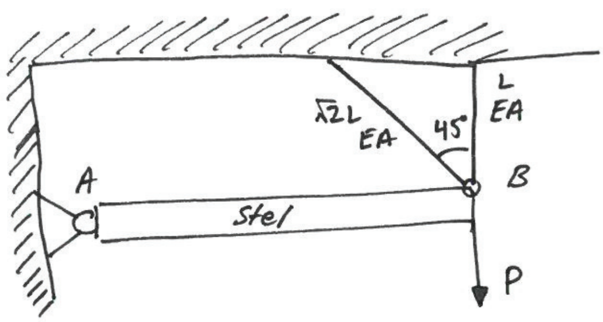
2.2.38 Givet Fackverk



× Icke-stela stänger är idealplastiska

Sökt

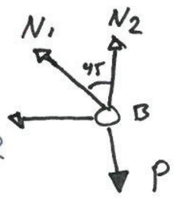
a) Bestäm σ i stänger
Bestäm förskjutning i vertikalt led i B

b) Egenspänning i stänger, om avlastning sker från fullt plastiskt tillstånd?

Lösning

(1. Frilägg, 2. Jmv)

3. Snitta



4. Jmv

$\uparrow: \; N_2 + \cos 45° \cdot N_1 - P = 0 \implies \frac{1}{\sqrt{2}} N_1 + N_2 = P$

$\rightarrow: \; -R - \sin 45° \cdot N_1 = 0 \implies N_1 = -\sqrt{2} \cdot R$

dvs 2 okända, 1 ekvation

5. Normalspänning

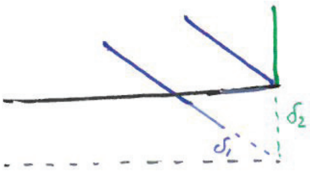
$\sigma_1 = \frac{N_1}{A} \qquad \sigma_2 = \frac{N_2}{A}$

6. Konstitutivt

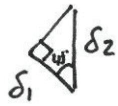
Lin. område $[\sigma = E\varepsilon]$ och $[\varepsilon = \frac{\delta}{L}] \implies \begin{cases} \varepsilon_1 = \frac{\delta_1}{\sqrt{2}L} \\ \varepsilon_2 = \frac{\delta_2}{L} \end{cases} \implies \begin{cases} \delta_1 = \sqrt{2}L \\ \delta_2 = L\varepsilon_2 \end{cases}$

Pl. område $[\sigma = \sigma_s]$

7. Kompabilitet



$\implies$



$\cos 45° = \frac{\delta_1}{\delta_2} \iff \delta_1 = \frac{\delta_2}{\sqrt{2}}$

$\implies \sqrt{2} L \varepsilon_1 = \frac{L \varepsilon_2}{\sqrt{2}} \iff \varepsilon_1 = \frac{1}{2} \cdot \varepsilon_2$

$\sigma = E\varepsilon \longrightarrow \varepsilon = \frac{\sigma}{E} \implies \frac{\sigma_1}{E} = \frac{1}{2} \cdot \frac{\sigma_2}{E} \longrightarrow \left[\sigma = \frac{N}{A}\right] \implies \frac{N_1}{EA} = \frac{1}{2} \frac{N_2}{EA}$

$\implies N_1 = \frac{1}{2} N_2$ dvs 2 okända, 2 ekvationer $\implies \begin{cases} N_1 = \frac{2P}{\sqrt{2}+4} \\ N_2 = \frac{4P}{\sqrt{2}+4} \end{cases}$

$\implies \left[\sigma = \frac{N}{A}\right]$ ger $\begin{cases} \sigma_1 = \frac{2P}{A(\sqrt{2}+4)} \approx 0.37 \cdot \frac{P}{A} & \text{SVAR a)} \\ \sigma_2 = \frac{4P}{A(\sqrt{2}+4)} \approx 0{,}74 \cdot \frac{P}{A} & \text{SVAR a)} \end{cases}$

