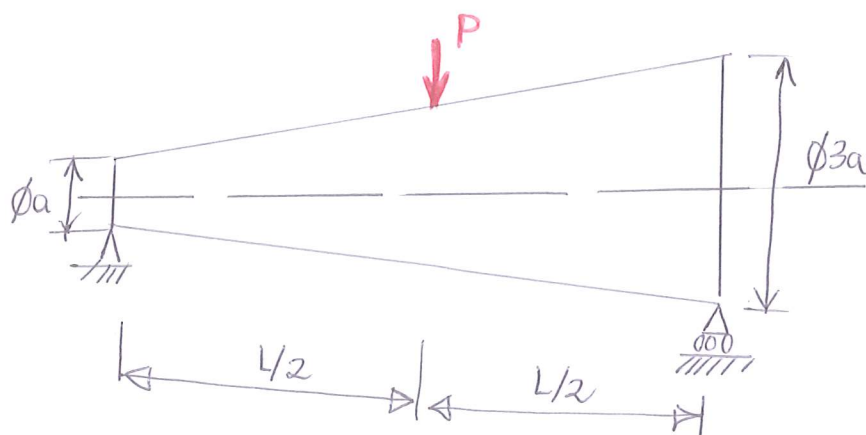


2.4.42

GIVET:

En fritt upplagd konisk balk är på mitten belastad med kraften P enligt figur.



Sektionen är cirkulär och diametern varierar linjärt.

SÖKT: Bestäm största spänningen till storlek och läge.

LÖSNING:

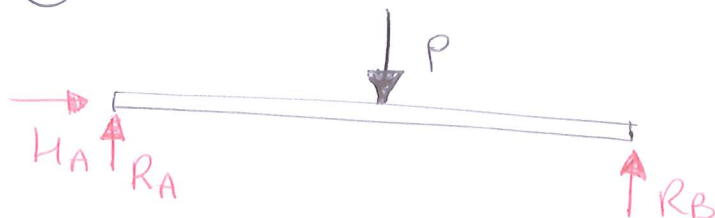
① Global jmv: Reaktionskrafter

② Smitta & jmv: Inre krafter och moment.

③ Spänning: $\sigma = \frac{N}{A} + \frac{M}{I} z$ [F.S.6.7]

④ Bestäm största spänningen $\frac{d}{dx}(\sigma_{\max}) = 0$

①.



$$\rightarrow: H_A = 0 \quad (1)$$

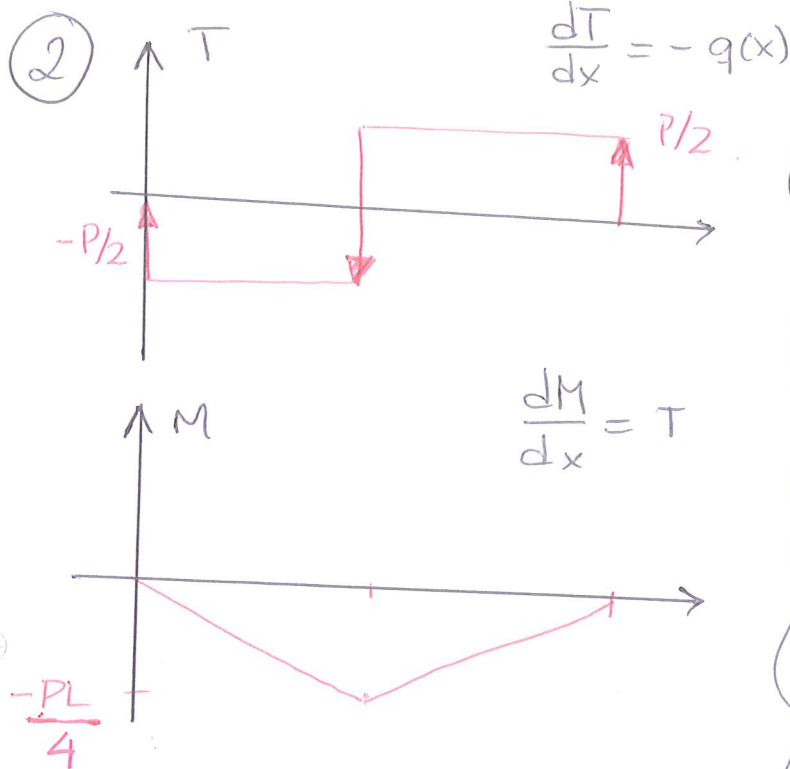
$$\uparrow: R_A + R_B - P = 0 \quad (2)$$

$$\curvearrowright A: \frac{L}{2} P - L R_B = 0$$

$$R_B = P/2 \quad (3)$$

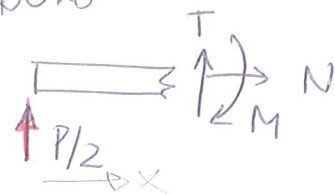
(3) i (2) $\Rightarrow R_A = P/2 \Rightarrow$ STATISKT
BESTÄMT.

②



②

$0 < x < L/2$
BÖRJA NÄR.

