Transitional Behavior of Millimeter Wave Networks

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Lemma 1: Given sector s, $1 \leq s \leq k-1$, the number of interferers $n_I \geq 1$, and the number of obstacles $n_o \geq 1$, randomly deployed on the sector, the probability of having at least one LoS interferer is given by Equation (9) on the top of page 2.

Proof: A proof is given in Appendix B.

Appendix A: Characterization of the Interference Range

In the interference range model, a packet loss occurs if there is some interfering device inside the interference range, that is, the maximum distance an interferer can be from the receiver and still causes a collision. Let $d_{\max}$ be the interference range. Let p, ν, and f be the signal transmission power, phase speed, and frequency, respectively. In free space, the phase speed of electromagnetic signals is almost the speed of light. The power that the typical receiver receives from a transmitter, with LoS condition and without deafness, located at distance d, is

$$p\left(\frac{2\pi-(2\pi-\theta)\epsilon}{\theta}\right)^2\left(\frac{\nu}{4\pi d f}\right)^\alpha , \tag{1}$$

where α is the path-loss exponent. Let β be the minimum SNR at the typical receiver due to transmission of an interferer that causes a strong interference or collision. Let σ be the noise power. The typical receiver can receive strong interference from that transmitter at maximum distance $d_{\max}$, where

$$d_{\max} = \frac{\nu}{4\pi f}\left(\frac{p\big(2\pi-(2\pi-\theta)\,\epsilon\big)^2}{\sigma\beta\theta^2}\right)^{1/\alpha} , \tag{2}$$

which can be reduced to

$$d_{\max} = \frac{\nu}{4\pi f}\left(\frac{4\pi^2 p}{\sigma\beta\theta^2}\right)^{1/\alpha} , \tag{3}$$

as $\epsilon \to 0$. For sake of simplicity, we assume that the transmission range and the interference range are equal. However, the analyses provided in this paper can be easily extended for the general interference range model and also for the protocol model [1]. Further, the channel model affects only the interference range. Hence, to consider a fading channel, we should only replace deterministic interference range $d_{\max}$ with a random variable following a given fading distribution. Then, we can apply the framework developed in this paper to compute the collision probability and MAC throughput.

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Appendix B: LoS Interference Given $n \geq 1$ and $m \geq 1$

In this appendix, we find probability of having at least one LoS interferer given the number of interferers $n_I \geq 1$ and the number of obstacles $n_o \geq 1$. We have the following Lemma:

Lemma 2: Let $\{x_1, x_2, \ldots, x_{n_I}\}$ be a set of n_I random variables, where n_I is a zero-truncated Poisson random variable with density λ_I, and x_is are uniformly distributed in $[0, d_{\max}]$. Define $X = \min\{x_1, x_2, \ldots, x_{n_I}\}$. Given $n_I = n \geq 1$, the joint PDF of X and n_I is given by Equation (4) on the top of page 2.

Proof: Given $n_I = n \geq 1$, cumulative distribution function (CDF) of the minimum of X is

$$1-\left(1-F\{x\}\right)^n , \tag{5}$$

where $F\{x\}$ is the CDF of x_i. Taking derivative of (5), the conditional PDF of X given $n_I = n \geq 1$ is

$$f_{X|n_I}\left(X=x|n_I=n, n\geq 1\right) = \frac{n}{d_{\max}}\left(1-\frac{x}{d_{\max}}\right)^{n-1} . \tag{6}$$

$n_I = n \geq 1$ is a random variable with zero-truncated Poisson distribution, thus [2]

$$\Pr\left[n_I = n|n\geq 1\right] = \frac{e^{-\lambda_I}}{1-e^{-\lambda_I}}\frac{\lambda_I^n}{n!} .$$

Therefore, we have (7). This concludes the proof. ■

Due to mutual independency of blockage and interferer processes and using Lemma 2, we have (8). Applying Lemma 2 to $f_{X_s,n_I}\left(X_s=x, n_I=n|n\geq 1\right)$ and $f_{Y_s,n_o}\left(Y_s=y, n_o=m|m\geq 1\right)$, the first part of Lemma 1 is straightforward. All we need to do is plugging the effective densities $\lambda_I A_{d_{\max}}$ and $\lambda_o A_{d_{\max}}$ into (4).

The next step is finding the probability of having at least one LoS interferer given $n_I \geq 1, n_o \geq 1$, which we denote by $\mathbb{T}$. We have (9), where $(\star)$ follows from the Taylor series of the exponential function. This completes the proof of Lemma 1. ■

References

[1] A. Iyer, C. Rosenberg, and A. Karnik, "What is the right model for wireless channel interference?" *IEEE Trans. Wireless Commun.*, vol. 8, no. 5, pp. 2662–2671, May 2009.

