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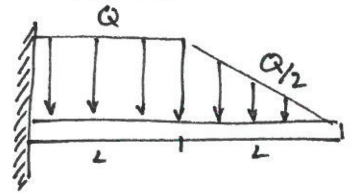
Givet

Konsolbalk



- Last $Q + \frac{Q}{2}$

Sökt Rita T- och M-diagram

Lösning

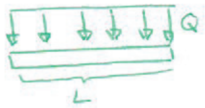
Alt. 1: Frilägga + jmv globalt och sedan snitta & jmv för varje "del"... jobbigt!

Alt. 2: Använda jämviktsambanden enl. FS. 6.7 och integrera fram T och M för varje del... något mer straight-forward.

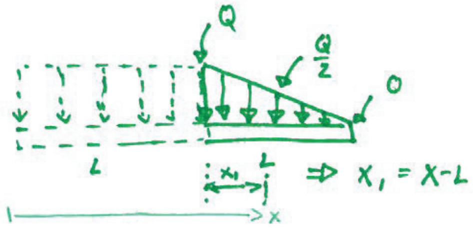
$\Longrightarrow$ Alt. 2

1. Dela in områden / gränser $\left[\frac{\text{kraft}}{\text{längd}}\right]$

Del 1: $0 \leq x \leq L \; : \; q_1(x) = -\frac{Q}{L}$ $\Leftarrow$

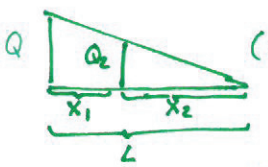


Del 2: $L \leq x \leq 2L \; : \; q_2(x) = -\frac{Q(2L-x)}{L^2}$



$\Rightarrow x_1 = x - L$

$\Longrightarrow$



$x_1 + x_2 = L$

$(x-L) + x_2 = L \Longrightarrow x_2 = L + L - x$

$x_2 = 2L - x$

$\frac{Q}{L} = \frac{Q_2}{2L-x} \Longrightarrow Q_2 = \frac{Q(2L-x)}{L}$

dvs om $q_2(x) = -\frac{Q_2}{L_2} \Rightarrow q_2(x) = -\frac{Q(2L-x)}{L^2}$

$\Longrightarrow$ Jämviktssamband enligt FS. 6.7:

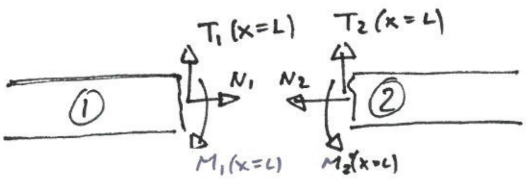
$$\left[\frac{dT(x)}{dx} = -q(x)\right]$$

$$\left[\frac{dM(x)}{dx} = T(x)\right]$$

$\Longrightarrow$ Randvärden här är:

$M_2(x=2L) = 0$ och $T_2(x=2L) = 0$ ty fri kant på höger sida

$M_1(x=L) = M_2(x=L)$ och $T_1(x=L) = T_2(x=L)$ ty samma snittmoment och snitt-tvärkraft i samma snitt:



$\Longrightarrow$

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forts. 1

Således har vi

$$\begin{cases} T_1(x) = \int -q_1(x)\,dx = \int \frac{Q}{L}\,dx = \frac{Q}{L}\cdot x + C_1 \\ M_1(x) = \int T_1(x)\,dx = \int\left(\frac{Q}{L}\cdot x + C_1\right)dx = \frac{Q}{2L}\cdot x^2 + C_1\cdot x + C_2 \\ T_2(x) = \int -q_2(x)\,dx = \int \frac{Q(2L-x)}{L^2}\,dx = \frac{2Q}{L}\cdot x - \frac{Qx^2}{2L^2} + C_3 \\ M_2(x) = \int T_2(x)\,dx = \int\left(\frac{2Q}{L}\cdot x - \frac{Q}{2L^2}\cdot x^2 + C_3\right)dx = \frac{Q}{L}\cdot x^2 - \frac{Q}{6L^2}\cdot x^3 + C_3\cdot x + C_4 \end{cases}$$

