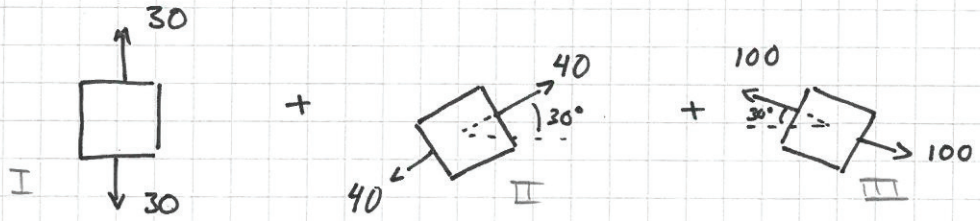
1.1.14 Spänningstillstånd i en punkt i en plåt

Givet



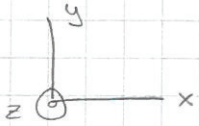
MPa

Sökt Beräkna huvudspänningar o deras riktningar

Lösning

1. Transformera till ett koordinatsystem

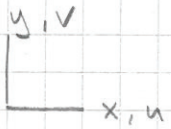
$\Rightarrow$ Väljer x-y-z enligt



FS 1.17 $\left[\sigma(\varphi) = \sigma_x \cos^2(\varphi) + \sigma_y \sin^2(\varphi) + 2\tau_{xy} \cdot \sin(\varphi)\cos(\varphi)\right]$

FS 1.18 $\left[\tau(\varphi) = \frac{\sigma_y - \sigma_x}{2} \cdot \sin(2\varphi) + \tau_{xy} \cdot \cos(2\varphi)\right]$

I)



Här är $\varphi = 0°$

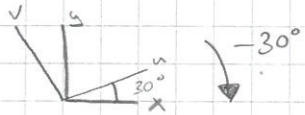
$\Rightarrow \sigma_x^I = 0$ MPa

$\sigma_y^I = 30$ MPa

$\tau_{xy}^I = 0$ MPa

Anm I en plåt råder typiskt plant sp. tillst. varpå $\begin{cases} \sigma_z = 0 \\ \tau_{yz} = 0 \\ \tau_{xz} = 0 \end{cases}$

II)



$\sigma_x^{II} = \sigma(\varphi = -30°) = 40 \cdot \cos^2(-30°) + 0 + 0$ ($\sigma_v = 0$, $\tau_{uv} = 0$)

$\Rightarrow \underline{\sigma_x^{II} = 30 \text{ MPa}}$

$\sigma_y^{II} = \sigma(\varphi = -30° + 90°) = 40 \cdot \cos(60°) + 0 + 0$

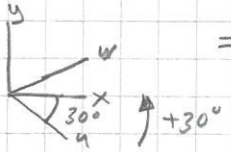
$\Rightarrow \underline{\sigma_y^{II} = 10 \text{ MPa}}$

Obs Anv. samma φ som för σ_x

$\tau_{xy}^{II} = \sigma(\varphi = -30°) = \frac{0 - 40}{2} \cdot \sin(2 \cdot -30°) + 0$ ($\tau_{uv} = 0$)

$\Rightarrow \underline{\tau_{xy}^{II} = 17.32 \text{ MPa}}$

III)



$\sigma_x^{III} = \sigma(\varphi = 30°) = 100 \cdot \cos^2(30) + 0 + 0$ ($\sigma_v = 0$, $\tau_{uv} = 0$)

$\underline{\sigma_x^{III} = 75 \text{ MPa}}$

$\sigma_y^{III} = \sigma(\varphi = 30 + 90°) = 100 \cdot \cos^2(120°) + 0 + 0$

$\underline{\sigma_y^{III} = 25 \text{ MPa}}$

$\underline{\tau_{xy}^{III} = \frac{0 - 100}{2} \cdot \sin(2 \cdot 30°) + 0 = -43.32 \text{ MPa}}$

1.1.14

forts. 1

2. Lägg ihop spänningstillstånd

$$\sigma_x^{tot} = \sigma_x^{I} + \sigma_x^{II} + \sigma_x^{III} = 0 + 30 + 75 = 105 \text{ MPa}$$

$$\sigma_y^{tot} = \sigma_y^{I} + \sigma_y^{II} + \sigma_y^{III} = 30 + 10 + 25 = 65 \text{ MPa}$$

$$\tau_{xy}^{tot} = \tau_{xy}^{I} + \tau_{xy}^{II} + \tau_{xy}^{III} = 0 + 17.32 + -43.32 = -26 \text{ MPa}$$

3. Mohr's cirkel

Obs. Plant spänningstillstånd $\rightarrow \sigma_3 = 0$; $\psi_3 = 0°$

FS. 1.19

$$\left[\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + R \right]$$

$$\left[\sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) - R \right]$$

$$\left[R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \right]$$

$$\left[\tan 2\psi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right]$$

Anm

se FS. s.7 för tolkning av vad R och ψ är...

$$\Rightarrow R = \sqrt{\left(\frac{105-65}{2}\right)^2 + (-26)^2} = 32,80 \text{ MPa}$$

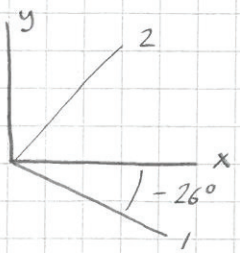
$$\Rightarrow \sigma_1 = \frac{1}{2}(105+65) + 32,80 = \underline{117,8 \text{ MPa}}$$

$$\sigma_2 = \frac{1}{2}(105+65) - 32,80 = \underline{52,2 \text{ MPa}}$$

Beräkna huvudspänningsriktning...

$$\psi_1 = \frac{1}{2}\tan^{-1}\left(\frac{2 \cdot -26}{105 - 65}\right) \cong \underline{-26,2°}$$

dvs



$$\Rightarrow \underline{\psi_2 = -26,2° + 90° = 63,8°}$$

SVAR

$\sigma_1 = 117,8$ MPa

$\sigma_2 = 52,2$ MPa

($\sigma_3 = 0$ MPa

$\psi_1 = -26,2°$

$\psi_2 = 63,8°$

$\psi_3 = 0°$)