[2] N. L. Johnson, A. W. Kemp, and S. Kotz, *Univariate discrete distributions*. John Wiley & Sons, 2005, vol. 444.

$$f_{X,n_I}\left(X=x,n_I=n|n\geq 1\right)=\frac{n}{d_{\max}}\left(1-\frac{x}{d_{\max}}\right)^{n-1}\frac{e^{-\lambda_I}}{1-e^{-\lambda_I}}\frac{\lambda_I^n}{n!} \tag{4}$$

$$\begin{aligned} f_{X,n_I}\left(X=x,n_I=n|n\geq 1\right) &= f_{X|n_I}\left(X=x|n_I=n,n\geq 1\right)\Pr\left[n_I=n|n\geq 1\right] \\ &= \frac{n}{d_{\max}}\left(1-\frac{x}{d_{\max}}\right)^{n-1}\frac{e^{-\lambda_I}}{1-e^{-\lambda_I}}\frac{\lambda_I^n}{n!} \end{aligned} \tag{7}$$

$$f_{X_s,Y_s,n_I,n_o}\left(X_s=x,Y_s=y,n_I=n,n_o=m|n,m\geq 1\right)=f_{X_s,n_I}\left(X_s=x,n_I=n|n\geq 1\right)f_{Y_s,n_o}\left(Y_s=y,n_o=m|m\geq 1\right) \tag{8}$$

$$\begin{aligned} \mathbb{T} &= \Pr[x<y|n\geq 1,m\geq 1] \\ &= \int_{y=0}^{d_{\max}}\int_{x=0}^{y}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} f_{X_s,n_I}\left(X_s=x,n_I=n|n\geq 1\right)f_{Y_s,n_o}\left(Y_s=y,n_o=m|m\geq 1\right)\mathrm{d}x\mathrm{d}y \\ &\overset{(4)}{=} \frac{\lambda_I\lambda_o A_{d_{\max}}^2}{d_{\max}^2}\int_{y=0}^{d_{\max}}\int_{x=0}^{y}\frac{e^{-(\lambda_I+\lambda_o)A_{d_{\max}}}}{\left(1-e^{-\lambda_I A_{d_{\max}}}\right)\left(1-e^{-\lambda_o A_{d_{\max}}}\right)} \\ &\qquad\times\sum_{n=1}^{\infty}\frac{\left(\left(1-\frac{x}{d_{\max}}\right)\lambda_I A_{d_{\max}}\right)^{n-1}}{(n-1)!}\sum_{m=1}^{\infty}\frac{\left(\left(1-\frac{y}{d_{\max}}\right)\lambda_o A_{d_{\max}}\right)^{m-1}}{(m-1)!}\mathrm{d}x\mathrm{d}y \\ &\overset{(\star)}{=} \frac{\lambda_I\lambda_o A_{d_{\max}}^2}{d_{\max}^2\left(1-e^{-\lambda_I A_{d_{\max}}}\right)\left(1-e^{-\lambda_o A_{d_{\max}}}\right)}\int_{y=0}^{d_{\max}}\int_{x=0}^{y}e^{-(\lambda_I+\lambda_o)A_{d_{\max}}}e^{(1-x/d_{\max})\lambda_I A_{d_{\max}}}e^{(1-y/d_{\max})\lambda_o A_{d_{\max}}}\mathrm{d}x\mathrm{d}y \\ &= \frac{\lambda_I\lambda_o A_{d_{\max}}^2}{d_{\max}^2\left(1-e^{-\lambda_I A_{d_{\max}}}\right)\left(1-e^{-\lambda_o A_{d_{\max}}}\right)}\int_{y=0}^{d_{\max}}e^{-\lambda_o A_{d_{\max}}y\big/d_{\max}}\int_{x=0}^{y}e^{-\lambda_I A_{d_{\max}}x\big/d_{\max}}\mathrm{d}x\mathrm{d}y \\ &= \frac{\lambda_o}{\left(1-e^{-\lambda_I A_{d_{\max}}}\right)\left(1-e^{-\lambda_o A_{d_{\max}}}\right)}\left(\frac{1-e^{-\lambda_o A_{d_{\max}}}}{\lambda_o}-\frac{1-e^{-(\lambda_o+\lambda_I)A_{d_{\max}}}}{\lambda_o+\lambda_I}\right) \end{aligned} \tag{9}$$
