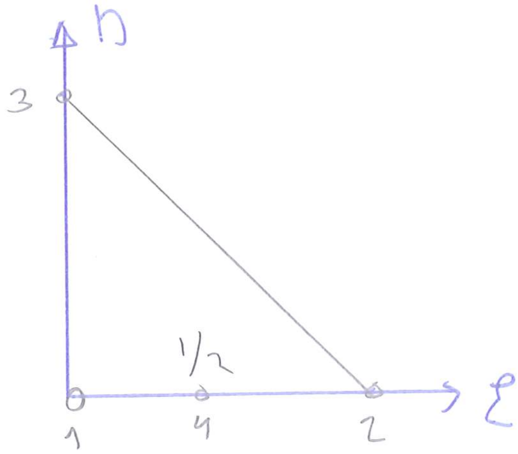
6.2



Transitionselement

$N_1 = 1 - \xi[3 - 2(\xi + \eta)] - \eta$

$N_2 = \xi[2(\xi + \eta) - 1)$

$N_3 = \eta$

Bestäm N_4

Använd $\Sigma N = 1$

$N_4 = 1 - N_1 - N_2 - N_3 = 1 - (1 - \xi[3 - 2(\xi + \eta)] - \eta) -$

$- (\xi(2(\xi + \eta) - 1) - \eta =$

$= 3\xi - 2\xi(\xi + \eta) + \eta - 2\xi(\xi + \eta) + \xi - \eta$

$= 4\xi - 4\xi(\xi + \eta) = 4\xi(1 - \xi - \eta)$

Krav: N_4 i nod $4 = 1$ N_4 i nod $1,2,3 = 0$

nod 1,3 $\xi = 0 \Rightarrow N_4 = 0$ nod 2: $\eta = 0$, $\xi = 1$

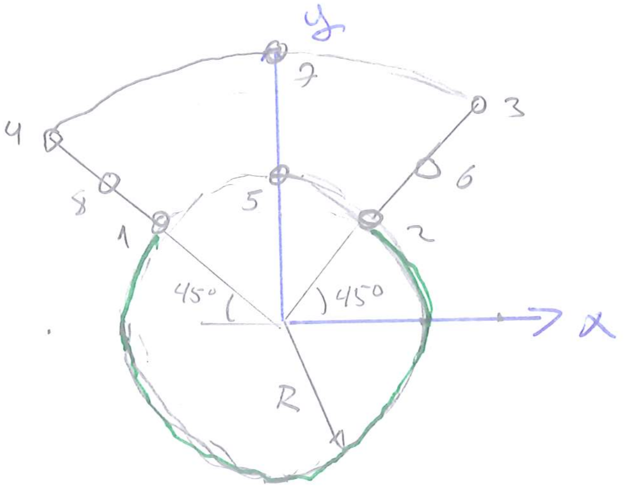
$\Rightarrow N_4 = 0$

nod 4: $\eta = 0$ $\xi = 1/2$

$N_4 = 2 \cdot (1 - \frac{1}{2} - 0) = 1$ OK!

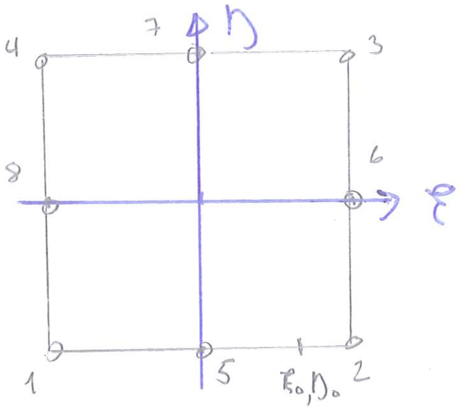
6.3

Skiva med hål, radie R
8 noders element



Bestäm avståndet från hålets mittpunkt till punkten x_0, y_0 som ges av $\xi_0 = 1/\sqrt{2}$, $\eta_0 = -1$.

Iso-parametriskt element



Koordinaterna avbildas med formfunktionerna

$$x = \sum x_i N_i$$

$$y = \sum y_i N_i$$

$\eta = -1 \Rightarrow \quad N_3 = N_4 = N_6 = N_7 = N_8 = 0$

$$x_0 = -\frac{R}{\sqrt{2}} N_1(\xi = 1/\sqrt{2}, \eta = -1) + 0 \cdot N_5 + \frac{R}{\sqrt{2}} N_2(\xi = 1/\sqrt{2}, \eta = -1)$$

$$y_0 = \frac{R}{\sqrt{2}} N_1(1/\sqrt{2}, -1) + R\, N_5(1/\sqrt{2}, -1) + \frac{R}{\sqrt{2}} (1/\sqrt{2}, -1)$$

