case the tenant will be indifferent between staying in the current apartment or moving to another housing unit. Therefore, the total payoff equals the payoff given by the "standard" option contract, i.e. $(R_{1,2} - K)$, implying that $F(o)$ and $F(s)$ will be of equal size.

Note that the rental option will not be exercised if the market rent for the second period ends up below the strike price K, implying that the option will be worthless. Also note that when $R_{1,2}$ is only slightly above K, a tenant's *net* profit (payoff from option minus option premium paid) may be negative (see numerical simulations below).

3.2 Explicit pricing formula for the extra option premium

The pricing formula (29) is general in the sense that it is valid under different assumptions of the stochastic rental process governing the evolution of the free market rent R_t. If we are only concerned about finding the size of the extra premium π, we do not need to assume that the risk neutral rental process is given by GBM. But in order to compare the levels of $F(o)$ and $F(s)$ with each other, it is necessary that $F(o)$ and $F(s)$ are derived using the same assumption about the stochastic rental process.

Since the pricing formula for $F(s)$ above is based on the assumption that R_t follows a stochastic differential equation given by the GBM (2) above, it is natural to continue to model the evolution of rents using this stochastic differential equation. Given this assumption for the rental process, we now obtain the explicit pricing formula for the extra premium assuming that M is a known constant (and including the annuity factor B):

[16] To simplify notation, I denote the payoff with $R_{1,2} - K$, instead of $\max(R_{1,2} - K, 0)$, thus assuming that $R_{1,2} > K$.

$$\pi = e^{-rT_1} A(MN(z_2) - R_0 e^{\alpha T} N(z_1)) \qquad (31)$$

where

$$z_1 = \frac{\ln(R_0 / B) + (\alpha + \sigma^2 / 2)T}{\sigma \sqrt{T}}$$

$$z_2 = \frac{\ln(R_0 / B) + (\alpha - \sigma^2 / 2)T}{\sigma \sqrt{T}} = z_1 - \sigma\sqrt{T}.$$

and where N is the cumulative distribution function for the N[0, 1] distribution.

At time $t = 0$, R_0 is equal to $R_{0,1}$ in the two-period model above. As usual, if rental payments of equal size are evenly distributed during the second rental period, it is appropriate to use the annuity factor A (see also section 4 below).

We can observe that the premium is equivalent to a long position in a binary (or digital) cash-or-nothing call with cash payoff *M* and a short position in a binary asset-or-nothing call. Hence, we can compute the value of the premium given by (31) as the difference between the value of a cash-or-nothing call and a asset-or-nothing call (see appendix C for an explicit derivation of the formula for the extra premium).

The total price of the rental option, *F*(*o*), is given by

$$F(o) = F(s) + \pi \qquad (32)$$

where *F*(s) and π are given by (15) and (31) respectively.

It might be interesting to see when the option premium π will be zero. That is, for which $M^* = M$, the standard option price *F*(*s*) will equal *F*(*o*). By setting (31) equal to zero and solving for *M*, we obtain that

$$M^* = R_t e^{\alpha T} \frac{N(z_1)}{N(z_2)}. \qquad (33)$$

For all M > *M**, *F*(*o*) will exceed *F*(*s*), and vice versa.[17]

3.2 Special case when *B* = *K*

From general option theory, it is well-known that the price of a European call option becomes more valuable as the strike price decreases, holding all other variables affecting the call option price fixed, and vice versa. Thus there exist a tradeoff between paying a low option premium

[17] It can be shown that ratio of the two distribution functions, $N(z_1)$ over $N(z_2)$, will always exceed one. This was pointed out to me by Fredrik Armerin.

now and enjoying low rents in the future. We cannot expect that a tenant will have a possibility to choose an arbitrary strike price that will fit his or her moving threshold exactly, but for the sake of the analysis, suppose now that the moving threshold and the strike price in fact coincides. This means that a tenant will pay the lowest possible premium for obtaining a rental option with the highest possible strike price, given the size of the moving threshold *B*. If the strike price exceeds the moving threshold, the tenant will naturally avoid buying an option.

Consider again the strategy when a tenant chooses to not buy an option. Then the expected present value of costs is given by (24) above, i.e.

$$R_{0,1} + e^{-rT_1} E^Q[R_{1,2}\,\mathbf{1}(R_{1,2} < B) + M\,\mathbf{1}(R_{1,2} \geq B)].$$

Now, observe that the expression within the expectation can be rewritten as

$$\min(R_{1,2}, B) + (M - B)\,\mathbf{1}(R_{1,2} \geq B). \tag{34}$$

implying that the expected present value of costs can be expressed as

$$R_{0,1} + e^{-rT_1}\left\{E^Q[\min(R_{1,2}, B)] + E^Q[(M - B)\,\mathbf{1}(R_{1,2} \geq B)]\right\}. \tag{35}$$

A tenant who buys an option at time $t = 0$ will face the expected present value of costs given by formula (25) above. But since we now consider a strike price *K* that equals the moving threshold *B*, (25) becomes

$$F(o) + R_{0,1} + e^{-rT_1} E^Q[\min(R_{1,2}, B)]. \tag{36}$$

Therefore, by equating (35) with (36) we obtain that the price of the option, $F(o)$ when $B = K$ is determined by the formula

$$F(o) = e^{-rT_1} E^Q[(M - B)\,\mathbf{1}(R_{1,2} \geq B)] = e^{-rT_1}(M - B) Q(R_{1,2} \geq B) \tag{37}$$

where both *M* and *B* can be left outside the expectation since they are assumed to be known at time $t = 0$. Recall that the expected value of the indicator function above equals the risk neutral probability that that the market rent for the second period ends up above the moving threshold, $Q(R_{1,2} \geq B)$. As mentioned above, in order to evaluate this probability, we can assume that the evolution of the market rent follows another process than the GBM given by (2) above. But in order to compare $F(o)$ with the pricing formula for $F(s)$, given by (15) above, we must assume that rents follow GBM specified by (1). In this case, $Q(R_{1,2} \geq B)$ is given by $N(z_2)$ (see appendix C). Thus, when $K = B$, and including the annuity factor *A*, the closed-form pricing formula for $F(o)$ when $K = B$ is given by[18]

$$F(o) = e^{-rT_1} A(M - B) N(z_2) \tag{38}$$

[18] Note that we can directly obtain (38) by summing the formula for $F(s)$, given by (15), with the formula for π_t given by (29). To get the desired result, just replace *K* with *B* in (15) (and set the factors *S* and *A* to one).

where z_2 is (as usual) given by

$$z_2 = \frac{\ln(R_0 / B) + (\alpha - \sigma^2 / 2)T}{\sigma\sqrt{T}}.$$

Of course, (38) will yield the same result as (32) when $K = B$.

Above, we argued that the transaction cost M couldn't be lower than the moving threshold B, otherwise the tenant will always move. In formula (38), we see directly that M must be greater than B in order to obtain a positive option value, since $N(z_2)$ cannot be less than zero. Indeed, when $M = B$, then the option value will have an intrinsic value of zero.

