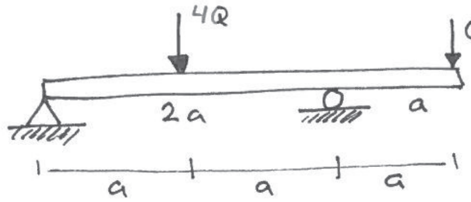
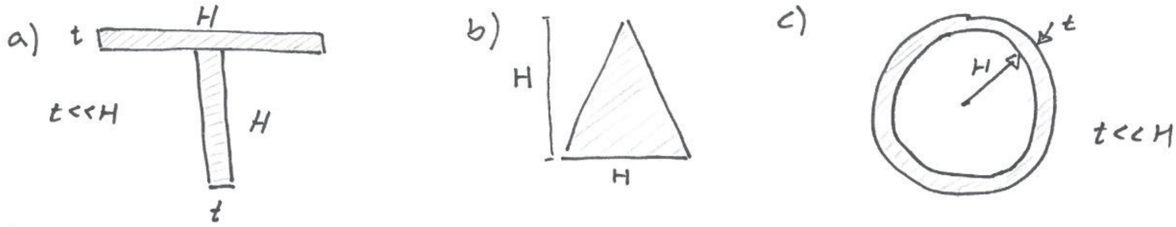
2.4.39 Balkar med olika tvärsnitt

<u>Givet</u>

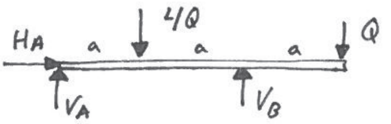


<u>Sökt</u> Bestäm τ_{max} och τ_{min}



<u>Lösning</u>

1. Frilägg



2. Jmv

$\uparrow: V_A - 4Q + V_B - Q = 0 \implies \underline{V_A + V_B = 5Q}$

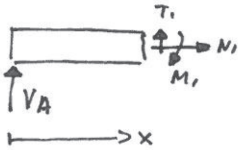
$\rightarrow: H_A = 0$

$\overset{\curvearrowright}{A}: -4Q\cdot a + V_B \cdot 2a - Q\cdot 3a = 0$

$\implies \underline{V_B = \frac{7Q}{2}} \implies \underline{V_A = \frac{3Q}{2}}$

3. Snitta

Del 1: $0 \le x \le a$



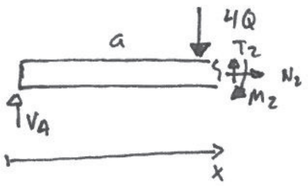
4. Jmv

$\uparrow: V_A + T_1 = 0 \implies T_1 = -V_A = -\frac{3Q}{2}$

$\rightarrow: N_1 = 0$

$\overset{\curvearrowright}{X}: -V_A\cdot x - M_1 = 0 \implies M_1(x) = -\frac{3Q}{2}\cdot x$

Del 2: $a \le x \le 2a$



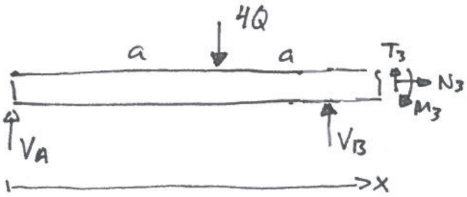
$\uparrow: V_A - 4Q + T_2 = 0 \implies T_2 = 4Q - V_A = \frac{5}{2}Q$

$\rightarrow: N_2 = 0$

$\overset{\curvearrowright}{X}: -V_A\cdot x + (x-a)\cdot 4Q - M_2 = 0$

$\implies M_2(x) = \frac{5Qx}{2} - 4Qa$

Del 3: $2a \le x \le 3a$



$\uparrow: V_A - 4Q + V_B + T_3 = 0 \implies T_3 = 4Q - V_A - V_B = -Q$

$\rightarrow: N_3 = 0$

$\overset{\curvearrowright}{X}: -V_A\cdot x + (x-a)\cdot 4Q - (x-2a)\cdot V_B - M_3 = 0$

$\implies M_3(x) = \frac{3Q\cdot x}{2} - 4Q(x-a) - (x-2a)\cdot\frac{7Q}{2}$

$\therefore$ $T_1 = -\frac{3Q}{2}$; $M_1(x) = -\frac{3Q}{2}\cdot x$ $\quad \forall\ 0 \le x \le a$

$T_2 = \frac{5Q}{2}$; $M_2(x) = \frac{5}{2}Q\cdot x - 4Q\cdot a$ $\quad \forall\ a \le x \le 2a$

