

**SF3674 DIFFERENTIAL GEOMETRY,  
GRADUATE COURSE, FALL 2016,  
READING INSTRUCTIONS AND EXERCISES**

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LECTURE 1, TUESDAY SEPTEMBER 6

**Reading instructions.** A basic familiarity with smooth manifolds is part of the background knowledge for this course. A thorough treatment of this material can be found in Chapters 1–11 of Lee’s book [2]. A more concise presentation is in Chapters 1–2 of O’Neill’s book [3]. An even shorter introduction can be found in Chapter 1 of Bär’s lecture notes [1].

In the first lecture we covered the Levi-Civita connection, parallel translation, and geodesics corresponding to Chapter 3, pages 54–73 in O’Neill [3]. This material can also be found in Chapters 1–3 of the lecture notes [4], and in Chapter 2 of [1].

**Exercises.**

- (1) (Background on smooth manifolds: O’Neill [3] problems 1.14 (p.33); 2.3, 2.6 (p.52).)
- (2) The Levi-Civita connection and parallel translation: O’Neill [3] problems 3.2, 3.3, 3.17, 3.19 (p.92-96).
- (3) Geodesics in the hyperbolic plane.
  - a) Calculate the Christoffel symbols of  $g_{\mathbb{H}} = y^{-2}(dx^2 + dy^2)$  defined on  $\mathbb{H} = \{(x, y) \in \mathbb{R}^2 \mid y > 0\}$  and find at least one geodesic.
  - b) Prove that translations in  $x$  are isometries; that dilations  $(x, y) \mapsto c(x, y)$ ,  $0 < c \in \mathbb{R}$ , are isometries; and that the inversion  $(x, y) \mapsto \frac{(x, y)}{x^2 + y^2}$  is an isometry.
  - c) Prove that isometries map geodesics to geodesics, and use a) and b) to construct all geodesics on  $(\mathbb{H}, g_{\mathbb{H}})$ .

REFERENCES

1. Christian Bär, *Differential geometry, summer term 2013*, [http://www.math.uni-potsdam.de/fileadmin/user\\_upload/Prof-Geometrie/Dokumente/Lehre/Lehrmaterialien/skript-DiffGeo-engl.pdf](http://www.math.uni-potsdam.de/fileadmin/user_upload/Prof-Geometrie/Dokumente/Lehre/Lehrmaterialien/skript-DiffGeo-engl.pdf).
2. John M. Lee, *Introduction to smooth manifolds*, second ed., Graduate Texts in Mathematics, vol. 218, Springer, New York, 2013. MR 2954043
3. Barrett O’Neill, *Semi-Riemannian geometry*, Pure and Applied Mathematics, vol. 103, Academic Press, Inc. [Harcourt Brace Jovanovich, Publishers], New York, 1983, With applications to relativity. MR 719023
4. Hans Ringström, *A brief introduction to semi-riemannian geometry and general relativity*, [https://people.kth.se/~hansr/Semi-Riemannian\\_Geometry.pdf](https://people.kth.se/~hansr/Semi-Riemannian_Geometry.pdf).