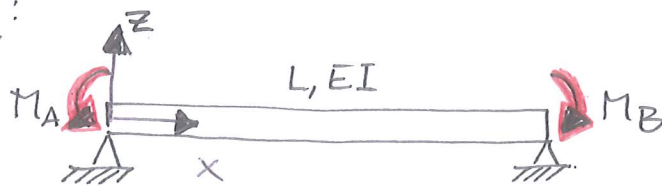


2.4.97

GIVET:



SÖKT: Bestäm (m.h.a elastiska linjens differentialekvation) utböjningen för balken ($w(x)$)

LÖSNING:

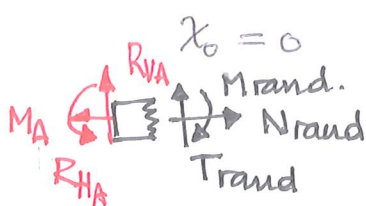
[F.S. 6.20] \Rightarrow Elastiska linjens ekvation.

$$\textcircled{1} \begin{cases} [EA u'(x)]' + K_x(x)A = [EA \alpha T(x)]' \Rightarrow \text{förskjutning i x-led.} \\ \underline{[EI w''(x)]'' = q(x) + [MT]''} \Rightarrow \text{utböjningen} \end{cases}$$

$$\frac{\partial}{\partial x^2} \left(EI \frac{\partial^2 w(x)}{\partial x^2} \right) = \underbrace{q(x)}_{\substack{\text{transv.} \\ \text{belastning} \\ \text{(Kraft/längd)}}} + \underbrace{[MT]''}_{\text{termisk last.}}$$

$$\Rightarrow \frac{\partial}{\partial x^2} \left(EI \frac{\partial^2 w(x)}{\partial x^2} \right) = 0 \quad \textcircled{1}$$

② Randvillkor: [F.S. 6.22]

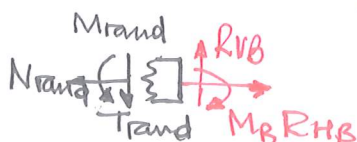


$x_0 = 0$

$$\begin{cases} w(x_0) = w_{\text{rand}} = 0 \end{cases} \quad \textcircled{2}$$

$$\begin{cases} -EI w''(x_0) = M_{\text{rand}} = M_A \end{cases} \quad \textcircled{3}$$

$x_0 = L$



$$\begin{cases} w(x_0) = w_{\text{rand}} = 0 \end{cases} \quad \textcircled{4}$$

$$\begin{cases} -EI w''(x_0) = M_{\text{rand}} = M_B \end{cases} \quad \textcircled{5}$$

③ Inledning (1):

1 gång: $\frac{\partial}{\partial x} \left(EI \frac{\partial^2 w(x)}{\partial x^2} \right) = C_1$

2 gånger: $EI \frac{\partial^2 w(x)}{\partial x^2} = C_1 x + C_2$

(6)

3: $EI \frac{\partial w(x)}{\partial x} = C_1 \frac{x^2}{2} + C_2 x + C_3$

4: $EI w(x) = C_1 \frac{x^3}{6} + C_2 \frac{x^2}{2} + C_3 x + C_4$ (7)

④ Lös C_1, C_2, C_3, C_4 m.h.a. ② i ③:

(3) i (6): $EI w''(0) = C_2 = -M_A$

$$\boxed{C_2 = -M_A} \quad (8)$$

(8), (5) i (6): $EI w''(L) = C_1 L - M_A = -M_B$

$$\boxed{C_1 = \frac{M_A - M_B}{L}} \quad (9)$$

(9), (8), (2) i (7):

$$w(0) = 0 = \frac{M_A - M_B}{L(EI)} \cdot 0 - \frac{M_A}{EI} \cdot 0 + C_3 \cdot 0 + C_4$$

$$\boxed{C_4 = 0} \quad (10)$$

(9), (8), (10), (4) i (7):

$$w(L) = 0 = \frac{M_A - M_B}{L(EI)} \cdot \frac{L}{6} - \frac{M_A}{EI} \cdot \frac{L}{2} + C_3 \cdot \frac{L}{6}$$

$$C_3 = L \left(\frac{M_B}{6} + \frac{M_A}{3} \right)$$

$$w(x) = \frac{M_A - M_B}{6EI} x^3 - \frac{M_A}{2EI} x^2 + \frac{Lx}{6EI} (M_B + 2M_A)$$

Dim. Kontrol.

$$\left[\frac{N \cdot m \cdot m^3}{N/m^4 \cdot m^4} \right] = [m] \quad \text{ok!}$$

$$\left[\frac{N \cdot m \cdot m^2}{N/m^4 \cdot m^4} \right] = [m] \quad \text{ok!}$$

$$\left[\frac{m \cdot m \cdot N \cdot m}{N/m^4 \cdot m^4} \right] = [m] \quad \text{ok!}$$