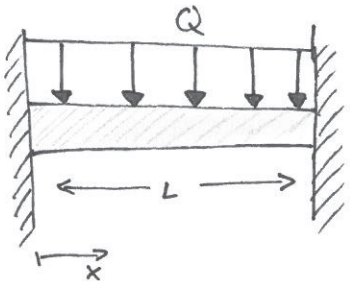
2.4.120 Givet Fast inspänd balk – Böjstyvhet EI



Sökt Bestäm reaktionsmoment
nedböjning i mitten

Lösning

1. Elastiska linjens ekvation

$$\left[EI\frac{\partial^4 w}{\partial x^4} = q(x) + \frac{d^2 M_T}{dx^2} \right] \Longrightarrow \underline{EI\frac{\partial^4 w}{\partial x^4} = q(x) = -\frac{Q}{L}}$$

2. Randvillkor

x Symmetri $M(x=0) = M(x=L)$

	Vänster ände	Höger ände
FS. 6.22	Fast inspänd, dvs ingen vinkelförändring → $\underline{w'(x=0)=0}$	... $\underline{w'(x=L)=0}$
FS. 6.23	Fast inspänd, dvs ingen förskjutning → $\underline{w(x=0)=0}$	... $\underline{w(x=L)=0}$

3. Beräkna

$$EI\cdot\frac{\partial^4 w(x)}{\partial x^4} = -\frac{Q}{L}$$

$$EI\cdot\frac{\partial^3 w(x)}{\partial x^3} = -\frac{Q}{L}\cdot x + C_1$$

$$EI\cdot\frac{\partial^2 w(x)}{\partial x^2} = -\frac{Q}{2L}\cdot x^2 + C_1\cdot x + C_2$$

$$EI\cdot\frac{\partial w(x)}{\partial x} = -\frac{Q}{6L}\cdot x^3 + \frac{C_1}{2}\cdot x^2 + C_2\cdot x + C_3$$

$$EI\cdot w(x) = -\frac{Q}{24L}\cdot x^4 + \frac{C_1}{6}\cdot x^3 + \frac{C_2}{2}\cdot x^2 + C_3\cdot x + C_4$$

$$\Longrightarrow w(x) = -\frac{Q}{24EIL}\cdot x^4 + \frac{C_1}{6EI}\cdot x^3 + \frac{C_2}{2EI}\cdot x^2 + C_3\cdot x + C_4$$

$\Longrightarrow$ Anv. R.V för att bestämma C_1 C_2 C_3 C_4

2.4.120

forts. 1

i) $w(x=0) = 0 = -\frac{Q}{24EIL}\cdot 0^4 + \frac{C_1}{6EI}\cdot 0^3 + \frac{C_2}{2EI}\cdot 0^2 + C_3\cdot 0 + C_4 \longrightarrow \underline{C_4 = 0}$

ii) $w'(x=0) = 0 = -\frac{Q}{6EIL}\cdot 0^3 + \frac{C_1}{2EI}\cdot 0^2 + \frac{C_2}{EI}\cdot 0 + C_3 \longrightarrow \underline{C_3 = 0}$

iii) $w(x=L) = 0 = -\frac{Q}{24EIL}\cdot L^4 + \frac{C_1}{6EI}\cdot L^3 + \frac{C_2}{2EI}\cdot L^2$

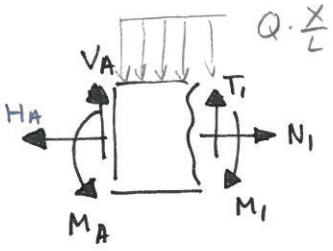
$\Longleftrightarrow C_1 = \frac{Q}{4} - \frac{3}{L}C_1$

iv) $w'(x=L) = 0 = -\frac{Q}{6EIL}\cdot L^3 + \frac{C_1}{2EI}\cdot L^2 + \frac{C_2}{EI}\cdot L$

$\Longleftrightarrow C_2 = \frac{QL}{6} - \frac{L}{2}\cdot C_1 \longrightarrow \begin{cases} C_1 = \frac{Q}{2} \\ C_2 = -\frac{QL}{12} \end{cases}$

$\therefore$ $w(x) = -\frac{Q}{24EIL}\cdot x^4 + \frac{Q}{12EI}\cdot x^3 - \frac{QL}{24EI}\cdot x^2$

4. Snitta



5. JMV

$A^B: M_A + x\cdot T_1 - Q\cdot\frac{x}{L}\cdot\frac{x}{2} - M_1 = 0$

$\Longrightarrow M_A = Q\cdot\frac{x^2}{2L} + M_1 - T_1\cdot x$

Då $x \to 0$ fås

$M_A = 0 + \left(-EI\frac{\partial^2 w(x=0)}{\partial x^2}\right) - \left(-EI\frac{\partial^3 w(x=0)}{\partial x^3}\right)\cdot \underbrace{x}_{=0}$

$= -EI\frac{\partial^2 w(x=0)}{\partial x^2} = \frac{Q}{2L}\cdot 0^2 - \frac{Q}{2}\cdot 0 + \frac{QL}{12}$

$M_A = \frac{QL}{12} = M_B$ pga symmetri

Utböjning vid $x = \frac{L}{2}$ fås genom...

$\delta_{mitt} = w(x=\frac{L}{2}) = -\frac{Q}{24EIL}\cdot\left(\frac{L}{2}\right)^4 + \frac{Q}{12EI}\cdot\left(\frac{L}{2}\right)^3 - \frac{QL}{24EI}\cdot\left(\frac{L}{2}\right)^2 = \ldots$

$\Longrightarrow \delta_{mitt} = -\frac{QL^3}{384EI}$