$$x_0 = R/2 \qquad y_0 = R\,\frac{2+\sqrt{2}}{4}$$

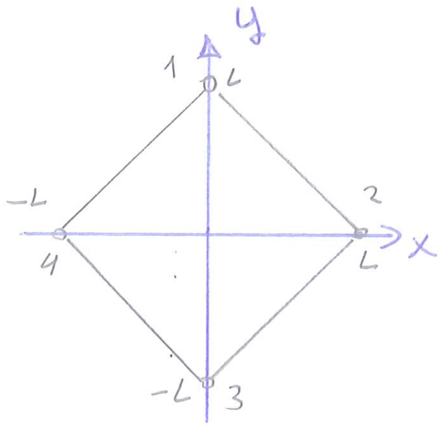
Avståndet från centrum: $\sqrt{x_0^2 + y_0^2} = R\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{2+\sqrt{2}}{4}\right)^2}$

$$= 0.989\,R$$

6.4

Iso-parametriskt element med omvänd nodnumrering

Beräkna determinanten av jacobi-matrisen



Jacobi-matrisen ges av

$$\underset{\sim}{J} = \begin{bmatrix} \partial x/\partial \xi & \partial y/\partial \xi \\ \partial x/\partial \eta & \partial y/\partial \eta \end{bmatrix} \qquad x = \sum_i x_i N_i \qquad y = \sum y_i N_i$$

Alltså med $x_1 = x_3 = 0$ & $y_2 = y_4 = 0$

$$\frac{\partial x}{\partial \xi} = x_2 \frac{dN_2}{\partial \xi} + x_4 \frac{\partial N_4}{\partial \xi} \qquad \frac{\partial y}{\partial \xi} = y_1 \frac{\partial N_1}{\partial \xi} + y_3 \frac{\partial N_3}{\partial \xi}$$

$$\frac{\partial x}{\partial \eta} = x_2 \frac{dN_2}{\partial \eta} + x_4 \frac{\partial N_4}{\partial \eta} \qquad \frac{\partial y}{\partial \eta} = y_1 \frac{dN_1}{\partial \eta} + y_3 \frac{\partial N_3}{\partial \eta}$$

$$\frac{\partial x}{\partial \xi} = \frac{L}{4} \cdot (1-\eta) + -\frac{L}{4} \cdot (1+\eta) = \frac{L}{2}$$

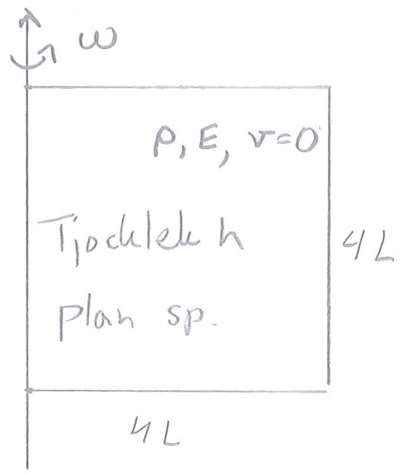
$$\frac{\partial y}{\partial \xi} = \frac{L}{4} \cdot -(1-\eta) + \frac{-L}{4}(1+\eta) = -\frac{L}{2}$$

Pss. $\frac{\partial x}{\partial \eta} = -\frac{L}{2}$

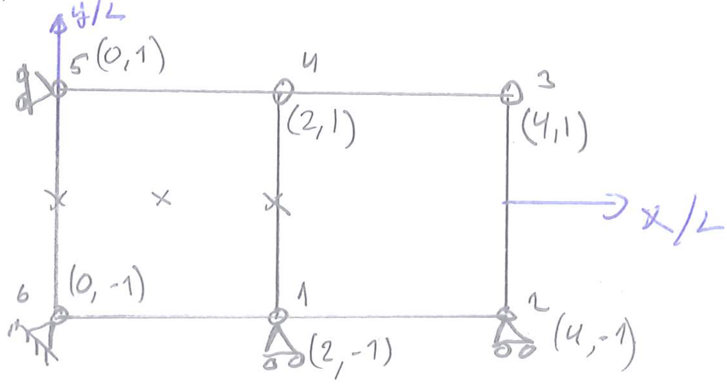
$\frac{\partial y}{\partial \eta} = -\frac{L}{2}$

$$\underset{\sim}{J} = \frac{L}{2} \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix} \qquad \det(\underset{\sim}{J}) = \left(\frac{L}{2}\right)^2 \cdot (1 \cdot (-1) - (-1)^2 = \underline{\underline{-\frac{L^2}{2}}}$$

6.19



FEM - modell



rotationen ger upphov till volymskraft $K_x = \rho \omega^2 x$
$K_y = 0$

a) Bestäm bidraget till nodlastvektorn för element 2 från volymslasten! Ledning $x = (3+\xi)L$
$y = \eta L$

b) Beräkna spänningarna i punkterna

$(0,0)$, $(L,0)$ o $(2L,0)$

$$\sigma_{xx}(x) = \frac{\rho\omega^2 L^2}{2}\left[16 - \left(\frac{x}{L}\right)^2\right] \quad \text{exakt}$$

$$\underline{D} = \frac{4}{3}\frac{\rho\omega^2 L^3}{E}\begin{bmatrix} 11 & 0 & 16 & 0 & 16 & 11 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

