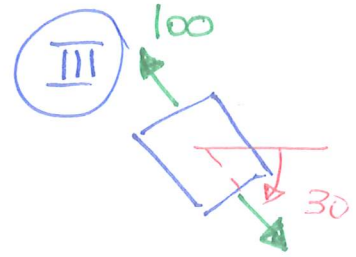
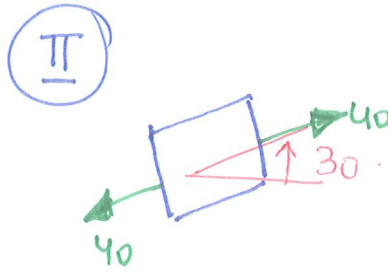
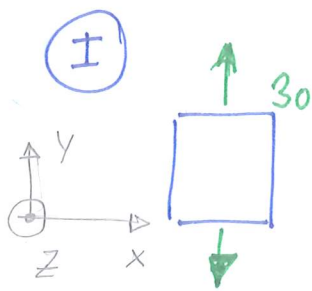


1.1.14

GIVET: [MPa]



SÖKT : Beräkna huvudspänningar och riktningar för det sammanlagda spänningstillståndet.

LÖSNING: [F.S. 1.6-7]

1. Transformera alla lokala spänningar till ett gemensamt koordinatsystem.

2. Lägg ihop spänningarna

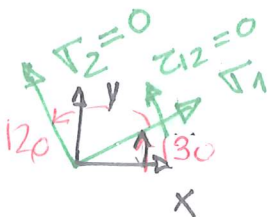
3. Beräkna huvudspän. samt riktning.

1.- Välj gemensamt koordinat-system: XYZ.

I).-  $\sigma_x^I = 0 \text{ MPa}; \sigma_y^I = 30 \text{ MPa}; \tau_{xy}^I = 0 \text{ MPa}$

Plant spänning  $\Rightarrow \tau_z^I = \tau_{xz}^I = \tau_{yz}^I = 0$

II) Plant spänning  $\Rightarrow \tau_z^{II} = \tau_{xz}^{II} = \tau_{yz}^{II} = 0$



$$\sigma_1 = 40 = \sigma_x \cos^2(30) + \sigma_y \sin^2(30) + 2\tau_{xy} \sin(30)\cos(30)$$

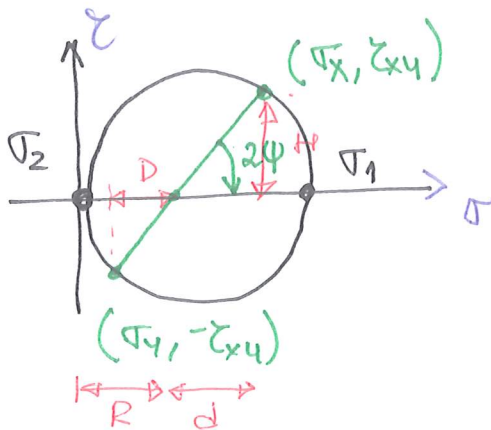
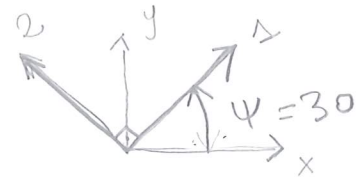
$$\sigma_2 = 0 = \sigma_x \cos^2(120) + \sigma_y \sin^2(120) + 2\tau_{xy} \sin(120)\cos(120)$$

$$\tau_{12} = 0 = \frac{\sigma_y - \sigma_x}{2} \sin 60 + \tau_{xy} \cos 60.$$

$$\begin{bmatrix} 40 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} c^2(30) & s^2(30) & 2sc(30) \\ c^2(120) & s^2(120) & 2sc(120) \\ -\frac{s(60)}{2} + \frac{s(60)}{2} & c60 & \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \rightarrow$$