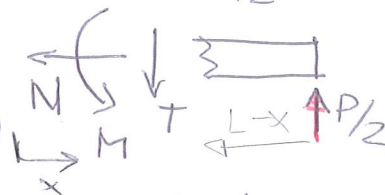


$\uparrow: T = -P/2$

$\rightarrow: N = 0$

$M = -\frac{P}{2}x$

$L/2 < x < L$



$\rightarrow: N = 0$

$\uparrow: T = P/2$

$M = -\frac{P}{2}(L-x)$

③

Spänningar

$\sigma = \frac{N}{A} + \frac{M \cdot z}{I_y} \Rightarrow$ Circular tvärsnitt: $A = \pi r^2$
och $I_y = \frac{\pi}{4} r^4 \Rightarrow$

\uparrow
[F.S. 31.1]

(Hur varierar radien?)

$r = c_1 x + c_2$

$r(x=0) = c_2 = \frac{a}{2}$

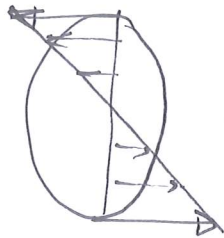
$r(x=L) = c_1 L + \frac{a}{2} = \frac{3a}{2}$

$c_1 = \frac{a}{L}$

$r = \frac{ax}{L} + \frac{a}{2} = a \left(\frac{x}{L} + \frac{1}{2} \right)$

$I_y = \frac{\pi}{4} a^4 \left(\frac{x}{L} + \frac{1}{2} \right)^4$

$$|\tau|_{\max} = \frac{|M| z_{\max}}{I_y} = \frac{|M| a \left(\frac{x}{L} + \frac{1}{2}\right)}{\frac{\pi}{4} a^4 \left(\frac{x}{L} + \frac{1}{2}\right)^4} = \frac{|M|}{\frac{\pi a^3}{4} \left(\frac{x}{L} + \frac{1}{2}\right)^3} = \frac{|M|}{W}$$

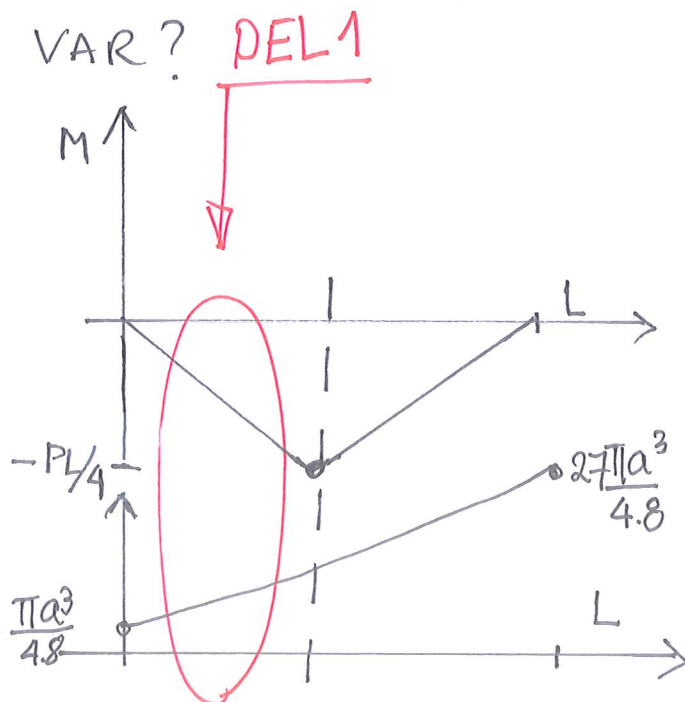


\Rightarrow symmetrisk tvärsnitt i y $\Rightarrow |\tau_{\max}| \rightarrow z = \pm r$

Del 1 $\Rightarrow |\tau_{1\max}| = \frac{P/2 x}{\frac{\pi a^3}{4} \left(\frac{x}{L} + \frac{1}{2}\right)^3}$

Del 2 $\Rightarrow |\tau_{2\max}| = \frac{P/2 (L-x)}{\frac{\pi a^3}{4} \left(\frac{x}{L} + \frac{1}{2}\right)^3}$

(4) \rightarrow Hitta max av $\tau_{\max} \Rightarrow$



$$\frac{d}{dx} (|\sigma_{1\max}|) = 0$$

$$\frac{d}{dx} \left(\frac{2Px}{\pi a^3 \left(\frac{x}{L} + \frac{1}{2} \right)^3} \right) = 0$$

$$0 = \frac{d}{dx} (2Px) \frac{1}{\pi a^3 \left(\frac{x}{L} + \frac{1}{2} \right)^3} + (2Px) \frac{d}{dx} \left(\frac{1}{\pi a^3 \left(\frac{x}{L} + \frac{1}{2} \right)^3} \right)$$

$$0 = 2P \left(\frac{1}{\pi a^3 \left(\frac{x}{L} + \frac{1}{2} \right)^3} \right) + 2Px \frac{1}{\pi a^3 \left(\frac{x}{L} + \frac{1}{2} \right)^4} \left(-\frac{3}{4L} \right)$$

$$\frac{2P}{\pi a^3 \left(\frac{x}{L} + \frac{1}{2} \right)^3} \left(1 + x \left(\frac{-3}{L \left(\frac{x}{L} + \frac{1}{2} \right)} \right) \right) = 0$$

$$\left(x + \frac{L}{2} \right) = 3x \Rightarrow \underline{\underline{x = L/4}}$$

$$\underline{\underline{|\sigma_{\max}| = \frac{32}{27} \frac{PL}{\pi a^3}}}$$