Ledning Jacobi-matrisen är $\underline{J} = L\underline{I}$

a) $$\underline{F}_b^{(2)} = \int_{V_e} \underline{N}^T \underline{K}\, dV = h \int_{-1}^{1}\int_{-1}^{1} \underline{N}^T \underline{K} \det(\underline{J})\, d\xi\, d\eta$$

med $\underline{J} = \begin{bmatrix} L & 0 \\ 0 & L \end{bmatrix}$ så är $\det(\underline{J}) = L^2$

$$\underline{K} = \begin{bmatrix} \rho \omega^2 (3+\xi) L \\ 0 \end{bmatrix}$$

$$\underline{F}_b^{(2)} = h\rho\omega^2 L^3 \int_{-1}^{1}\int_{-1}^{1} \begin{bmatrix} N_1 (3+\xi) \\ 0 \\ N_2 (3+\xi) \\ 0 \\ N_3 (3+\xi) \\ 0 \\ N_4 (3+\xi) \\ 0 \end{bmatrix} d\xi \, d\eta =$$

= Analytisk lösning eller 2×2 Gauss =

$$\underline{F}_b^{(2)} = \frac{h\rho\omega^2 L^3}{3} [8, 0, 10, 0, 10, 0, 8, 0]^T$$

b) Spänningarna i ett element ges av

$$\underline{\sigma} = \underline{C}\,\underline{\varepsilon} = \underline{C}\,\underline{B}\,\underline{d}^e$$

Plan sp. och $\nu = 0$ ger

$$\underline{C} = E \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \qquad \underline{B} = [\underline{B}_1\ \underline{B}_2\ \underline{B}_3\ \underline{B}_4]$$

$$\underline{B}_i = \begin{bmatrix} \partial N_i/\partial x & 0 \\ 0 & \partial N_i/\partial y \\ \partial N_i/\partial y & \partial N_i/\partial x \end{bmatrix} = \left\{ \begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix} = \underline{J}^{-1} \begin{bmatrix} \partial N_i/\partial \xi \\ \partial N_i/\partial \eta \end{bmatrix} \right\} =$$

$$= \frac{1}{L} \begin{bmatrix} \partial N_i/\partial \xi & 0 \\ 0 & \partial N_i/\partial \eta \\ \partial N_i/\partial \eta & \partial N_i/\partial \xi \end{bmatrix}$$

Nodförskjutningsvektorn för elem (1)

$$\underset{\sim}{d}_e^{(1)} = [D_{6x}\ D_{6y},\ D_{1x}, D_{1y},\ D_{4x}, D_{4y},\ D_{5x}\ D_{5y}]^T =$$

$$= \frac{4}{3}\frac{\rho\omega^2 L^3}{E}[0, 0, 11, 0, 11, 0, 0, 0]^T$$

$$\underset{\sim}{\varepsilon}^{(1)} = [\underset{\sim}{B}_1\ \underset{\sim}{B}_2\ B_3\ B_4]\underset{\sim}{d}_e^{(1)} = \frac{4}{3}\frac{\rho\omega^2 L^3}{E}\left[\underset{\sim}{B}_2\begin{bmatrix}11\\0\end{bmatrix} + B_3\begin{bmatrix}11\\0\end{bmatrix}\right] =$$

$$= \frac{4}{3}\frac{\rho\omega^2 L^3}{E}\frac{1}{L}\left(11\begin{bmatrix}\partial N_2/\partial\xi\\0\\\partial N_2/\partial\eta\end{bmatrix} + 11\begin{bmatrix}\partial N_3/\partial\xi\\0\\\partial N_3/\partial\eta\end{bmatrix}\right) =$$

$$= \frac{11}{3}\frac{\rho\omega^2 L^2}{E}\begin{pmatrix}(1-\eta) + (1+\eta)\\0 + 0\\-(1+\xi) + (1+\xi)\end{pmatrix} = \frac{11}{3}\frac{\rho\omega^2 L^2}{E}\begin{bmatrix}2\\0\\0\end{bmatrix} =$$

$$= \frac{22}{3}\frac{\rho\omega^2 L^2}{E}\begin{bmatrix}1\\0\\0\end{bmatrix}$$ överallt i element (1)

Spänningen $\underset{\sim}{\sigma} = \underset{\sim}{C}\,\underset{\sim}{\varepsilon} = \frac{22}{3}\rho\omega L^2\,[1, 0, 0]^T$

Felet: $\dfrac{\sigma_{xx}^{FEM} - \sigma_{xx}^{exakt}}{\sigma_{xx}^{exakt}}$ är

$x=0,\ y=0$ -8.3%

$x=L,\ y=0$ 2%

$x=2L,\ y=0$ 22%