$$\underline{\underline{\sigma_x^{\text{II}} = 30 \text{ MPa}}}; \quad \underline{\underline{\sigma_y^{\text{II}} = 10 \text{ MPa}}}; \quad \underline{\underline{\tau_{xy}^{\text{II}} = 17,3 \text{ MPa}}}$$

m.h.a. Mohr's circle:



$$\sigma_1 = 40 \text{ MPa}$$

$$\sigma_2 = 0 \text{ MPa.}$$

$$\sigma_x^{\text{II}} = R + d = R + R \cdot \cos(2\psi)$$

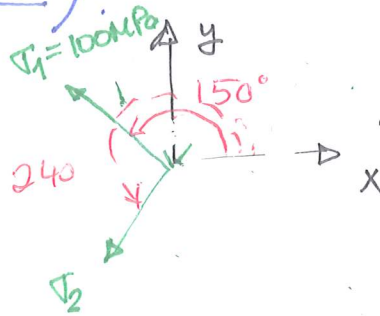
$$\underline{\underline{\sigma_x^{\text{II}}}} = \left\{ \begin{array}{l} \sigma_1 = 40 \text{ MPa} \\ \sigma_2 = 0 \text{ MPa} \\ \psi = 30^\circ \end{array} \right\} = \underline{\underline{30 \text{ MPa}}}$$

$$\underline{\underline{\sigma_y^{\text{II}}}} = R - d = R - R \cos(2\psi) = R(1 - \cos(2\psi)) = \underline{\underline{10 \text{ MPa}}}$$

$$\underline{\underline{\tau_{xy}^{\text{II}}}} = H = R \cdot \sin(2\psi) = \underline{\underline{17,3 \text{ MPa.}}}$$

$$\sigma_z^{\text{III}} = \tau_{xz}^{\text{III}} = \tau_{yz}^{\text{III}} = 0$$

III)

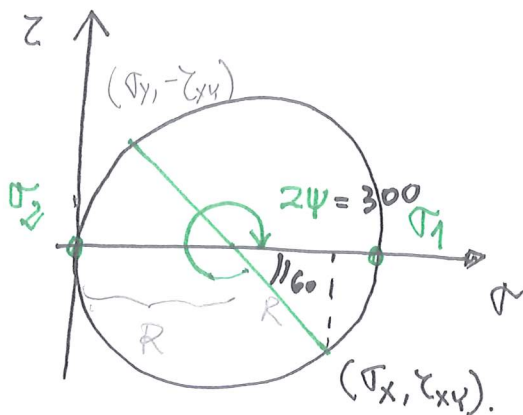


$$\begin{cases} 100 = \sigma_x \cos^2(150) + \sigma_y \sin^2(150) + 2\tau_{xy} \sin(150)\cos(150) \\ 0 = \sigma_x \cos^2(240) + \sigma_y \sin^2(240) + 2\tau_{xy} \sin(240)\cos(240) \\ 0 = \frac{\sigma_y - \sigma_x}{2} \sin(300) + \tau_{xy} \cos(300) \end{cases}$$

$$3\text{ekv} + 3\text{obek} \Rightarrow \text{L\u00f5s:}$$

eller

$$\sigma_x = 75 \text{ MPa}; \quad \sigma_y = 25 \text{ MPa}; \quad \tau_{xy} = -43,3 \text{ MPa}$$



$$R = \frac{100}{2} = 50 \text{ MPa}$$

$$\begin{cases} \sigma_x^{\text{II}} = R + R \cos(60) = 75 \text{ MPa} \\ \sigma_y^{\text{II}} = R - R \cos(60) = 25 \text{ MPa} \\ \tau_{xy}^{\text{II}} = -R \sin(60) = -43,3 \text{ MPa} \end{cases}$$

