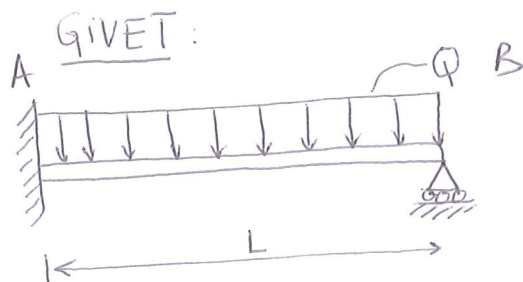


2.4.117



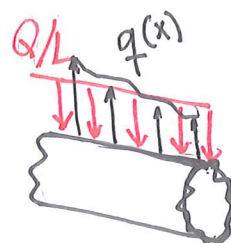
Balkens böjstyvhet är EI .

SÖKT: Bestäm reaktionsmoment och reaktionskrafter samt vinkeländringen vid stödet

LÖSNING:

① Elastiska linjens ekv:

$$EI w'''' = q(x) \quad \text{där } q(x) = -\frac{Q}{L}$$



$$EI w'''' = -\frac{Q}{L} \quad (1)$$

② Randvillkor:

$$x_0 = 0 \quad \begin{cases} w(x_0) = 0 & (2) \text{ (ingen rörelse i z-led)} \\ w'(x_0) = 0 & (3) \text{ (ingen vinkeländring)} \end{cases}$$

$$x_0 = L \quad \begin{cases} w(x_0) = 0 & (4) \text{ (ingen rörelse i z-led)} \\ EI w''(x_0) = 0 & (5) \text{ (ingen moment)} \end{cases}$$

③ Integrera (1) 4 gånger:

$$EI w'''' = -\frac{Q}{L} x + C_1 \quad (6)$$

$$EI w''' = -\frac{Q}{2L} x^2 + C_1 x + C_2 \quad (7)$$

$$EI w'' = -\frac{Q}{6L} x^3 + \frac{C_1}{2} x^2 + C_2 x + C_3 \quad (8)$$

$$EI w = -\frac{Q}{24L} x^4 + \frac{C_1}{6} x^3 + \frac{C_2}{2} x^2 + C_3 x + C_4$$

$$w = \frac{-Qx^4}{24EI} + C_1 \frac{x^3}{6EI} + C_2 \frac{x^2}{2EI} + C_3 \frac{x}{EI} + \frac{C_4}{EI} \quad (9)$$

→ (2) i (9) :

$$w(0) = \frac{C_4}{EI} = 0 \Rightarrow \boxed{C_4 = 0} \quad (10)$$

(3) i (8) :

$$w'(0) = C_3 = 0 \Rightarrow \boxed{C_3 = 0} \quad (11)$$

→ (10), (11), (4) i (9) :

$$w(L) = \left(C_1 \frac{L^3}{6} + C_2 \frac{L^2}{2} - \frac{QL^4}{24L} \right) \frac{1}{EI} = 0$$

$$\underline{\underline{C_1 \frac{L^3}{6} + C_2 \frac{L^2}{2} - \frac{QL^3}{24} = 0}} \quad (12)$$

→ (10), (11), (5) i (7)

$$EI w''(L) = -\frac{QL^2}{2L} + C_1 L + C_2 = 0$$

$$-\frac{QL}{2} + C_1 L + C_2 = 0$$

$$\therefore \underline{\underline{C_2 = \frac{QL}{2} - C_1 L}} \quad (13)$$

→ (13) i (12) : $C_1 \frac{L^3}{6} + \left(\frac{QL}{2} - C_1 L \right) \frac{L^2}{2} - \frac{QL^3}{24} = 0$

$$\boxed{C_1 = \frac{5Q}{8}} \quad (14) \quad (14) \text{ i } (13) \Rightarrow$$

$$(14) \text{ i } (13) : \quad \boxed{C_2 = -\frac{LQ}{8}} \quad (15)$$

$$w(x) = \frac{-Qx^4}{24LEI} + \frac{5Qx^3}{48EI} - \frac{QLx^2}{16EI}$$

$$EIw'''(x) = -\frac{Q}{L}x + \frac{5Q}{8}$$

$$EIw''(x) = -\frac{Qx^2}{2L} + \frac{5Q}{8}x + \left(-\frac{LQ}{8}\right)$$

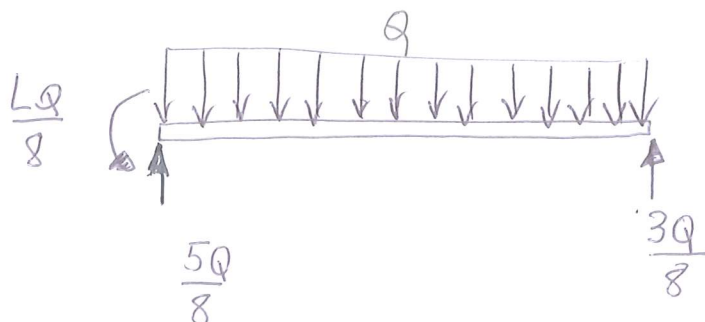
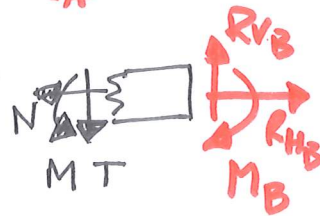
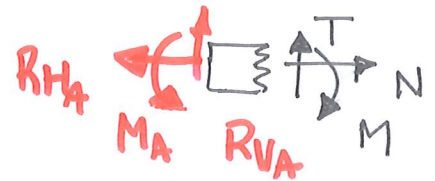
$$M_A = M(0) = -EIw''(0) = \frac{LQ}{8}$$

$$R_{VA} = -T(0) = +EIw'''(0) = +\frac{5Q}{8}$$

$$R_{VB} = T(L) = -EIw'''(L) = -\left(-\frac{QL}{L} + \frac{5Q}{8}\right) = \frac{3Q}{8}$$

$$\theta_B = w'(L) = \frac{1}{EI} \left(\frac{5QL^2}{16} - \frac{LQ}{8}L - \frac{QL^3}{24L} \right)$$

$$\theta_B = w'(L) = \frac{QL^2}{48EI}$$



Kontrollera m h a jmv
att det stämmer.