Förskjutning i B ges av δ_2, dvs

$\Delta_{vert, B} = \delta_2 = \varepsilon_2 \cdot L = \frac{\sigma_2}{E} \cdot L = \frac{4PL}{EA(\sqrt{2}+4)}$

Svar a)

forts $\longrightarrow$

2.2.38 b)

forts 1

– Full plasticering fås då $|\sigma_1| = |\sigma_2| = \sigma_s$

$$\left(\text{Jmv-sb}: \frac{N_1}{\sqrt{2}} + N_2 = P\right) \Longrightarrow \frac{\sigma_s A}{\sqrt{2}} + \sigma_s A = P_f$$

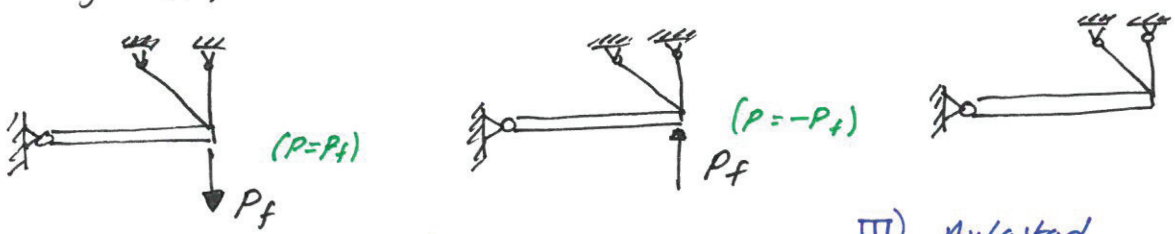
$$\Longleftrightarrow \underline{P_f = \sigma_s A\left(\frac{2+\sqrt{2}}{2}\right)} \approx 1.71 \cdot \sigma_s A$$

– Begynnande plasticering fås i stång 2 ty $\sigma_2 > \sigma_1$,

dvs $\sigma_2 = \sigma_s = \dfrac{4P_s}{A(\sqrt{2}+4)} \Longrightarrow P_s = \sigma_s A\left(\dfrac{4+\sqrt{2}}{4}\right) \approx 1.35\,\sigma_s A$

$\Longrightarrow$ Flytlastförhöjningen är då alltså $\left|\beta = \dfrac{P_f - P_s}{P_s}\right| \Rightarrow \beta = \ldots = \dfrac{\sqrt{2}}{4+\sqrt{2}} \approx 0{,}26$

Enligt uppgift är lastscenariot följande:



I) Full plasticering II) Elastisk avlastning III) Avlastad

$\Longrightarrow$ I + II = III

I) $\sigma_1^{I} = +\sigma_s$

$\sigma_2^{I} = +\sigma_s$

II) $\sigma_1^{II} = \dfrac{2}{\sqrt{2}+4} \cdot \dfrac{-P_f}{A} = \left\{P_f = \sigma_s A\left(\dfrac{2+\sqrt{2}}{2}\right)\right\} = \dfrac{-2+\sqrt{2}}{4+\sqrt{2}} \cdot \sigma_s$

$\sigma_2^{II} = \dfrac{4}{\sqrt{2}+4} \cdot \dfrac{-P_f}{A} = -\dfrac{2(2+\sqrt{2})}{4+\sqrt{2}} \cdot \sigma_s$

III) $\sigma_1^{III} = \sigma_1^{I} + \sigma_1^{II} = \sigma_s - \dfrac{2+\sqrt{2}}{4+\sqrt{2}} \cdot \sigma_s = \underline{\dfrac{2}{4+\sqrt{2}} \cdot \sigma_s}$ obs. dragspänning

$\sigma_2^{III} = \sigma_2^{I} + \sigma_2^{II} = \sigma_s - \dfrac{2(2+\sqrt{2})}{4+\sqrt{2}} = \underline{\dfrac{-\sqrt{2}}{4+\sqrt{2}} \cdot \sigma_s}$ obs. tryckspänning

Svar b)