4 Numerical results and sensitivity analysis

This chapter looks at the input parameters (factors) that affect the value of the rental option according to the pricing models discussed above. These parameters are the current rent or rent for the first period $R_{0,1}$, strike rent K, time to maturity/rent review T_1, length of lease period L, risk-free interest rate r, Annuity factor A, risk neutral drift rate α, market rental volatility σ, moving threshold B, and finally transaction cost M.

A short discussion of each of the parameters that enters the pricing formulas for $F(s)$ and $F(o)$ is presented in section 4.1. In section 4.2, the results the simulation of the standard option price $F(s)$ are presented. Section 4.3 presents how the option price $F(o)$ changes with different assumptions of B and M

All numerical figures and calculations are in real terms. All monetary figures are in SEK per square metre and year.

4.1 Assumption of the parameters affecting option prices

We again consider a two-period model as we did in chapter three; at time $t = 0$, a tenant both starts to rent a housing unit, and buys a rental option to obtain a hedge against high rental payments for the second lease period. The rent levels for the two periods are given by $R_{0,1}$ and $\min(R_{1,2}, K)$ respectively. The rent will be reviewed at time T_1, L years from $t = 0$.

Time to maturity/rent review and length of lease period

The maturity date of the option, T_1, will naturally coincide with the date for the rent review. The value of the option is supposed to increase as L increases. One obvious reason is that as L increases, so does also the insured second period, since the length of each of the two lease periods are assumed to be equal. Furthermore, the probability for the future rent level to exceed the strike rent is an increasing function of the time to maturity. Here, we consider the lease length of both periods, L, to be five years. Since the tenant is supposed to buy the option at time $t = 0$, T_1 will be equal to L.

A large expected positive yearly payoff ($R_{1,2} - K$) during the second lease period may result in a high option value because the effect of the option will last for $L = 5$ years. Therefore, it may be necessary to spread out the cost of the option. A natural alternative is to spread out the cost of the option over the first lease period by including the part-payments in the ordinary rental payments. For instance, when the lease length is five years, the cost of the option will be divided into five payments (assuming rents are paid annually in advance as we do in this paper). Since the calculated option prices, $F(s)$ and $F(o)$, are present values, the landlord should be compensated for the time value of money. To achieve this, the sizes of the future yearly option payments during the first period are simply compounded using the real interest rate r. The average option payments per year are denoted $F_{avg}(s)$ and $F_{avg}(o)$ respectively.

The following simple example clarifies the discussion above. A tenant signs a new rental contract today (at time $t = 0$) for a 70 square metre apartment. The rent that amounts to $R_{0,1} = 1000$ SEK per square metre and year for the first five years, shall be paid annually in advance. The rent will be reviewed after five years for another five-year period. The tenant also buys a rental option with a maturity date that naturally equals the date of the rent review (i.e. five years from $t = 0$). Assume furthermore that the price of the rental option $F(s)$ is set to 140 SEK per square meter. This implies that the total cost of the option amounts to 9800 SEK (140x70). Consequently, the total size of the first part-payment will amount to 1960 SEK (9800/5), or 28 SEK per square metre (140/5). This part-payment will be paid with the first rental payment in the beginning of year 1. The second part-payment will come about with the second rental payment and so on. To compensate the landlord for the time value of money, the size of the second part-payment will equal $28(1 + r)$, or $28\text{EXP}(r)$ with continuous compounding, and so on, where r is the interest rate. The average size of the part-payments, $F_{avg}(s)$ or $F_{avg}(o)$, is simply the arithmetic mean of the five part-payments. For instance, with $r = 3\%$, $F_{avg}(s)$ will equal 29,8 SEK per square metre, assuming continuous compounding. Consequently, the ratio of the average size of the part-payments over the annual rental payment, $F_{avg}(s) / R_{0,1}$, is in this example 3.0%.

Current rent

The value of the option increases as the current rent $R_{0,1}$ increases (given that the exercise price K is held constant). The reason is simply because the higher the current rent is, the higher is the probability that the rental option will be exercised, i.e. that $R_{1,2} > K$. Furthermore, the probability that the payoff $\max(R_{1,2} - K, 0)$ will be larger will also increase with higher $R_{0,1}$ (given that the exercise price is held constant).

Here, the rent for the first period, $R_{0,1}$, is set to 1000 SEK per square meter and year, assumed to be paid annually in advance. It is assumed that the rent is adjusted annually according to the change in the consumer price index (CPI) until next rent review. I assume that rents are 100% CPI adjusted. Since all quantities are expressed in real terms, this implies that the rent during each of the two lease periods will stay constant in real terms. This also implies that we can use an annuity factor A to discount the eventually yearly payoffs of the option during the second lease period to time T_1 (see below).

Strike price

At time $t = 0$, a tenant has the opportunity to buy a rental option with strike price K for the second rental period. Using same reasoning as above, we can establish that the option becomes less valuable as the strike prices increases.

Naturally, there must be some rules regarding strike price alternatives. Firstly, we cannot expect a system where tenants have a possibility to choose any level of the strike rent. Secondly, landlords must offer options with strike prices that can be considered to be within a reasonable range. Thus, there is a need for more or less standardized options. One alternative is to establish that landlords must offer option alternatives with at least three different strike prices. For instance, the three different strike rents offered can be calculated as the current rent grossed up with 2%, 2.5% or 3% annual real growth.

Here, following three different strike rent will be used: 1100, 1150, and 1200. Table 1 below presents the different strike price alternatives and the corresponding yearly real growth rates.

Table 1. Relationship between strike price K and annual real growth rate in rents, when current rent is 1000.

	Strike prices and corresponding annual real growth rates in rents.
K	Annual real growth rate
1100	1.9%
1150	2.8%
1200	3.6%

Risk-free interest rate

The effect of an increase in the real interest rate on the option value may be difficult to infer. An increase in the risk-free real interest rate affects the option value negatively through the discount factor in the pricing formula, all other factors remained fixed. But from an macroeconomic point of view, an increase in the real interest rate may reflect some kind of boom or expected boom, and thus, higher expected growth rate in rents, which will have a positive effect on the option price. Here, the real risk-free interest rate is assumed to be 3% throughout all numerical calculations below.

Annuity factor

The annuity factor A is used to calculate the value of the yearly payoffs $R_{1,2} - K$ that occurs during the second lease period to time T_1. Since it is supposed that rents are paid annually in advance, the annuity factor is calculated as

$$A = \sum_{i=0}^{L-1} e^{-ri}.$$

Risk neutral drift rate

A higher expected actual real world growth rate μ, or drift, in rents will obviously increase the expected rental growth. When risk neutral valuation is applied, what matters is the risk neutral drift α. As mentioned in section 2 above, the expected risk neutral growth rate in rents can be calculated by reducing the expected actual real growth rate to the risk neutral expected growth rate by incorporating a risk premium of size $\lambda\sigma$, where λ is the so called market price of risk.

