Sound absorption by turbulence in pipe flow

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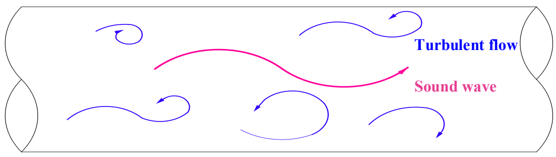
Abstract

We developed a frequency dependent eddy-viscosity model for sound waves propagation in fully developed turbulent pipe flow with low Mach number. The model is applied to predict the wave damping caused by turbulent absorption. Unlike other quasi-static eddy viscosity models, this new formulation takes into account the nonequilibrium effects in sound-turbulence interaction. The new model shows that the turbulent flow in such interaction process acts as viscoelastic fluid to the acoustic waves, namely the stress does not only depend on the present straining by the sound waves, but also has the "memory" of the past of it. The predicted acoustic damping shows good agreement with experimental data.

Objective



When sound waves propagate in turbulent pipe flow, turbulent stress can result in extra acoustic attenuation, and becomes more dominant at sufficient low frequencies. If we decompose the flow field into mean, turbulent and acoustic quantities, we can define the turbulent stress acting on sound waves as the difference between the phase and time averages of Reynolds stress of the background turbulence, i.e.,

$$\tilde{r}_{ij} = \left\langle u_i' u_j' \right\rangle - \overline{u_i' u_j'}$$

Our aim is to derive an analytical model for this perturbation Reynolds stress so to calculate the attenuation of sound due to turbulent absorption.

Method

A quasi-linear model for the Reynolds stress anisotropy is used, i.e.,

$$\frac{\partial \overline{\tau}_{ij}}{\partial t} + \overline{u}_k \frac{\partial \overline{\tau}_{ij}}{\partial x_k} = -C_1 \frac{\overline{\varepsilon}}{\overline{k}} \overline{\tau}_{ij} + \left(C_2 - \frac{4}{3} \right) \overline{k}\, \overline{S}_{ij}$$

where the time averaged dimensional Reynolds stress anisotropy is defined by

$$\overline{\tau}_{ij} = \overline{u_i' u_j'} - \frac{2}{3} \overline{k} \delta_{ij}$$

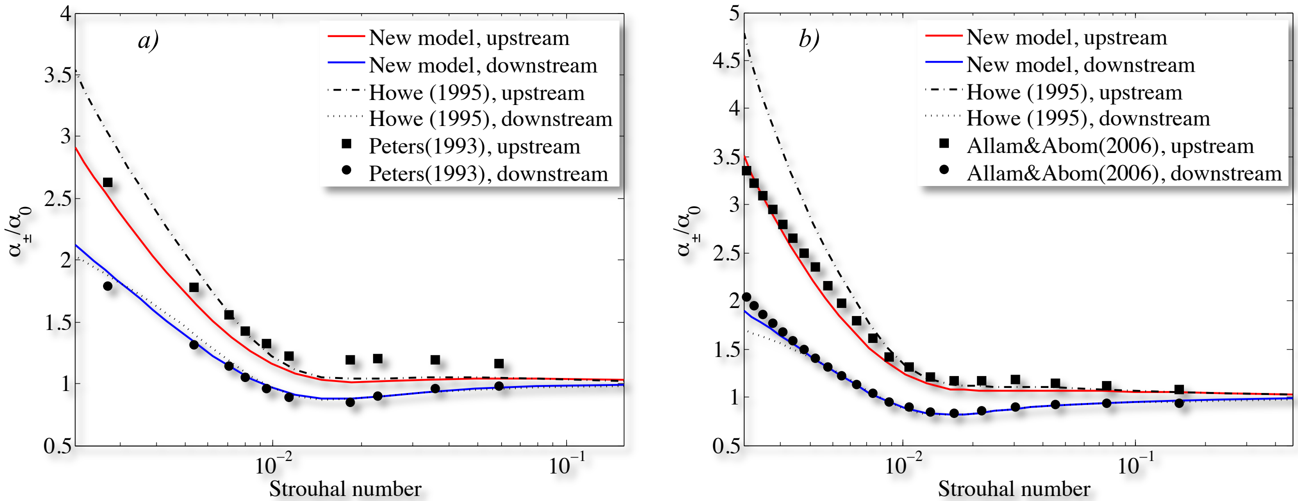
C_1 and C_2 are constants, $\overline{\varepsilon}$ and $\overline{k}$ are turbulent dissipation and kinetic energy respectively, and $\overline{S}_{ij}$ is shear strain rate of the mean flow.
Applying this quasi-linear model to the phase average of Reynolds stress anisotropy, and some further assumptions, we derived an model for $\tilde{r}_{ij}$ in the frequency domain:

$$\tilde{r}_{ij} = \frac{1}{1 + i\omega t^*} \cdot \left(-4 \nu_T \tilde{S}_{ij} \right)$$

where t^* is turbulent relaxation time scale, ν_T is an eddy viscosity model in equilibrium state, and $\tilde{S}_{ij}$ is shear strain rate of sound wave.

Results

The sound attenuation is calculated numerically in Matlab by solving the wave equations in frequency domain, where the new model for the Reynolds stress is used. The calculated damping α (imaginary part of the complex wavenumber), normalized by the damping in a quiescent fluid, is plotted against the Strouhal number.
The plus/minus sign denotes down/up-stream propagating waves. The results are compared with the experimental data by Peters et al. (1993) and Allam and Åbom (2006), as well as the semi-empirical model proposed by Howe (1995).



Relation between damping and Strouhal number for Helmholtz number a) 0.024, and b) 0.08.