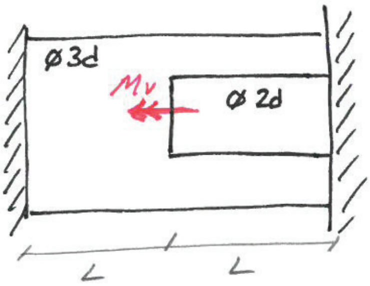
2.6.15 Axel med urborrat hål



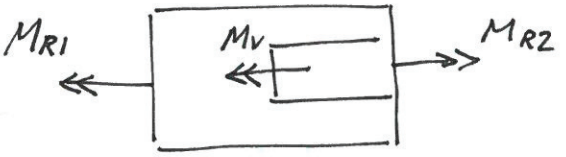
× Fast inspännt

× Lin. el. - idealpl. mtrl

Sökt: M_v då begynnande plasticering sker $\longrightarrow$ dvs M_{vs}

Lösning

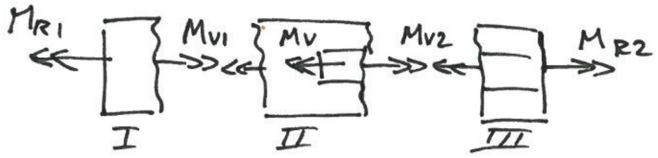
1. Frilägg



2. Jmv

$\rightarrow$: $-M_{R1} - M_v + M_{R2} = 0$

3. Snitta



4. Jmv

$\rightarrow_{I}$: $-M_{R1} + M_{v1} = 0 \Rightarrow M_{v1} = M_{R1}$

$\rightarrow_{II}$: $-M_{v1} - M_v + M_{v2} = 0 \Rightarrow M_{v2} - M_{v1} = M_v$

$\rightarrow_{III}$: $-M_{v2} + M_{R2} = 0 \Rightarrow M_{v2} = M_{R2}$

Dvs 2 obekanta, 1 ekvation

5. Skjuvspänning/Vridmoment

$$\left[M_v = \int_a^b 2\pi \tau r^2 dr\right]$$

6. Konstitutivt sb

Lin. el. område: $[\tau = G\gamma]$ "Hookes lag"

7. Kompabilitet

$[\gamma \cdot L = r \cdot \theta]$ Det. sb. vridning av rör ; samt fast inspänt dvs $\theta_1 + \theta_2 = 0$

$\Rightarrow \underline{\theta_1 = -\theta_2}$ $\Rightarrow$ Behöver rotationen för del 1 och 2

Alt. 1

$\left[\text{FS } 6.75 \quad \theta = \frac{M_v L}{GK}\right]$

Alt. 2 "Härled" genom 5. + 6. + 7. :

$$M_v = \int_a^b 2\pi \cdot \tau \cdot r^2 dr = \int_a^b 2\pi \cdot (G\gamma) \cdot r^2 dr = \int_a^b 2\pi G \left(\frac{r\theta}{L}\right) r^2 dr$$

$$= \frac{2\pi G \theta}{L} \int_a^b r^3 dr = \frac{2\pi G \theta}{4 \cdot L}(b^4 - a^4)$$

$$\Leftrightarrow \quad \theta = \frac{2 M_v L}{G\pi(b^4 - a^4)} \qquad K = \frac{\pi}{2}(b^4 - a^4)$$

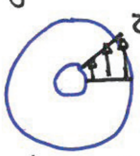
$\therefore \quad \theta_1 = \frac{2 M_{v1} \cdot L}{G\pi\left(\left(\frac{3d}{2}\right)^4 - 0^4\right)} = \frac{32 M_{v1} L}{81 \pi G d^4}$; $\theta_2 = \frac{2 M_{v2} \cdot L}{G\pi\left(\left(\frac{3d}{2}\right)^4 - \left(\frac{2d}{2}\right)^4\right)} = \frac{32 M_{v2} \cdot L}{65 \pi G d^4}$

$$\Rightarrow \theta_1 = -\theta_2 \Leftrightarrow \frac{32 M_{v1} \cdot L}{81 \pi G d^4} = -\frac{32 M_{v2} \cdot L}{65 \pi G d^4} \Leftrightarrow \underline{M_{v1} = -\frac{81}{65} M_{v2}}$$

Alltså har vi nu

$$\begin{cases} M_{v2} - M_{v1} = M_v \\ M_{v1} = -\frac{81}{65} M_{v2} \end{cases} \Longrightarrow \begin{cases} M_{v1} = -\frac{81}{146} \cdot M_v \\ M_{v2} = \frac{65}{146} \cdot M_v \end{cases}$$

Skjuvspänningen är störst längst ut på röret (dvs då $r = r_{max}$)
(tjockväggigt)
(se Fig. 6.12 i FS) $\longrightarrow$



$\Longrightarrow$ $r_{max} = \frac{3d}{2}$ för båda delarna.

Alt. 1

$$\left[\tau_{max} = \frac{M_v}{W_v}\right] \text{FS. 6.76}$$

$$W_v \leftarrow \left[\frac{\pi}{2b}(b^4 - a^4)\right] \text{FS. 6.78}$$

Alt. 2.

Vi har härlett att $\theta = \frac{2 M_v \cdot L}{G \cdot \pi (b^4 - a^4)}$ samt har att $\tau(r) = G \cdot \frac{r\theta}{L}$

från 7. $\Longrightarrow$ Lös ut map θ:

$$\frac{\tau(r) \cdot L}{G \cdot r} = \frac{2 \cdot M_v \cdot L}{\pi G (b^4 - a^4)} \Longleftrightarrow \tau(r) = \frac{2 \cdot M_v}{\pi (b^4 - a^4)} \cdot r$$

Dvs samma som Alt. 1

$$\tau_{1,max}\left(r = r'_{max} = \frac{3d}{2}\right) = \frac{2 \cdot M_{v1}}{\pi\left(\left(\frac{3d}{2}\right)^4 - 0^4\right)} \cdot \frac{3d}{2} = \ldots = \frac{16 M_{v1}}{27\pi \cdot d^3} = \frac{-24 M_v}{73 \pi d^3}$$

$$\tau_{2,max}\left(r = r^2_{max} = \frac{3d}{2}\right) = \frac{2 \cdot M_{v2}}{\pi\left(\left(\frac{3d}{2}\right)^4 - \left(\frac{2d}{2}\right)^4\right)} \cdot \frac{3d}{2} = \ldots = \frac{48 M_{v2}}{65 \pi \cdot d^3} = + \frac{24 M_v}{73 \pi d^3}$$

$\Longrightarrow$ Rören plasticerar alltså samtidigt!

$$\tau_s = \frac{24}{73} \cdot \frac{M_{vs}}{\pi d^3} \Longleftrightarrow \underline{\underline{M_{vs} = \frac{73 \pi d^3 \cdot \tau_s}{24}}}$$