It is outside the scope of this paper to calculate different values of α based on the continuous time CAPM.[19] Instead, following Clapham (2004), I just suppose that the expected risk neutral drift will range from some more or less arbitrary minimum value to a maximum value, though not greater than the real interest rate r. Nevertheless, the higher expected the risk neutral drift, the greater is the expected option value. Here, given that the real interest rate is 3%, I let α range from 0% to 2%.

Another expression of the risk neutral drift is the (real) risk-free interest rate minus the yield; $r - d$, where the yield d can be interpreted as the initial yield of for instance a multi-family

[19] See Hendershott and Ward (2003) for some examples on how risk neutral drift can be calculated using CAPM.

property. Note that the risk neutral drift may be negative as well (see Clapham 2004) and this might occur if the initial yield d exceeds r.

Market rental volatility

The more volatile market rents, denoted by σ, the higher the chance for the option to have a high value. Note that an owner of a rental option is not interested in the probability of the rent to decrease as much as he or she is interested in the probability of the rent level to rise, since the downside risk is limited to the option premium paid, while the potential gains from higher rents are unlimited (at least in theory).

Since I do not have access to any data on volatility of residential rents (given a non-regulated environment), I use estimates of volatility for commercial rents and of IPD return figures as a proxy for volatility σ. For instance, based on a data set consisting of nominal office rents from 1981 to 2003 at prime locations in the three major Swedish metropolises, Badur and Edwardsson (2004) find that the annual rental volatility is about 11.7%. Even if it is difficult to establish whether the volatility for residential rents in Sweden will be below or above this figure, it is likely that residential rents will be less volatile than market rents for offices at prime locations.

Furthermore, based on IPD annual rental data set over the period 1976 to 1997, Booth and Walsh (2001b) estimate the volatility to be just over 11%. Here, I will use three volatility values: 7.5%, 10% and 15%.

Transaction cost and moving threshold

The transaction cost and moving threshold parameters enter the pricing formula for the rental option, $F(o)$, which considers the so called "outside option". According to the two-period model developed in section 3.2 above, an uninsured tenant will at time T_1 either choose to stay in the current apartment and pay a rent of size $R_{1,2}$, or choose to move to another apartment, depending on whether $R_{1,2}$ ends up below the moving threshold or not. In case the tenant chooses to move, a transaction cost of size M will arise.

As mentioned above, the transaction cost parameter is supposed to include both the rent for the alternative housing unit (with lease period L) as well as other direct and indirect costs related to a move (see discussion in section 3.2 above). Here, three different levels of the transaction cost will be applied: 1200, 1300, and 1400 (SEK per square meter and year). As usual, the costs are supposed to be paid annually in advance during the second lease period.[20] Naturally, the option price $F(o)$ increases as M increases.

It is as argued above that there might exist large differences in transaction costs between different households group. In order to get some feeling for what are interesting magnitudes it might therefore be more interesting to focus on what a change in M implies for a change in direct and indirect transactions cost. If we are looking at a household that rents a 70 square metre apartment, an increase in M with 100 SEK (for instance from 1200 to 1300) means over a 5-year period roughly an increase in transactions cost of 100x70x5= 35 000 SEK.

[20] This assumption simplifies the calculations. Naturally, transaction costs, especially direct moving costs arise at the time of a move. Other components of the transaction cost may decline over the years.

To simplify the presentation below, I will concentrate on cases for which the moving threshold B equals the strike rent (i.e. $B = K$, see table 1 above). Recall that this will result in the lowest possible option premium. Table 2 below summarizes the discussion above.

Table 2. Key input parameter values.

Key input parameters	Parameter values
Lease period, L	5 years
Time to maturity, T_1	5 years
Initial/current rent, $R_{0,1}$	1000 (SEK per square meter and year)
Strike rent, K	1100, 1150 and 1200 (SEK per square meter and year)
Interest rate, r	3%
Drift rate (risk neutral), α	0%, 1% and 2%
Volatility, σ	7.5%, 10% and 15%
Moving threshold, B	$B = K$ (SEK per square meter and year)
Transaction cost, M	1200, 1300 and 1400 (SEK per square meter and year).

4.2 Simulation of the standard option price, *F*(*s*).

Table 3 below presents our numerical results of the simulation of the standard option price $F(s)$, given by formula (14) or (15) above, for different drift and volatility environments and various strike price assumptions. This table shows the average yearly payments of the standard option premium in relation to the size of the yearly rental payments during the first rental period, given by the ratio $F_{avg}(s) / R_{0,1}$.

Table 3. Average yearly part-payments of option premium in relation to yearly rental payments during the first rental period; $F_{avg}(s)/R_{0,1}$. The length of the two lease periods equals time to maturity; $L = T_1 = 5$ years. The rent for the first lease period is $R_{0,1} = 1000$. Strike rent is K. Real risk-free interest is $r = 3\%$. Spot rent follows a geometric Brownian motion with constant risk neutral drift α and volatility σ.

K	$F_{avg}(s) / R_{0,1}$ (Average per year)		
	σ=7.5%	σ=10%	σ=15%
Panel A: $\alpha = 0\%$			
1100	2.7%	4.5%	8.2%
1150	1.7%	3.3%	6.9%
1200	1.1%	2.5%	5.8%
Panel B: $\alpha = 1\%$			
1100	4.3%	6.3%	10.4%
1150	3.0%	4.9%	8.8%
1200	2.0%	3.7%	7.5%
Panel C: $\alpha = 2\%$			
1100	6.6%	8,7%	12.9%
1150	4.7%	6.9%	11.1%
1200	3.3%	5.4%	9.5%

As expected, we can see that the ratio $F_{avg}(s) / R_{0,1}$ increases with higher values of the expected risk neutral drift α and higher values of the volatility σ, but decreases with higher strike prices K. Note that the share of the average yearly option (insurance) premiums in relation to yearly rental payments seem to be of reasonable size (say less than 5% or 6% of rental payments), as long as α does not exceed 1% and σ does not exceed 10%. For larger values of α or σ, the ratio $F_{avg}(s)/R_{0,1}$ probably becomes too large to be an attractive alternative for a majority of the tenants.

4.3 Simulation of option price including the outside option, *F*(*o*)

The analysis in chapter three shows that there may be some theoretical justification for the idea that the existence of an outside option may affect the price of a rental option. In order to perform simulations of the rental option price $F(o)$, that is, the option price that includes the effect of the outside option, formula (32) will be applied. This pricing formula contains the moving threshold parameter B, and the transaction cost parameter M.

In order to keep the analysis as simple as possible, I consider the special case when the moving threshold equals the strike rent ($K = B$). As explained above, this implies that the option price will be lowest possible with respect to K (recall that a tenant will not sign an option contract if $K > B$).

