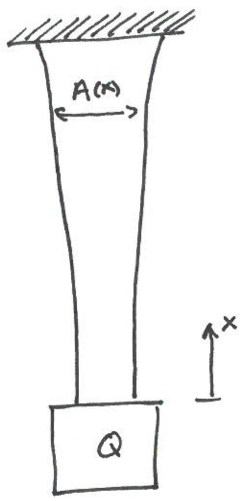
2.1.9 **Givet** Gruvlina med varierande area

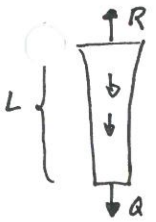


- Tvärsnittsarea $A(x)$
- Tyngd Q + egenvikt
- Tillåten spänning σ_{till}

Sökt $A(x)$ så att $\sigma(x) = \sigma_{till}$ (konstant) "Metod 1"

Lösning

1. Friläggning

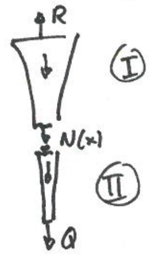


2. Jmv

$$\uparrow: R - \rho g \int_0^L A(x)\,dx - Q = 0$$

$$\Leftrightarrow R = Q + \rho g \int_0^L A(x)\,dx$$

3. Snittning



4. Jmv

$$\uparrow_{I}: R - \rho g \int_x^L A(x)\,dx - N(x) = 0$$

$$\Leftrightarrow N(x) = \rho g \left(\int_0^L A(x)\,dx - \int_x^L A(x)\,dx \right)$$

$$\underline{N(x) = \rho g \int_0^x A(x)\,dx + Q}$$

$$\uparrow_{II}: N(x) - \rho g \int_0^x A(x)\,dx - Q = 0$$

$$\Leftrightarrow \underline{\underline{N(x) = \rho g \int_0^x A(x)\,dx + Q}}$$

5. Normalspänning

$$\left[\sigma(x) = \frac{N(x)}{A(x)} \right] \Rightarrow \sigma(x) = \{\text{konstant}\} = \sigma_{till}, \text{ dvs } N(x) = \sigma_{till} \cdot A(x)$$

$$\Rightarrow \sigma_{till} \cdot A(x) = \rho g \int_0^x A(x)\,dx + Q \xrightarrow{} \text{Lös ut } A(x) \Rightarrow \frac{d}{dx}\left(\sigma_{till} \cdot A(x)\right) = \frac{d}{dx}\left(\rho g \int_0^x A(x)\,dx\right) + \frac{d}{dx}(Q)$$

$$\Leftrightarrow \underline{\underline{\frac{dA}{dx} = \frac{\rho g}{\sigma_{till}} \cdot A(x)}} \xrightarrow{\text{Diff. ekv!}} A(x) = C \cdot \exp\left(\frac{\rho g}{\sigma_{till}} \cdot x \right)$$

R.V. $\sigma(x=0) = \dfrac{N(x=0)}{A(x=0)} = \dfrac{Q}{A(0)} = \sigma_{till} \Rightarrow A(0) = \dfrac{Q}{\sigma_{till}}$

$$\therefore \underline{\underline{A(x) = \frac{Q}{\sigma_{till}} \cdot \exp\left(\frac{\rho g}{\sigma_{till}} \cdot x \right)}}$$

Dim-utrl!

$$\frac{N}{\frac{N}{m^2}} \cdot \exp\left(\frac{\frac{kg}{m^3} \cdot \frac{m}{s^2} \cdot m}{\frac{N}{m^2}} \right)$$

m^2 ok!

$$\Leftrightarrow \frac{\frac{kg}{m \cdot s^2}}{\frac{kg \cdot m}{s^2} \cdot \frac{1}{m^2}} = \frac{kg}{m^2 \cdot s} \cdot \frac{m^2 s}{kg} = 1 \Leftrightarrow \exp([-]) \text{ ok!}$$