$\therefore$ 4 konstanter $(C_1\ C_2\ C_3\ C_4)$

$\Rightarrow$ Lös ut C_3 sedan C_4, C_1, C_2

C_3) $T_2(x=2L) = \frac{2Q}{L}\cdot 2L - \frac{Q}{2L^2}(2L)^2 + C_3 = 0$

$= 4Q - \frac{Q}{2L^2}\cdot 4\cdot L^2 + C_3 = 0 \iff \underline{C_3 = -2Q}$

C_4) $M_2(x=2L) = \frac{Q}{L}\cdot(2L)^2 - \frac{Q}{6L^2}\cdot(2L)^3 - 2Q\cdot 2L + C_4 = 0$

$= \frac{Q}{L}\cdot 4L^2 - \frac{Q}{6L^2}\cdot 8L^3 - 4QL + C_4 = 0 \iff \underline{C_4 = \frac{4}{3}\cdot QL}$

C_1) $T_1(x=L) = \frac{Q}{L}\cdot L + C_1 = T_2(x=L) = \frac{2Q}{L}\cdot L - \frac{Q}{2L^2}\cdot L^2 - 2Q$

$\iff \underline{C_1 = -\frac{3}{2}Q}$

C_2) $M_1(x=L) = \frac{Q}{2L}\cdot L^2 - \frac{3}{2}Q\cdot L + C_2 = M_2(x=L) = \frac{Q}{L}\cdot L^2 - \frac{Q}{6L^2}\cdot L^3 + -2Q\cdot L + \frac{4}{3}Q$

$\iff \underline{C_2 = \frac{7}{6}QL}$

$$\therefore \begin{cases} T_1(x) = \frac{Q}{L}\cdot x - \frac{3}{2}Q & \text{Dim ok} \quad \forall\ 0 \le x \le L \\ T_2(x) = \frac{2Q}{L}\cdot x - \frac{Qx^2}{2L^2} - 2Q & \text{Dim ok} \quad \forall\ L \le x \le 2L \\ M_1(x) = \frac{Q}{2L}\cdot x^2 - \frac{3Q}{2}\cdot x + \frac{7}{6}QL & \text{Dim ok} \quad \forall\ 0 \le x \le L \\ M_2(x) = \frac{Q}{L}\cdot x^2 - \frac{Q}{6L^2}\cdot x^3 - 2Q\cdot x + \frac{4}{3}\cdot QL & \text{Dim ok} \quad \forall\ L \le x \le 2L \end{cases}$$

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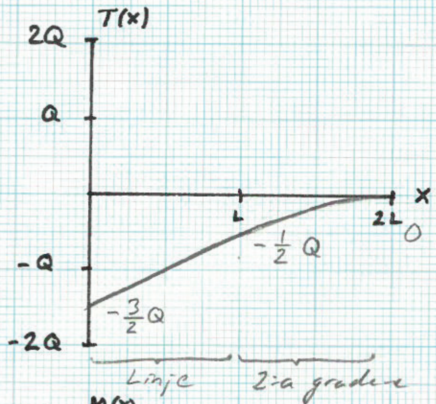
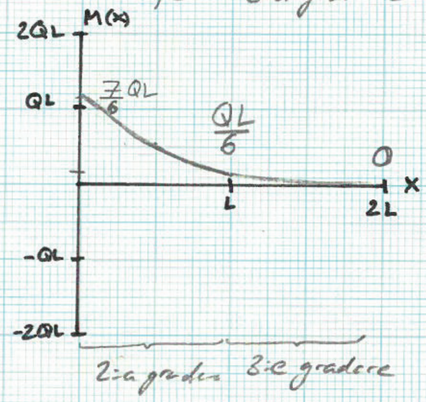
forts. 2



T-diagram



M-diagram