Base results

Table 4 below presents the numerical results for a base case scenario ($K = B = 1150$; $\alpha = 1\%$) under different volatility assumptions and where the transaction cost M varies from 1200 to 1400. For comparison, I also present average option expenses based on the calculations of $F(s)$ in table 3 above (within parenthesis). The size of M^*, i.e. the size of the transaction cost that will make the standard option price $F(s)$ to equal $F(o)$, is also presented (see formula 33 above). Appendix D presents sensitivity analysis for other strike price alternatives and other drift assumptions ($\alpha = 0\%$ and $\alpha = 2\%$).

Table 4. Average yearly part-payments of option premium in relation to yearly rental payments during the first rental period; $F_{avg}(o) / R_{0,1}$ and $F_{avg}(s)/R_{0,1}$ respectively (latter within parenthesis). The length of the two lease periods equals time to maturity; $L = T_1 = 5$ years. The rent for the first lease period is $R_{0,1} = 1000$. Strike rent $K = B = 1150$. Real risk-free interest is $r = 3\%$. Spot rent follows a geometric Brownian motion with risk neutral drift $\alpha = 1\%$ and volatility σ. When $M = M^*$, $F(s) = F(o)$, i.e. the premium $\pi_t = 0$.

$K = B$	M	$F_{avg}(o) / R_{0,1}$ (Average per year) Within parenthesis: $F_{avg}(s)/R_{0,1}$ from table 2 above		
$\alpha = 1\%$		σ=7.5%	σ=10%	σ=15%
		(3.0%)	(4.9%)	(8.8%)
1150	1200	1.2%	1.3%	1.4%
1150	1300	3.5%	3.9%	4.3%
1150	1400	5.8%	6.6%	7.1%
		$M^* = 1280$	$M^* = 1340$	$M^* = 1460$

The numerical results show that the as M increases, $F(o)$ also increases, which is quite intuitive. What may be less clear-cut is the relationship between $F(s)$ and $F(o)$. For instance, consider the case when the volatility is 7.5%. Then as already shown above in table 3, the ratio $F_{avg}(s) / R_{0,1}$ amounts to 3.0%. Furthermore, when a household's transaction cost amounts to $M^* = 1280$, then $F(s)$ and $F(o)$ will be equal (i.e. $F_{avg}(s) / R_{0,1} = F_{avg}(o) / R_{0,1}$)). For all other values of M, the extra premium $\pi = F(o) - F(s)$ will either be positive (when $M > 1280$), or negative (when $M < 1280$). Thus the implementation of the outside option by introducing the moving threshold parameter (B) and the transaction cost parameter (M) in the option pricing model (compare figure 1 and 2) reveals rather interesting price effects; given that the moving threshold equals the strike price ($B = K$), the option price determined the "standard way", $F(s)$, may either be lower or higher than the option price that takes into account the outside option $F(o)$, depending on the size of a household's transaction cost. In other words, the numerical results suggest that it may be important for a household to take into consideration the size of the household's moving threshold and transaction cost in order to form an opinion of what a reasonable option price might be. In particular, if rental option prices are determined the "standard way" by the insurer (i.e. the landlord), then the standard option price $F(s)$ may either be lower or higher than a household's willingness to pay for the option $F(o)$, depending on the household's estimation of the size of its moving threshold and transaction cost.

It is not likely that a household will have the opportunity to choose a strike rent K that will perfectly fit the household's moving threshold B. Instead it is more likely that there will be some standard alternatives offered. For instance, consider the case in which a household's moving threshold amounts to 1150, but that only two strike price alternatives are available: $K = 1100$ and $K = 1200$. Since the higher strike price alternative is not an interesting alterative (a household will not buy an insurance where $K > B$), the only available option alternative is the one with strike price 1100. This means that the option price will be higher, holding all other factors constant, since the option price increases with lower strike prices. As mentioned above, the minimum option price will be obtained when $K = B$.

Table 5 below differs from table 4 above in only one way: the strike price is now 1100 SEK per square foot and year and not 1150. First, as expected when the strike price becomes lower, we can see that the standard option price increases (as usual presented as the ratio $F_{avg}(s)/R_{0,1}$; see values within parenthesis). For instance, while the price ratio $F_{avg}(s)/R_{0,1}$ amounts to 3.0% when $K = 1150$ and $\sigma = 7.5\%$, the price ratio is now 4.3% due to a lower strike price. Consequently, $F(o)$ and therefore each value of the price ratio $F_{avg}(o)/R_{0,1}$ in table 5 is higher than each of the corresponding values in table 4. To conclude, we may say that it might be important for a landlord to offer a minimum number of different standard strike price alternatives with, say at most 100 SEK between each alternative.

Table 5. Average yearly part-payments of option premium in relation to yearly rental payments during the first rental period; $F_{avg}(o)/R_{0,1}$ and $F_{avg}(s)/R_{0,1}$ respectively (latter within parenthesis). The length of the two lease periods equals time to maturity; $L = T_1 = 5$ years. The rent for the first lease period is $R_{0,1} = 1000$. Strike rent $K = 1100$. Moving threshold $B = 1150$. Real risk-free interest is $r = 3\%$. Spot rent follows a geometric Brownian motion with risk neutral drift $\alpha = 1\%$ and volatility σ.

K (B)	M	$F_{avg}(o)/R_{0,1}$ (Average per year) Within parenthesis: $F_{avg}(s)/R_{0,1}$ from table 2 above		
$\alpha = 1\%$		σ=7.5%	σ=10%	σ=15%
		(4.3%)	(6.3%)	(10.4%)
1100 (1150)	1200	2.5%	2.8%	3.0%
1100 (1150)	1300	4.8%	5.4%	5.8%
1100 (1150)	1400	7.1%	8.0%	8.7%

5 Conclusion and policy and research implications

While there exist a number of studies on the valuation of commercial lease options, only a few articles have discussed residential rental options, mainly because there exist no real world applications with option contracts on the residential sector. The few articles written on residential options have not focused on valuation problems, but on how different types of voluntary lease agreements with option like features can be used as attractive alternatives to traditional rent regulation policies. Indeed, the rental option policy discussed in this paper is supposed to be a "market based" risk mitigating instrument that gives a protection similar to rent regulation, in a sense that it provides sitting and insured tenants with protection against sharp increases in market rent.

One of the most interesting and important questions to study when discussing new rent insurance instruments like the rent option studied here, is whether the insurance can be sold to households at a reasonable price. Based on financial derivatives methods, which has been commonly applied in the commercial lease option valuation literature the last ten years, this paper shows that it is possible to obtain prices on rental options that seem to reasonable.

