Propagation of higher order acoustic modes in turbulent internal flows

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Introduction

The subject of plane wave propagation in turbulent internal flows has been investigated both theoretically and experimentally since the 1970s, and various theoretical models for computing the wave propagation constants are available. However, there is a lack of knowledge on how higher order acoustic modes propagate in turbulent flows, and such knowledge is becoming more important in nowadays-industrial applications. For example, the size of a turbofan aeroengine determines that the higher modes can be excited even with relatively low frequency sound source, which necessitates the need for taking these modes into account in the liner design process. The present work aims for providing a sound-turbulent interaction model in the presence of the higher order modes. In addition, an efficient numerical scheme for computing the propagation constants for the higher order modes are proposed, where the mean flow convection, refraction, and the turbulent absorption effects are all taken into account.

Governing equations

The numerical formulation is based on the compressible linearized Navier-Stokes equations (LNSE), i.e., the conservation of mass, the conservation of momentum, and the conservation of (internal) energy. A mean flow velocity profile is computed from a different incompressible solver, which is used in the LNSE to count for the mean flow convection and refraction effects. In addition, the turbulent stresses and heat fluxes are also included in the LNSE, in the preliminary results shown here, however, these two quantities are set to be zero, which is the so-called "quasi-laminar" treatment.

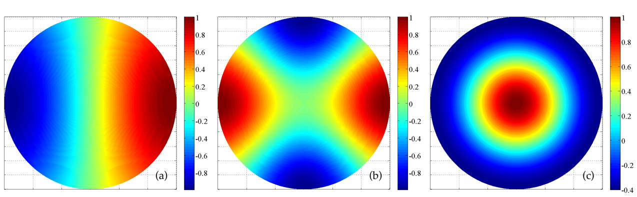

Fig. 1. Demonstration of acoustic pressure field of (a) the first non-axisymmetric mode, (b) the second non-axisymmetric mode, and (c) the second axisymmetric mode, in a circular pipe. The mode shapes are computed from the LNSE, without mean flow.

Methodology

For an axis-symmetric circular duct, each mode should be periodic in the azimuthal (θ) direction, therefore the ansatz of the solution reads

$$\tilde{\varphi} = \varphi \exp(\mathrm{i}\omega t - \mathrm{i}kx + \mathrm{i}m\theta), \; m = 0, \pm 1, \pm 2... \quad (1)$$

Then LNSE turn into an eigenvalue problem, with respect to the eigenvalue k, in the form

$$(\mathbb{A} + k\mathbb{B})\varphi = 0, \quad (2)$$

With proper boundary conditions, the wavenumber, i.e., the propagation constant, k, can be obtained for each mode.

Preliminary results for the first non-axisymmetric ($m > 0$) modes propagation in hard-wall pipe

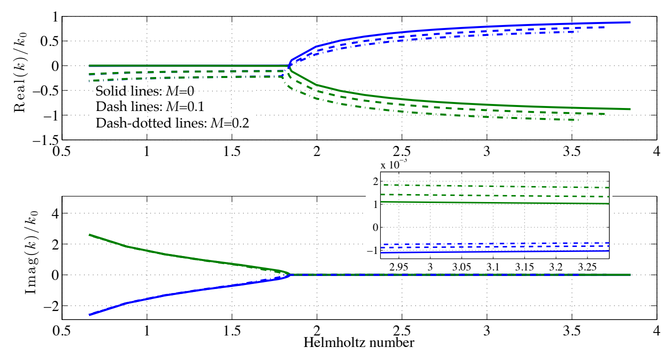

Fig. 2. Dispersion relation for the first non-axisymmetric modes, showing the mean flow convection effects, and possibly the refraction effects, on the phase velocity and attenuation of the sound wave. Since the attenuation factors (the imaginary part of the wavenumber) of the propagated modes have small values, part of the graph for Imag(k) is zoomed in. The blue and green curves denote the downstream and upstream propagating waves, respectively.

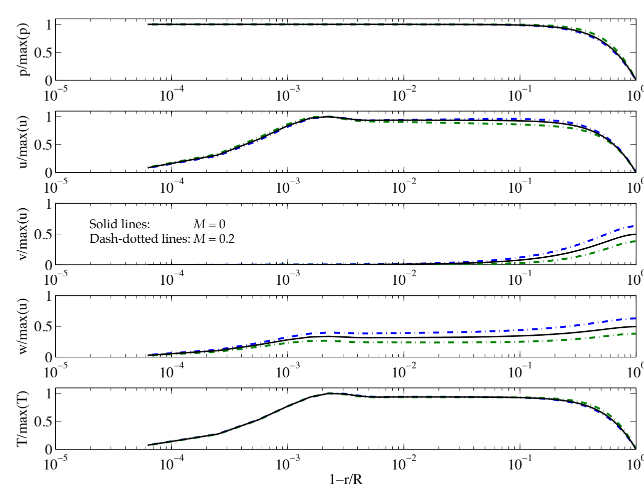

Fig. 3. Mode shapes for the first non-axisymmetric modes, with Helmholtz number 3.5, showing the mean flow refraction effects on the sound wave mode shapes. The blue and green curves denote the downstream and upstream propagating waves, respectively, and u, v, and w here denote the axial, radial, and azimuthal components of the perturbation velocity, respectively.