2. L\u00e4gg ihop sp\u00e4nningarna.

$$\begin{cases} \sigma_x = \sigma_x^{\text{I}} + \sigma_x^{\text{II}} + \sigma_x^{\text{III}} = 105 \text{ MPa} \\ \sigma_y = \sigma_y^{\text{I}} + \sigma_y^{\text{II}} + \sigma_y^{\text{III}} = 65 \text{ MPa} \\ \tau_{xy} = \tau_{xy}^{\text{I}} + \tau_{xy}^{\text{II}} + \tau_{xy}^{\text{III}} = -25,9 \text{ MPa} \end{cases}$$

$$\sigma_z = \tau_{xz} = \tau_{yz} = 0 \text{ MPa}$$

3. Ber\u00e4kna huvudsp\u00e4nningar samt riktn\u00e4ng

Ans 1:

$$\underline{\underline{S}} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ & \sigma_y & \tau_{yz} \\ & & \sigma_z \end{bmatrix} \begin{cases} \det(\underline{\underline{S}} - \lambda \underline{\underline{I}}) = 0 \dots \\ (\underline{\underline{S}} - \sigma \underline{\underline{I}}) \underline{\underline{n}} = 0 \end{cases}$$

Ant 2:

Plant spänning  $\Rightarrow \sigma_z = \tau_{xz} = \tau_{yz} = 0$

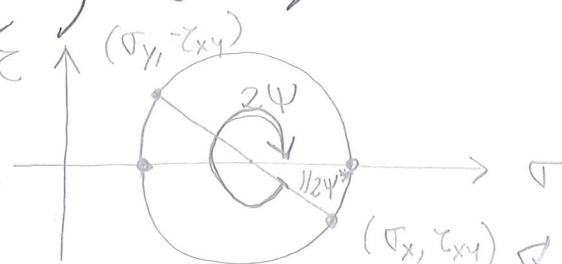
$\sigma_z = 0 \Rightarrow$  huvudspänning / huvudriktning  $[0 \ 0 \ 1]^T$

Mohr:

$$\sigma_{1,2} = \frac{1}{2} (105 + 65) \pm \sqrt{\left(\frac{105 - 65}{2}\right)^2 + (-25.9)^2}$$

$$\sigma_1^{2D} = 118 \text{ MPa}$$

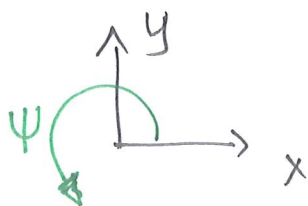
$$\sigma_2^{2D} = 52 \text{ MPa}$$



F.S. 1.19  $\Rightarrow 2\psi^* = \arctan \left( \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right) = -52.4^\circ$

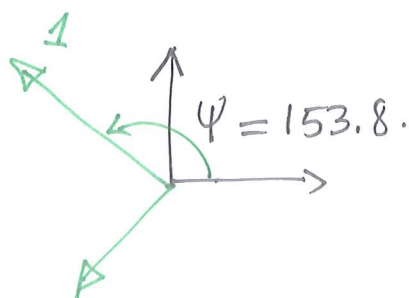
*värde som man får från miniräkaren.*

Skriv om till positiv.  $\begin{cases} \tau_{xy} < 0 \\ \sigma_x - \sigma_y > 0 \end{cases} \Rightarrow 270 \leq 2\psi \leq 360$



$$2\psi = 360 - 52.4 = 307.6$$

$$\psi = 153.8^\circ$$



$$\sigma_1 \geq \sigma_2 \geq \sigma_3$$

$$\begin{cases} \sigma_1^{3D} = 118 \text{ MPa} & \underline{n}_1 = [\cos \psi \quad \sin \psi \quad 0] \\ \sigma_2^{3D} = 52 \text{ MPa} & \underline{n}_2 = [\cos(\psi + 90^\circ) \quad \sin(\psi + 90^\circ) \quad 0] \end{cases}$$

$$\sigma_3^{3D} = 0 \text{ MPa} \quad \underline{n}_3 = [0 \ 0 \ 1]$$