This paper proposes a formula for pricing a residential option with respect to a tenant's so called outside option. We believe that the paper provides a potentially useful way of conceptualizing the way households might actually think at the time of deciding to buy an option. As could be expected, the proposed option valuation model predicts that the value of an option increases with higher transaction cost of moving, and decreases with higher moving threshold. These findings imply households to reflect over what could be a reasonable level of its moving threshold and transaction cost of moving. With a low moving threshold (i.e. a liquidity constrained household) and/or a high transaction cost of moving, the rental option may have a high value for this household. This value might therefore be higher than the option value calculated the standard way. Naturally, the opposite situation occurs for a household with a high moving threshold and/or a low transaction cost of moving.

If rent regulation is replaced with a system where it is mandatory for the landlord to offer a rental option, and where a ceiling of say 4% is introduced for the price for the option, the calculation above can also be used to find "reasonable" terms for the option. Using table 4 it could be argued that such an option would protect against real rent increases higher than roughly 3% per year. A lower strike rent would mean a subsidy from the landlord to the tenant, a subsidy that might be motivated taking into account the advantages for the landlord of a deregulated market.

Even if there exist no real world application of this kind of rental option, there are still a number of research questions that could be studied for preparatory purposes. Some examples concern the valuation model. For instance, the underlying rental process will probably be based on residential rent indices, mainly characterized by location, standard and size. One fundamental question that arises is whether the option price will depend on previous trends in rents or not. With mean reverting or trend reverting rents, it might be more appropriate to develop option pricing models based on some version of the Ornstein-Uhlenbeck (O-U) process instead of assuming geometric Brownian motion. Indeed, in the commercial lease option literature, O-U processes have been applied. Furthermore, estimation of important parameters like the risk neutral rental drift, rental volatility, households' moving thresholds and transaction costs are important. It is also interesting to conduct theoretical microeconomic studies in the fields of incentive and insurance economics (e.g. asymmetric information with adverse selection and moral hazard issues). Finally, this paper has discussed the valuation of single review option contracts. Going forward, further examination of valuing multiple rent review contracts, as well as valuing rental income streams (i.e. property valuation) with several (as well as single) review dates would be warranted.

References

Ambrose, B., P. Hendershott and M. Klosek (2002). Pricing upward-only adjusting leases. *Journal of Real Estate Finance and Economics* 25(1): 33-49.

Arnott, R. (1995). Time for revisionism on rent control. *Journal of Economic Perspectives* 9(1): 99-20.

Atterhög, M. and H-S Song. (2003). Policies to increase access to home ownership for low-income households. Licentiate paper.

Badur, S. and C. Edwardsson. (2004). *Hyresprognoser – en statistisk analys med makroekonomiska variabler*. Master of Scienc Thesis. Dept. Of Infrastructure, Royal Institute of Technology, Stockholm, Sweden.

Baum, A., N. Crosby and S. Murdoch (1998). The contribution of upwards-only rent reviews to the capital market value of commercial property in the UK. *Journal of Property Research* 15(2): 105-20.

Björk, T. (2004). *Arbitrage theory in continuous time*. Oxford.

Black, F. and M. Scholes (1973). The pricing of options and corporate liabilities. *Journal of Political Economy* 81: 617-59.

Booth, P.M. and Walsh, D.E. (2001a). The application of financial theory to the valuation of upward only rent reviews. *Journal of Property Research* 18(1): 69–83.

Booth, P.M. and Walsh, D.E. (2001b). An option pricing approach to valuing upward only rent review with multiple reviews. *Insurance: Mathematics and Economics* 28: 151–71.

Buetow G. and J. Albert (1998). The Pricing of Embedded Options in Real Estate Lease Contracts. *Journal of Real Estate Research* 15(3): 253–65.

Clapham, E. (2003). A note on embedded lease option. *Journal of Real Estate Research* 25(3): 347-60.

Clapham, E. (2004). Leases with upward-only characteristics. *Journal of Property Research* 21(1): 31- 49.

Clapham, E. and Å. Gunnelin (2003). Rental Expectations and the Term Structure of Lease Rates. *Real estate Economics* 31(4): 647-70.

Ellingsen, T. and P. Engund (2003). Rent regulation: An introduction. *Swedish Economic Policy Review*. 10: 3-9.

Fisher, S. (1978). Call option pricing when the exercise price is uncertain, and the valuation of indexed bonds. *The Journal of Finance* 33(1): 169-76.

Geltner and Miller (2001). *Commercial Real Estate Analysis and Investments.* South-Western Publishing.

Grenadier S. (1995). Valuing Lease Contracts: A Real-Options Approach, *Journal of FinancialEconomics* 38: 297–331.
——., Leasing and Credit Risk (1996). *Journal of Financial Economics* 42: 333–64.

Hendershott, P. (2002). A comparison of upward-only and turnover leases. *Journal of Property Investment and Finance* 20(6): 513-24.

Hendershott, P. and C. Ward (2003). Valuing and pricing retail leases with renewal and overage options. *Journal of Real Estate Finance and Economics* 26(2/3): 223-40.

Hull, J.C. (2003). *Options, futures and other derivatives.* Prentice Hall.

Lind, H. (1994). *Hur skulle en fri marknad för hyreslägenheter fungera?* Division of Building and Real Estate Economics. Stockholm, KTH.

Lind, H. (1999a). *Risk allocation on the housing market.* Division of Building and Real Estate Economics. Stockholm, KTH.

Lind, H. (1999b). *Mer sofistikerade hyreskontrakt som ett alternativ till hyresreglering.* Preliminär version. Division of Building and Real Estate Economics. Stockholm, KTH.

Lind, H. (2000). Bostadshyrorna och rimligheten - om behovet av en ny hyresreglering. Stockholm: Agora.

Lind, H. (2001). Rent regulation: A conceptual and comparative analysis. *European Journal of Housing Policy* 1: 41-57.

Merton, R. (1973). Theory of rational option pricing. *The Bell Journal of Economics and Management Science* 4(1): 141-83.

Nielsen, L. T. (1999). *Pricing and hedging of derivative securities.* Oxford.

Sing, T.F. (2003). *Valuing Default Risks In Percentage Leases In An Option Pricing Framework*. Presented at the 10th European Real Estate Society (ERES) conference, 10-13 June 2003, Helsinki, Finland.

Song, H-S. (2004). *Mortgage and Home equity insurances for home owners and rental insurance for tenants.* Licentiate paper. Division of Building and Real Estate Economics. Stockholm, KTH.

Ward, C. and N. French (1997). The valuation of upwards-only rent reviews; an option pricing model. J*ournal of Property Valuation and Investment* 15(2): 171-82.

Appendix A

Arnott (1995) distinguishes between first and second generation rent control. Lind (2001) argues that this distinction is too crude to be useful. Instead, Lind (2001) identifies five types of rent regulation (A-E).While the two first types (A-B) covers sitting tenants, the three other types (C-E) cover all tenants, both new tenants and sitting tenants. A very short summary of each of the five types of rent regulation is presented below. For a full discussion, see Lind (2001).

Type A is called “weak transaction cost-related rent regulation” and protects a sitting tenant against rents higher than the market rent.

