# SF3674 DIFFERENTIAL GEOMETRY, GRADUATE COURSE, FALL 2016, READING INSTRUCTIONS AND EXERCISES

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## Lecture 3, Tuesday September 20

**Reading instructions.** The central object of this lecture is the second fundamental form of a semi-Riemannian submanifold. It describes how a semi-Riemannian submanifold is shaped (or "curves") in an ambient semi-Riemannian manifold. Using it, the curvature of the ambient manifold can be related to that of the submanifold (via the Gauss and Codazzi equations). There is also a relation concerning the acceleration of a curve (as viewed in the submanifold/ambient manifold). Using these relations, the curvature and geodesics of the sphere, the hyperbolic space, and more general semi-Riemannian hypersurfaces called hyperquadrics can be computed.

The main material for the lecture is Chapter 4 of O'Neill's book [2]. However, Chapter 4 of Bär's lecture notes [1] is also highly recommended.

**Exercises.**

(1) O'Neill [2] problems 4.1, 4.2, 4.4 (pp. 123–125).

(2) Let $H^n_\nu(r)$ and $S^n_\nu(r)$ be the pseudohyperbolic space and pseudosphere respectively, cf. Definition 23, p. 110 of O'Neill's book. For a general semi-Riemannian manifold $(M, g)$, let

$$G = \mathrm{Ric} - \frac{1}{2} S g,$$

where Ric is the Ricci tensor and $S$ is the scalar curvature of $(M, g)$. $G$ is called the *Einstein tensor*. Compute the Einstein tensor of $H^4_1(r)$ and of $S^4_1(r)$.

Note that $S^4_1(r)$ is *de Sitter space*, a Lorentz manifold of interest in general relativity. The universal covering space of $H^4_1(r)$ is called *anti de Sitter* space.

## References

[1] Christian Bär. Differential geometry, summer term 2013. `http://www.math.uni-potsdam.de/fileadmin/user_upload/Prof-Geometrie/Dokumente/Lehre/Lehrmaterialien/skript-DiffGeo-engl.pdf`.

[2] Barrett O'Neill. *Semi-Riemannian geometry*, volume 103 of *Pure and Applied Mathematics*. Academic Press, Inc. [Harcourt Brace Jovanovich, Publishers], New York, 1983. With applications to relativity.