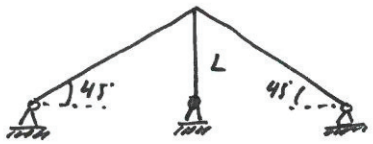
2.2.22 **Givet**

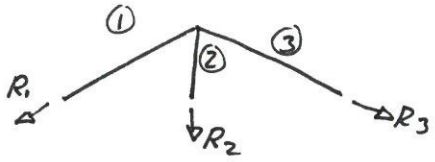


× Tvärsnittsdiameter $2a$, dvs $A = \pi a^2$
× Lin. termoel. mtrl (E, α)

Sökt Normalspänning i mittenstång pga ΔT; dvs $\sigma_2(\Delta T)$

Lösning

1. Frilägg



2. Jmv

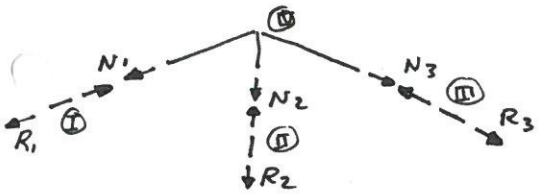
$\uparrow: -R_2 - \sin 45° \cdot R_1 - \sin 45° \cdot R_3 = 0$

$\Leftrightarrow -R_2 - \frac{1}{\sqrt{2}} \cdot R_1 - \frac{1}{\sqrt{2}} \cdot R_3 = 0$

$\rightarrow: -R_1 \cdot \cos 45° + R_3 \cdot \cos 45° = 0$

$\Leftrightarrow \underline{R_1 = R_3}$

3. Snitta



4. Jmv

$\nwarrow_{I}: -R_1 + N_1 = 0 \Rightarrow N_1 = R_1$

$\uparrow_{II}: N_2 - R_2 = 0 \Rightarrow N_2 = R_2$

$\searrow_{III}: -N_3 + R_3 = 0 \Rightarrow N_3 = R_3$

Alltså har vi att

$\underline{N_1 = N_3}$

$-N_2 - \frac{1}{\sqrt{2}} \cdot N_1 - \frac{1}{\sqrt{2}} \cdot N_3 = 0$

$\Leftrightarrow \underline{N_2 = -\sqrt{2} \cdot N_1}$

5. Normalspänning

$$\left[\sigma = \frac{N}{A}\right] \Rightarrow \begin{cases} \sigma_1 = \frac{N_1}{A} \\ \sigma_2 = \frac{N_2}{A} \\ \sigma_3 = \frac{N_3}{A} \end{cases}$$

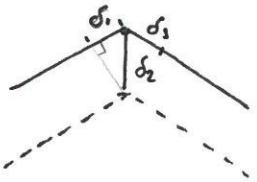
6. Konstitutivt samband

Lin. termoel. mtrl: $\left[\varepsilon = \frac{\sigma}{E} + \alpha\Delta T\right]$ se FS. 3.1

$$\Rightarrow \left[\varepsilon = \frac{N}{EA} + \alpha\Delta T\right] \quad \text{samt att} \quad \left[\delta = \varepsilon L\right]$$

7. Kompabilitet

Deformationsamband:



$\Rightarrow$ 

$\Leftrightarrow \sin 45° = \frac{\delta_1}{\delta_2} = \frac{1}{\sqrt{2}}$

$\Rightarrow \underline{\delta_1 = \frac{1}{\sqrt{2}} \cdot \delta_2}$

vilket med $\left.\begin{aligned} \delta_1 &= \varepsilon_1 \cdot L_1 = \varepsilon_1 \cdot \sqrt{2} \cdot L \\ \delta_2 &= \varepsilon_2 \cdot L_2 = \varepsilon_2 \cdot L \end{aligned}\right\}$ ger $\varepsilon_1 \cdot \sqrt{2} \cdot L = \frac{1}{\sqrt{2}} \cdot \varepsilon_2 \cdot L \Rightarrow$

$\Rightarrow \underline{\varepsilon_2 = 2\varepsilon_1}$ ... då fås tillsammans med uttryck från 6.

$\frac{N_2}{EA} + \alpha\Delta T = 2 \cdot \left(\frac{N_1}{EA} + \alpha\Delta T\right) \Leftrightarrow N_1 = \frac{N_2}{2} - \frac{EA\alpha\Delta T}{2}$; isatt jmv-ekv från 4.

$$N_2 = -\sqrt{2} \cdot N_1 = -\sqrt{2}\left(\frac{N_2}{2} - \frac{EA\alpha\Delta T}{2}\right) = -\frac{\sqrt{2}}{2} N_2 + \frac{\sqrt{2}}{2} \cdot EA\alpha\Delta T = -\frac{1}{\sqrt{2}} N_2 + \frac{1}{\sqrt{2}} EA\alpha\Delta T$$

$$\Leftrightarrow N_2 + \frac{1}{\sqrt{2}} \cdot N_2 = \frac{1}{\sqrt{2}} \cdot EA\alpha\Delta T \Leftrightarrow \sqrt{2} N_2 + N_2 = EA\alpha\Delta T \Rightarrow \underline{N_2 = \frac{EA\alpha\Delta T}{1+\sqrt{2}}}$$

och då $\sigma_2 = \frac{N_2}{A}$ fås $\underline{\sigma_2 = \frac{E\alpha\Delta T}{1+\sqrt{2}}}$

Dim-kontroll $\frac{\frac{N}{m^2} \cdot \frac{1}{K} \cdot K}{[-]} = \frac{N}{m^2}$ OK