Type B is called “strong transaction cost-related rent regulation” and protects a sitting tenant against certain types of increases in market rents.

Type C is called “monopoly-related rent regulation” and protects all tenants against rents higher than the market rent.

Type D is called “overshooting-related rent control” and aims at smoothing changes in rents.

Type E is called “segregation-related rent control” and aims at keeping the rents for all tenants below the market rent level in certain areas.

Appendix B

This appendix shows that a 90% confidence interval for R_T is given by (5) in section 2, i.e. by

$$\left[R_0 e^{(\mu-\sigma^2/2)T-1.64\sigma\sqrt{T}}, R_0 e^{(\mu-\sigma^2/2)T-164\sigma\sqrt{T}} \right]. \qquad \text{(B.1)}$$

The change in ln*R* between time $t = 0$ and *T*, is normally distributed so that

$$\ln R_T - \ln R_0 = \ln \frac{R_T}{R_0} \in N[m, s]$$

where

$$m = \left(\alpha - \frac{1}{2}\sigma^2 \right) T \text{ and } s = \sigma\sqrt{T}.$$

Therefore we have that lnR_T is normally distributed so that

$$\ln R_T \in N\left[\ln R_0 + \left(\alpha - \frac{1}{2}\sigma^2 \right) T, \sigma\sqrt{T} \right]. \qquad \text{(B.2)}$$

From (A.2) we now obtain that a 90% confidence interval for lnR_T is given by the interval

$$\left[\ln R_0 + \left(\alpha - \frac{1}{2}\sigma^2 \right) T - 1.64\sigma\sqrt{T}, \ln R \ + \left(\alpha - \frac{1}{2}\sigma^2 \right) T + 1{,}64\sigma\sqrt{T} \right].$$

That is, we have that a 90% confidence interval for R_T is

$$\left[e^{\ln R_t + \left(\alpha - \frac{1}{2}\sigma^2 \right)(T-t) - 1.64\sigma\sqrt{T}}, e^{\ln R_t + \left(\alpha - \frac{1}{2}\sigma^2 \right)(T-t) + 1.64\sigma\sqrt{T}} \right]. \qquad \text{(B.3)}$$

Simplifying (B.3) gives (B.1).

Note that an arbitrary confidence coefficient can be applied. For instance, if we want a 95% confidence interval for R_T, then just replace 1.64 with 1.96 above.

Appendix C

We shall show how the pricing formula for the option premium π given by (31) in the main text

$$\pi = e^{-rT_1}(MN(z_2) - R_0 e^{\alpha T} N(z_1)).$$

where

$$z_1 = \frac{\ln(R_0 / B) + (\alpha + \sigma^2 / 2)T)}{\sigma\sqrt{T}}$$

$$z_2 = \frac{\ln(R_0 / B) + (\alpha - \sigma^2 / 2)T}{\sigma\sqrt{T}} = z_1 - \sigma\sqrt{T}.$$

can be derived.

To start with, we have a risk neutral stochastic process for the market rent given by the following geometric Brownian motion (GBM) under the risk neutral probability measure *Q*.:

$$dR_t = \alpha R_t dt + \sigma R_t dV_t. \tag{C.1}$$

where α and σ are assumed to constants, and where *V* denotes the *Q*-Wiener process. We also consider an interest rate *r* to be a constant.

Now observe that the premium is equivalent to a long position in an cash-or-nothing call and a short position in an asset-or-nothing call. Thus we can express π_t as

$$\begin{aligned}
\pi &= F(o) - F(s) \\
&= e^{-rT_1} E^Q[(M - R_{1,2})\,\mathbb{1}(R_{1,2} \geq B)] \\
&= e^{-rT_1} E^Q[M\,\mathbb{1}(R_{1,2} \geq B)] - e^{-rT_1} E^Q[R_{1,2}\,\mathbb{1}(R_{1,2} \geq B)] \\
&= CC - AC
\end{aligned}$$

where *CC* denotes the expected present value of a long position in a binary cash-or-nothing call, and where *AC* denotes the expected present value of a short position a binary asset-or-nothing call. The payoff to the cash-or-nothing call happens at maturity (time T_1) and equals

$$M\,\mathbb{1}(R_{1,2} \geq B) = \begin{cases} M \text{ if } R_{1,2} \geq B \\ 0 \text{ otherwise} \end{cases}$$

where *M* denotes a prespecified amount of cash (payoff). The moving threshold *B* represents a non-negative *trigger* price (compare Nielsen, 1999).

Similarly, for the asset-or-nothing call, the payoff at time T_1 equals

$$R_{1,2}\ \mathbf{1}(R_{1,2} \geq B) = \begin{cases} R_{1,2} \text{ if } R_{1,2} \geq B \\ 0 \text{ otherwise} \end{cases}$$

where again B represents a non-negative *trigger* price. But now the payoff equals the rent level $R_{1,2}$ itself if $R_{1,2} \geq B$. To simplify notation below, T will from henceforth denote time of maturity, i.e. $T = T_1$, and similarly R_T will denote the rent for the second period, $R_{1,2}$, which is set at maturity.

Below, we will make use of the following two results (see Björk, 2004). If the market rent follows GBM then

$$R_T = R_0 e^{\left(\alpha - \frac{1}{2}\sigma^2\right)T + \sigma V_T} \tag{C.2}$$

and the expected value is given by

$$E^Q[R_T] = R_0 e^{\alpha T}. \tag{C.3}$$

Using the result (C.2) above, we can express R_T as

$$R_T = e^{\left(\alpha - \frac{1}{2}\sigma^2\right)T + \sigma V_T} = R\ e^X \tag{C.5}$$

where X is a stochastic variable with distribution

$$X \in N\left[\left(\alpha - \frac{1}{2}\sigma^2\right)T, \sigma\sqrt{T}\right]$$

since $V(T) - V(0)$ has the Gaussian distribution $N[0, T^{1/2}]$.[21] Now let

$$m = \left(\alpha - \frac{1}{2}\sigma^2\right)T$$

and

$$s = \sigma\sqrt{T}.$$

Hence, we may express the distribution of X as

$$X \in N[m, s]. \tag{C.6}$$

The probability density function (pdf) of the stochastic variable X with distribution as above, $\varphi(x)$, is given by

[21] We observe that X is a linear combination of the normally distributed stochastic variable V(T) – V(0) (see Björk, 2004).

$$\varphi(x) = \frac{1}{\sqrt{2\pi s^2}} e^{-(x-m)^2/(2s^2)}. \tag{C.7}$$

For stochastic variable X with distribution $N[m, s]$, recall that

$$Q(X < x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-W^2/2} dw = N\left(\frac{x-m}{s}\right) \tag{C.8}$$

where N denotes the cumulative distribution function (cdf) of the $N[0, 1]$ distribution. Recall also the following symmetry property of the standard normal pdf

$$1 - N(x) = N(-x). \tag{C.9}$$

Starting with the price of the cash-or-nothing call, CC, we have that

$$\begin{aligned} CC &= e^{-rT)} E^Q[M \mathbf{1}(R_T \geq B)] = e^{-rT} M E^Q[\mathbf{1}(R_T \geq B)] \\ &= \left\{E^Q\left[\mathbf{1}(R_T \geq B)\right] = Q(R_T \geq B)\right\} = e^{-rT} M Q(R_T \geq B) \\ &= e^{-rT} M(1 - Q(R_T < B)). \end{aligned}$$