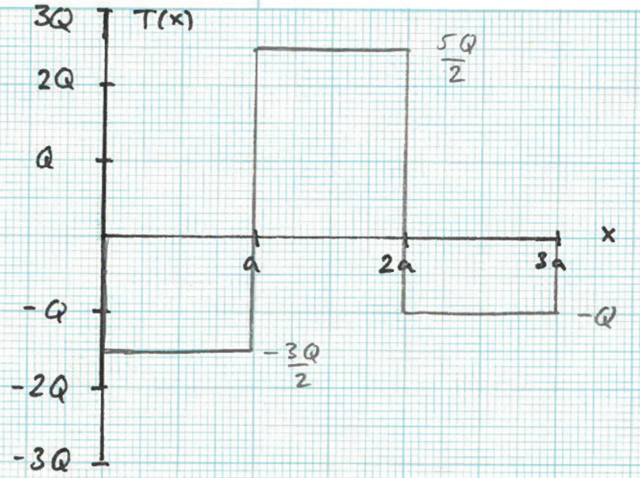
$T_3 = -Q$; $M_3(x) = 11Qa - 6Qx$ $\quad \forall\ 2a \le x \le 3a$

R.V.: $T(x=0) = 0 \quad T(x=3a) = 0$ (fria ändar)

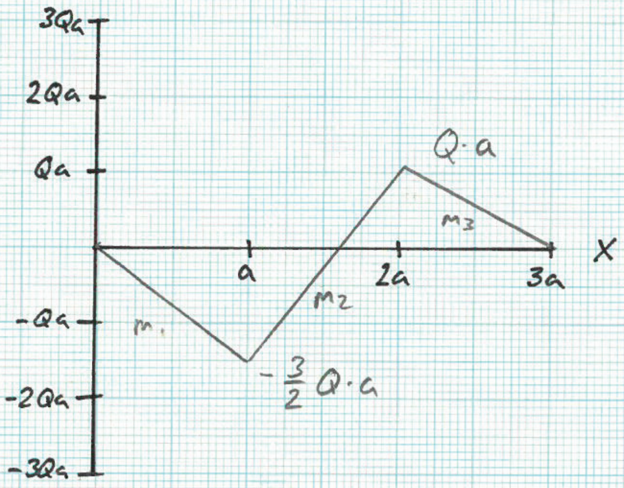
$M(x=0) = 0 \quad M(x=3a) = 0$

2.4.3a

forts. 1



T-diagram



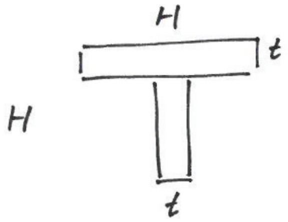
M-diagram

$$\Longrightarrow \begin{cases} M_{max} = Q\cdot a & \text{vid } x=2a \\ M_{min} = -\frac{3}{2}\cdot Q\cdot a & \text{vid } x=a \end{cases}$$

2.4.39

forts. 2

a)



Då $H >> t$ är tvärsnittet tunnväggigt

$$\left[\sigma_x(z) = \frac{M}{I_y} \cdot z \right] \longrightarrow \text{Vi behöver } I_y!$$

$\Longrightarrow$ FS. tab. 30.2.10 : $\left[I_y = \frac{t_1 \cdot h^3}{12} + h \cdot t_1 \left(e_z - \frac{h}{2}\right)^2 + b t_f \left(h - e_z\right)^2 \right]$

$$\left[e_z = h \cdot \frac{b t_f + h t_1/2}{b t_f + h t_1} \right]$$

Här är $\begin{cases} t_1 = t \\ t_f = t \\ h = H \\ b = H \end{cases}$

tyngdpunkts koordinat i z-led

Vilket ger $\quad e_z = H \cdot \left(\frac{H \cdot t + H \cdot t \cdot 0.5}{H \cdot t + H \cdot t} \right) = \frac{3H}{4}$ Dim ktrl!

$$I_y = \frac{t \cdot H^3}{12} + H \cdot t \cdot \left(\frac{3H}{4} - \frac{H}{2} \right)^2 + H \cdot t \cdot \left(H - \frac{3H}{4} \right)^2 = \frac{5H^3 \cdot t}{24}$$

Vi har extrempunkter : $x = a$ och $x = 2a$ ... utvärdera!

$\llcorner M = -\frac{3}{2} Qa \qquad \llcorner M = Q \cdot a$

$$x = a : \begin{cases} \sigma_x\left(z = \frac{-3H}{4}\right) = \dfrac{-\frac{3}{2} \cdot Q \cdot a}{\frac{5H^3 \cdot t}{24}} \cdot \left(\frac{-3H}{4} \right) = \dfrac{27 Q \cdot a}{5H^2 t} = \sigma_{max}^{x=a} \\[2ex] \sigma_x\left(z = \frac{H}{4}\right) = \dfrac{-\frac{3}{2} \cdot Q \cdot a}{\frac{5H^3 t}{24}} \cdot \left(\frac{H}{4} \right) = \dfrac{-9 \cdot Q \cdot a}{5H^2 t} = \sigma_{min}^{x=a} \end{cases}$$

