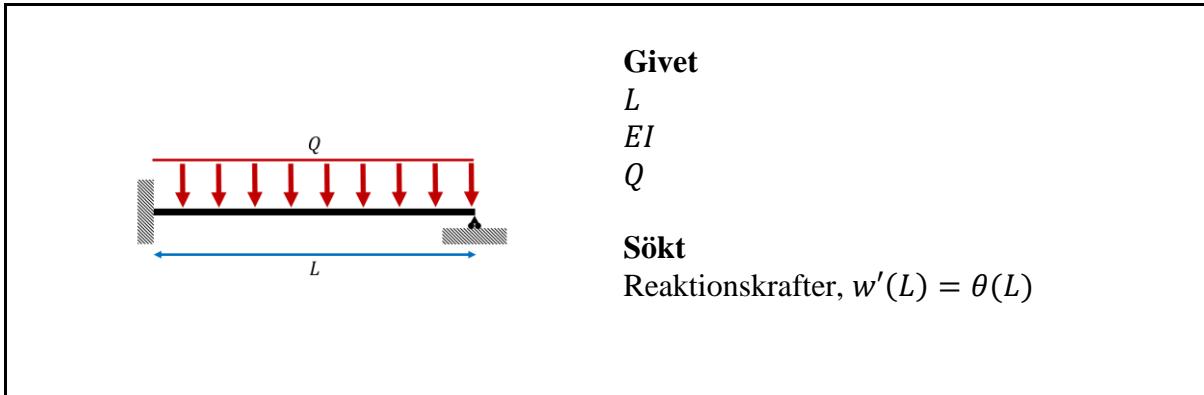


2.4.117



Lösning

Använd elastiska linjens ekvation

{FS 6.20}

$$[EIw''(x)]'' = q(x)$$

Integrera 4 gånger för att få utböjning $w(x)$

$$EIw'''(x) = -\frac{Q}{L}x + C_1$$

$$EIw''(x) = -\frac{Qx^2}{2L} + C_1x + C_2$$

$$EIw'(x) = -\frac{Qx^3}{6L} + \frac{C_1x^2}{2} + C_2x + C_3$$

$$EIw(x) = -\frac{Qx^4}{24L} + \frac{C_1x^3}{6} + \frac{C_2x^2}{2} + C_3x + C_4$$

C_1, C_2, C_3, C_4 måste bestämmas med hjälp av randvillkor.

(I)

$$w(0) = 0$$

(II)

$$w(L) = 0$$

(III)

$$EIw''(0) = 0$$

(IV)

$$w'(0) = 0$$

Lös ut konstanterna

Villkor (I)

$$w(0) = 0 = -\frac{Q0^4}{24L} + \frac{C_10^3}{6} + \frac{C_20^2}{2} + C_30 + C_4$$

$$C_4 = 0$$

Villkor (IV)

$$w'(0) = 0 = -\frac{Q0^4}{24L} + \frac{C_10^2}{2} + C_20 + C_3$$

$$C_3 = 0$$

Villkor (III)

$$-EIw''(L) = 0 = -\frac{QL^2}{2L} + C_1L + C_2$$

$$C_2 = \frac{QL}{2} - C_1L$$

Villkor (II)

$$\begin{aligned} w(L) = 0 &= -\frac{Q\textcolor{red}{L}^4}{24\textcolor{red}{L}} + \frac{C_1\textcolor{red}{L}^3}{6} + \frac{\left(\frac{Q\textcolor{red}{L}}{2} - C_1\textcolor{red}{L}\right)L^2}{2} \\ 0 &= -\frac{Q}{24} + \frac{C_1}{6} + \frac{Q}{4} - \frac{C_1}{2} \\ \frac{C_1}{3} &= \frac{5Q}{24} \\ C_1 &= \frac{5Q}{8} \end{aligned}$$

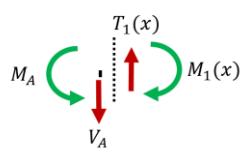
Vilket ger

$$C_2 = \frac{QL}{2} - \frac{5QL}{8} = -\frac{QL}{8}$$

Sätt ihop allt

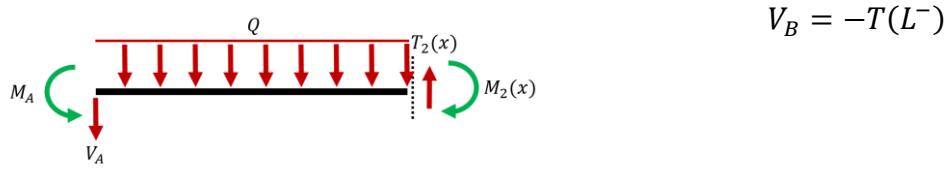
$$w(x) = \frac{1}{EI} \left(-\frac{Qx^4}{24L} + \left(\frac{5Q}{8}\right)\frac{x^3}{6} - \left(\frac{QL}{8}\right)\frac{x^2}{2} \right)$$

Nu kan vi ta ut alla de värden vi vill ha



$$\begin{aligned}\uparrow: \quad & -V_A + T_1(x) = 0 \\ \tilde{x}: \quad & M_A + V_A x - M_1(x) = 0\end{aligned}$$

$$\begin{aligned}V_A &= -T_1(0^+) \\ M_A &= M_1(0^+)\end{aligned}$$



$$V_B = -T(L^-)$$

$$M_A = M_1(0^+) = -EIw''(0) = -\left(-\frac{QL}{8}\right) = \frac{QL}{8}$$

$$V_A = EIw'''(0) = -\frac{5Q}{8}$$

$$V_B = EIw'''(L) = -\frac{Q}{L}L + \frac{15Q}{24}$$

$$V_B = -\frac{3Q}{8}$$

$$\theta(L) = \frac{1}{EI} \left(-\frac{QL^3}{6L} + \frac{15Q}{24} \cdot \frac{L^2}{2} - \frac{QL}{8} \cdot L \right)$$

$$\theta(L) = \frac{QL^2}{EI} \left(-\frac{1}{6} + \frac{5}{16} - \frac{1}{8} \right)$$

$$\theta(L) = \frac{QL^2}{48EI}$$