Using (C.5) we obtain

$$\begin{aligned} CC &= e^{-rT} M(1 - Q(R_0 e^X < B)) = \left\{\ln(R_0 e^X) < \ln(B) \Leftrightarrow X < \ln\left(\frac{B}{R_0}\right)\right\} \\ &= e^{-rT} M\left(1 - Q\left(X < \ln\left(\frac{B}{R_0}\right)\right)\right). \end{aligned}$$

Using (C.6), (C.8) and (C.9) we now obtain that

$$\begin{aligned} CC &= e^{-rT} M\left(1 - Q\left(X < \ln\left(\frac{B}{R}\right)\right)\right) = e^{-rT)} M\left(1 - N\left(\frac{x-m}{s}\right)\right) \\ &= e^{-rT} M\left(1 - N\left(\frac{1}{\sigma\sqrt{T}}\left\{\ln\left(\frac{B}{R_0}\right) - \left(\alpha - \tfrac{1}{2}\sigma^2\right)T\right\}\right)\right) \\ &= e^{-rT} M N\left(\frac{1}{\sigma\sqrt{T}}\left\{\ln\left(\frac{R_0}{B}\right) + \left(\alpha - \tfrac{1}{2}\sigma^2\right)T\right\}\right) \\ &= e^{-rT} M N(z_2). \end{aligned}$$

where we used that $-\ln(a/b) = \ln(b/a)$. Thus we have showed the cash-or-nothing call part.

For the asset-or-nothing call, AC, the price is given by

$$AC = e^{-rT} E^{Q}[R_T \mathbf{1}(R_T \geq B)] = \int_{\ln(B/R_0)}^{\infty} R_0 e^{X} \varphi(x) dx$$

where φ(x) denotes pdf given by (C.7) above and where we have made use of (C.5) again. Inserting the pdf (C.7) and using that $e^a e^b = e^{a+b}$ yields

$$AC = e^{-rT} \int_{\ln(B/R_0)}^{\infty} R_0 e^{x} \frac{1}{\sqrt{2\pi s^2}} e^{-(x-m)^2/(2s^2)} dx$$
$$= e^{-rT} R_0 \int_{\ln(B/R_0)}^{\infty} \frac{1}{\sqrt{2\pi s^2}} e^{-((x-m)^2 - 2s^2 x)/(2s^2)} dx$$
$$= e^{-rT} R_0 \int_{\ln(B/R_0)}^{\infty} \frac{1}{\sqrt{2\pi s^2}} e^{-(x^2 - 2(m+s^2)x + m^2)/(2s^2)} dx.$$

To evaluate this integral, the important trick now is to first complete the square in the exponent of the second-order *x*-polynomial, and then to simplify the two remaining terms, i.e.

$$x^2 - 2(m+s^2)x + m^2 = x^2 - 2(m+s^2)x + (m+s^2)^2 - (m+s^2)^2 + m^2$$
$$= (x-(m+s^2))^2 - (m+s^2)^2 + m^2 = (x-(m+s^2))^2 - m^2 - 2s^2 m - s^4 + m^2$$
$$= (x-(m+s^2))^2 - 2s^2 m - s^4.$$

Thus

$$AC = e^{-rT} R_0 \int_{\ln(B/R_0)}^{\infty} \frac{1}{\sqrt{2\pi s^2}} e^{-(x-(m+s^2)+2s^2 m - s^4)/(2s^2)} dx$$
$$= e^{-rT} R_0 \int_{\ln(B/R_0)}^{\infty} \frac{1}{\sqrt{2\pi s^2}} e^{(s^4+2s^2 m)/(2s^2)} e^{-(x-(m+s^2))/(2s^2)} dx$$
$$= e^{-rT} R_0 e^{(s^4+2s^2 m)/(2s^2)} \int_{\ln(B/R_0)}^{\infty} \frac{1}{\sqrt{2\pi s^2}} e^{-(x-(m+s^2))/(2s^2)} dx$$
$$= e^{-rT} R_0 e^{(s^2/2)+m} \int_{\ln(B/R_0)}^{\infty} \frac{1}{\sqrt{2\pi s^2}} e^{-(x-(m+s^2))/(2s^2)} dx$$
$$= e^{-rT} R_0 e^{(s^2/2)+m} Q\left(Y \geq \ln\left(\frac{B}{R_t}\right)\right)$$
$$= e^{-rT} R_0 e^{(s^2/2)+m} \left(1 - Q\left(Y < \ln\left(\frac{B}{R_t}\right)\right)\right)$$

where

$$Y \in N[m+s^2, s].$$

Now,

$$Q(Y < y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} e^{-W^2/2} dw = N\left(\frac{y - (m + s^2)}{s} \right)$$

where N denotes the cdf of the $N[0, 1]$ distribution. Thus

$$\begin{aligned}
AC &= e^{-rT} R_0 e^{(s^2/2)+m} \left(1 - Q\left(Y < \ln\left(\frac{B}{R_0} \right) \right) \right) \\
&= e^{-rT} R_0 e^{(s^2/2)+m} \left(1 - N\left(\frac{y - (m + s^2)}{s} \right) \right) \\
&= e^{-rT} R_0 e^{(s^2/2)+m} \left(1 - N\left(\frac{1}{\sigma\sqrt{T}} \left\{ \ln\left(\frac{B}{R_0} \right) - \left(\alpha - \tfrac{1}{2}\sigma^2 \right) T + \sigma^2 T \right\} \right) \right) \\
&= e^{-rT} R_0 e^{(s^2/2)+m} \left(1 - N\left(\frac{1}{\sigma\sqrt{T}} \left\{ \ln\left(\frac{B}{R_0} \right) - \left(\alpha + \tfrac{1}{2}\sigma^2 \right) T \right\} \right) \right) \\
&= e^{-rT} R_0 e^{(s^2/2)+m} N\left(\frac{1}{\sigma\sqrt{T}} \left\{ \ln\left(\frac{R_0}{B} \right) + \left(\alpha + \tfrac{1}{2}\sigma^2 \right) T \right\} \right) \\
&= e^{-rT} R_0 e^{\alpha T} N(z_1)
\end{aligned}$$

where the second exponent is obtained according to

$$e^{(s^2/2)+m} = e^{\sigma^2 T/2 + (\alpha - \sigma^2/2)T} = e^{\alpha T}.$$

Thus we have showed that the value of the extra premium is given by the formula

$$\pi \;= e^{-rT_1} (MN(z_2) - R_0 e^{\alpha T} N(z_1)).$$

where

$$z_1 = \frac{\ln(R_0/B) + (\alpha + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$z_2 = \frac{\ln(R_0/B) + (\alpha - \sigma^2/2)T}{\sigma\sqrt{T}} = z_1 - \sigma\sqrt{T}.$$