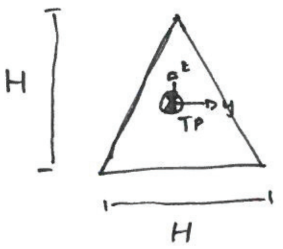
$$x = 2a : \begin{cases} \sigma_x\left(z = \frac{-3H}{4}\right) = \dfrac{Q \cdot a}{\frac{5H^3 \cdot t}{24}} \left(\frac{-3H}{4} \right) = \dfrac{-18 Q a}{5H^2 t} = \sigma_{min}^{x=2a} \\[2ex] \sigma_x\left(z = \frac{H}{4}\right) = \dfrac{Q \cdot a}{\frac{5H^3 \cdot t}{24}} \left(\frac{H}{4} \right) = \dfrac{6 Q a}{5H^2 t} = \sigma_{max}^{x=2a} \end{cases}$$

$$\therefore \begin{cases} \sigma_{max} = \dfrac{27 Qa}{5H^2 t} \\[2ex] \sigma_{min} = \dfrac{-18 Qa}{5H^2 t} \end{cases}$$

SVAR a)

2.4.39 b) Triangulärt tvärsnitt

forts 3



$$I_y = \left\{ \text{s. 344 tab 30.1.2} \right\} = \left[\frac{bh^3}{36}\right] \quad \text{och} \quad \left[e_z = \frac{2h}{3}\right]$$

Här är $b = H$, $h = H$

$$\Rightarrow I_y = \frac{H \cdot H^3}{36} = \frac{H^4}{36}$$

$$e_z = \frac{2H}{3}$$

Således fås:

$$x = a: \begin{cases} \sigma\left(z = \frac{2H}{3}\right) = \dfrac{-\frac{3}{2}Qa}{\frac{H^4}{36}} \cdot \frac{2H}{3} = \dfrac{-36\,Qa}{H^3} = \sigma_{min}^{x=a} \\ \sigma\left(z = -\frac{H}{3}\right) = \dfrac{-\frac{3}{2}Qa}{\frac{H^4}{36}} \cdot \left(-\frac{H}{3}\right) = \dfrac{18\,Qa}{H^3} = \sigma_{max}^{x=a} \end{cases}$$

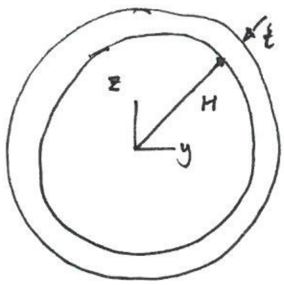
$$x = 2a: \begin{cases} \sigma\left(z = \frac{2H}{3}\right) = \dfrac{Q \cdot a}{\frac{H^4}{36}} \cdot \frac{2H}{3} = \dfrac{24\,Qa}{H^3} = \sigma_{max}^{x=2a} \\ \sigma\left(z = -\frac{H}{3}\right) = \dfrac{Q \cdot a}{\frac{H^4}{36}} \cdot \left(-\frac{H}{3}\right) = -\dfrac{12\,Qa}{H^3} = \sigma_{min}^{x=2a} \end{cases}$$

$$\Rightarrow \begin{cases} \sigma_{max} = \dfrac{24\,Qa}{H^3} \\ \sigma_{min} = -\dfrac{36Qa}{H^3} \end{cases}$$

Svar b)

2.4.39
forts 4

c) Ihåligt cirkulärt / rör - tvärsnitt



- Tunnväggigt antas

$I_y = \left\{ FS.\ s344\ tes\ 30.1.4 \right\} = \left[\pi a^3 t \right]$

Här är $\begin{cases} a = H \\ t = t \end{cases}$

$$\Rightarrow \quad I_y = \pi H^3 t$$

$$x = a: \begin{cases} \sigma(z = H) = \dfrac{-\frac{3Qa}{2}}{\pi H^3 t} \cdot (H) = \dfrac{-3Qa}{2\pi H^2 t} = \sigma_{min}^{x=a} \\[2ex] \sigma(z = -H) = \dfrac{-\frac{3Qa}{2}}{\pi H^3 t} (-H) = \dfrac{3Qa}{2\pi H^2 t} = \sigma_{max}^{x=a} \end{cases}$$

$$x = 2a: \begin{cases} \sigma(z = H) = \dfrac{Qa}{\pi H^3 t} (H) = \dfrac{Qa}{\pi H^2 t} = \sigma_{max}^{x=2a} \\[2ex] \sigma(z = -H) = \dfrac{Qa}{\pi H^3 t} \cdot (-H) = \dfrac{-Qa}{\pi H^2 t} = \sigma_{min}^{x=2a} \end{cases}$$

$$\therefore \begin{cases} \sigma_{max} = \dfrac{3}{2} \cdot \dfrac{Qa}{\pi H^2 t} \\[2ex] \sigma_{min} = -\dfrac{3}{2} \cdot \dfrac{Qa}{\pi H^2 t} \end{cases}$$

Svar c)