Note that a regular European call option is equivalent to long position in an asset-or-nothing call and a short position in a cash-or-nothing call where the cash payoff on the cash-or-nothing call equals the strike price (see Hull, 2003; Nielsen 1999). Thus we can use the calculations above to derive the closed-form pricing formula for a rent option with predetermined strike price K given by (9) in the main text, i.e.

$$F = e^{-rT} A(R_0 e^{\alpha T} N(d_1) - KN(d_2))$$

where d_1 and d_2 are given by

$$d_1 = \frac{\ln(R_0 / K) + (\alpha + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(R_0 / K) + (\alpha - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

Here, the asset-or-nothing call is the first term and the cash-or-nothing call is the second term.

Finally, note that by replacing the risk neutral drift α in (C.1) with the short rate of interest *r*, and by replacing the rent R_0 with the common notation for the stock price, S_0, and of course dropping the annuity factor *A*, we obtain the famous Black-Scholes formula for valuing a European call option on a stock with strike price *K* and date of maturity *T* (see Black and Scholes, 1973)

Appendix D

The three tables presented in this appendix show how option prices, (presented as average yearly part-payments of the option premium in relation to yearly rental payments during the first rental period; $F_{avg}(o) / R_{0,1}$ and $F_{avg}(s)/R_{0,1}$ respectively), change with different parameter assumptions. The tables are related to section 4.2 in the main text. All monetary figures are in SEK per square metre and year. Table D1 shows results for zero drift assumption (α = 0%), table D2 for α = 1%, and finally table D3 for α = 2%. As we did in section 4.2, we assume for simplicity that the household has an opportunity to buy a rental option with a strike price *K* that equals the household's moving threshold *B*.

As expected, the expenses for the option premium increases with higher rental drift assumptions. For example, when the risk neutral drift increases from 0% to 1% (assuming *K* = *B* = 1100, *M* = 1200, σ=7.5%), the option payment ratio $F_{avg}(o) / R_{0,1}$ increases from 2.2% to 3.1%. We also notice the relatively large decline in the expense ratio that follows an increase in the strike price *K* (and consequently the moving threshold *B*).

Table D1. Average yearly part-payments of option premium in relation to yearly rental payments during the first rental period; $F_{avg}(o) / R_{0,1}$ and $F_{avg}(s)/R_{0,1}$ respectively (latter within parenthesis). The length of the two lease periods equals time to maturity; $L = T_1$ = 5 years. The rent for the first lease period is $R_{0,1}$ = 1000. Strike rent is *K*. The moving threshold *B* is for simplicity set to equal *K*. Real risk-free interest is *r* = 3%. Spot rent follows a geometric Brownian motion with risk neutral drift α = 0% and volatility σ.

K = B	*M*	$F_{avg}(o) / R_{0,1}$ (Average per year) Within parenthesis: $F_{avg}(s)/R_{0,1}$		
α = 0%		σ=7.5%	σ=10%	σ=15%
		(2.7%)	(4.5%)	(8.2%)
1100	1200	2.2%	2.6%	2.8%
1100	1300	4.4%	5.1%	5.6%
1100	1400	6.7%	7.6%	8.4%
		(1.7%)	(3.3%)	(6.9%)
1150	1200	0.8%	1.0%	1.2%
1150	1300	2.3%	3.0%	3.6%

1150	1400	3.9%	5.0%	6.0%
		(1.1%)	(2.5%)	(5.8%)
1200	1200	0.0%	0.0%	0.0%
1200	1300	1.0%	1.5%	2.0%
1200	1400	2.1%	3.1%	4.1%

Table D2. Average yearly part-payments of option premium in relation to yearly rental payments during the first rental period; $F_{avg}(o) / R_{0,1}$ and $F_{avg}(s)/R_{0,1}$ respectively (latter within parenthesis). The length of the two lease periods equals time to maturity; $L = T_1 = 5$ years. The rent for the first lease period is $R_{0,1} = 1000$. Strike rent is K. The moving threshold B is for simplicity set to equal K. Real risk-free interest is $r = 3\%$. Spot rent follows a geometric Brownian motion with risk neutral drift $\alpha = 1\%$ and volatility σ.

$K = B$	M	$F_{avg}(o) / R_{0,1}$ (Average per year) Within parenthesis: $F_{avg}(s)/R_{0,1}$		
$\alpha = 1\%$		$\sigma=7.5\%$	$\sigma=10\%$	$\sigma=15\%$
		(4.3%)	(6.3%)	(10.4%)
1100	1200	3.1%	3.3%	3.3%
1100	1300	6.2%	6.5%	6.6%
1100	1400	9.4%	9.8%	9.9%
		(3.0%)	(4.9%)	(8.8%)
1150	1200	1.2%	1.3%	1.4%
1150	1300	3.5%	3.9%	4.3%
1150	1400	5.8%	6.6%	7.1%
		(2.0%)	(3.7%)	(7.5%)
1200	1200	0.0%	0.0%	0.0%
1200	1300	1.6%	2.1%	2.5%
1200	1400	3.3%	4.2%	5.0%

Table D3. Average yearly part-payments of option premium in relation to yearly rental payments during the first rental period; $F_{avg}(o) / R_{0,1}$ and $F_{avg}(s)/R_{0,1}$ respectively (latter within parenthesis). The length of the two lease periods equals time to maturity; $L = T_1 = 5$ years. The rent for the first lease period is $R_{0,1} = 1000$. Strike rent is K. The moving threshold B is for simplicity set to equal K. Real risk-free interest is $r = 3\%$. Spot rent follows a geometric Brownian motion with risk neutral drift $\alpha = 2\%$ and volatility σ.

$K = B$	M	$F_{avg}(o) / R_{0,1}$ (Average per year) Within parenthesis: $F_{avg}(s)/R_{0,1}$		
$\alpha = 2\%$		$\sigma=7.5\%$	$\sigma=10\%$	$\sigma=15\%$
		(6.6%)	(8.7%)	(12.9%)
1100	1200	4.1%	4.0%	3.8%
1100	1300	8.2%	8.0%	7.6%

1100	1400	12.4%	12.0%	11.4%
		(4.7%)	(6.9%)	(11.1%)
1150	1200	1.6%	1.7%	1.7%
1150	1300	4.7%	5.0%	5.0%
1150	1400	8.1%	8.3%	8.4%
		(3.3%)	(5.4%)	(9.5%)
1200	1200	0.0%	0.0%	0.0%
1200	1300	2.4%	2.7%	2.9%
1200	1400	4.9%	5.4%	5.9%