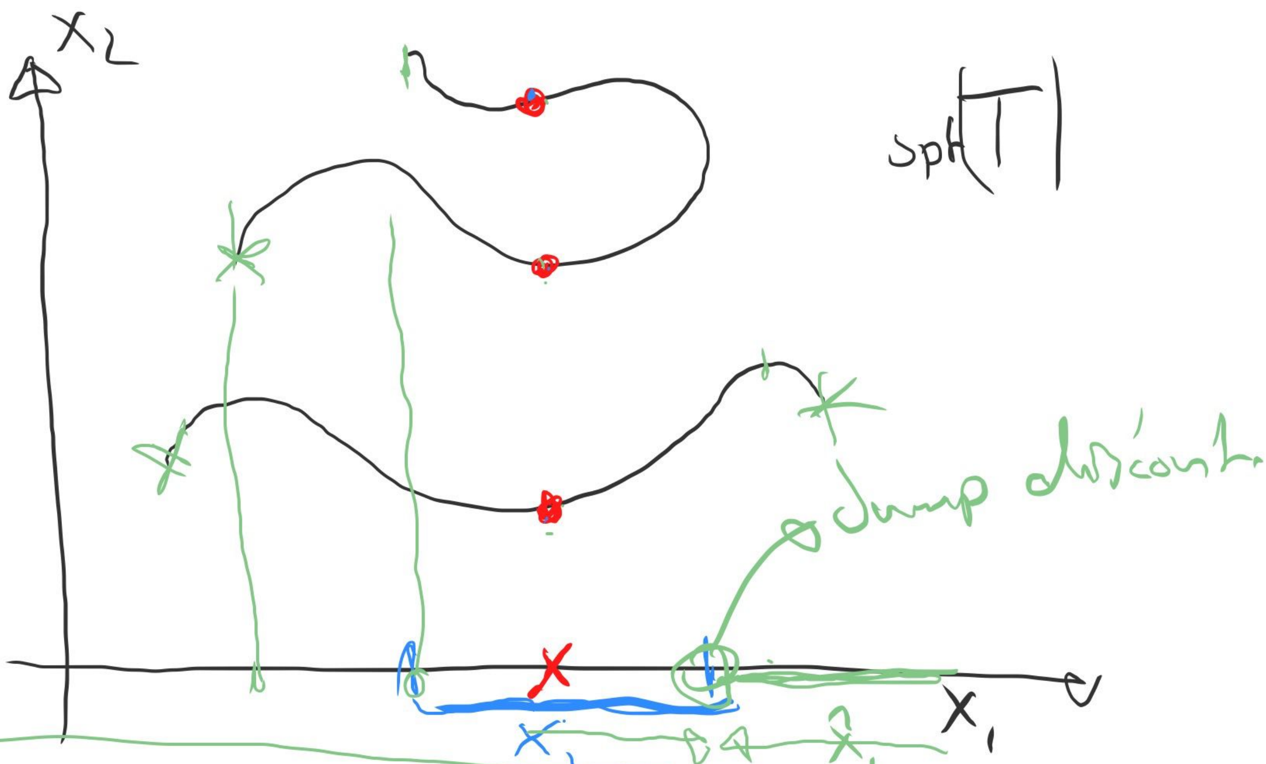
More or less if $u : \mathbb{R}^M \mapsto \mathcal{R}_0(\mathbb{R}^{M+k})$ is Lip.

$\bigcup_x \operatorname{spt}[u(x)]$ will be countably rectifiable.



$$\boxed{u(x_1) = \langle T, p, x_1)}$$

projection to x_1 axis

$u(x_1)$ will not be Lipschitz.

$\phi = 1$

$$\boxed{u(x_1)[\phi] - u(\hat{x}_1)[\phi]}$$

$= c_1(x_1) + c_2(x_1) + c_3(x_1)$ $\qquad c_1(\hat{x}_1)$

Def [MBV, 8.1.7] (Metric space valued functions of Bounded variation)

$$u : \mathbb{R}^M \longrightarrow \mathbb{R}^{M+k}$$

$$u(x) = \sum_{k=1}^{\infty} c_k(x) \, C_{P_k(x)}$$

1) $\Diamond$ if $\phi : \mathbb{R}^{M+k} \longmapsto \mathbb{R}$

then we can define

$$u \Diamond \phi(x) = \sum_{k=1}^{\infty} c_k(x) \, \phi[P_k(x)]$$

2) We will say that u is MBV if $u \Diamond \phi(x)$ is BV function for every $\phi \in C^{0,1}$

3) $$V_u(A) = \sup \left\{ \int_A (u \boxed{\Diamond} \phi) \, \mathrm{div}(g) \, d\mathcal{L}^M \, ; \, g \in C_c^1(A), \ |g| \le 1, \ |\phi|, |d\phi| \le 1 \right\}$$

set $A \subset \mathbb{R}^M$

Thm (8.1.8). $p : \mathbb{R}^{M+k} \longmapsto \mathbb{R}^M$, $T \in \mathcal{D}_M(\mathbb{R}^{M+k})$

$$\mathbb{M}(T) + \mathbb{M}(\partial T) < \infty.$$

define $u : \mathbb{R}^M \longmapsto \mathcal{R}_0(\underline{\mathbb{R}^{M+k}})$ by

$$u(x) = \langle T, p, x \rangle$$

Then $u(x) \in MBV$ and

$$V_u(A) \leq M \left(\|T\|(A) + \|\partial T\|(A) \right)$$

t dim in $\mathbb{R}^M$

Know that if $u(x)$ is <u>Lipschitz</u> then $\bigcup_x \sup[u(x)]$ is rectifiable.

<u>$u(x)$</u> is <u>MBV</u>.

Proof: $u(x) = \langle T, p, x \rangle$ show:

$$V_u(A) = \sup \left\{ \int_A (u \circ \phi) \operatorname{div} g \, d\mathcal{L}^m, \; |g| \le 1, \; |\phi|, |d\phi| \le 1 \right\}$$

$$\operatorname{div} g = \sum_{i=1}^{m} \frac{\partial g^i}{\partial x_i}$$

$$\le M \left(\|T\|(A) + \|\partial T\|(A) \right)$$

$\psi = g^i$. Take any ϕ, $|\phi|, |d\phi| \le 1$.

$$\left| \int (D_i \psi) \left(\langle T, p, x \rangle (\phi) \right) d\mathcal{L}^m(x) \right| =$$

$$= \left| T \left(\underbrace{\phi \left[D_i \psi \circ p \right] dx_1 \wedge dx_1 \ldots dx_m}_{d(\psi \circ p) \wedge dx_1 \wedge \ldots \wedge dx_i \ldots dx_m} \right) \right|$$

$$= \left| T \left(\phi \, d(\psi \circ p) \, dx_1 \wedge \ldots \widehat{dx_i} \wedge \ldots dx_m \right) \right|$$

$$\sum D_j (\psi \circ p) \, dx_j$$

$$d[\phi (\psi \circ p)] = (d\phi) \, \psi \circ p + \phi \, d(\psi \circ p)$$

$$\leq \left| T\left(d\left(\phi(\psi\circ p)\, dx_1\wedge\ldots\widehat{dx_i}\wedge dx_m\right)\right)\right| + \left|T\left((\psi\circ p)\, d\phi\wedge dx_1\wedge\ldots\widehat{dx_i}\wedge\ldots dx_m\right)\right|$$

$$= \left|\partial T\left(\phi(\psi\circ p)\, dx_1\wedge\ldots\widehat{dx_i}\ldots dx_m\right)\right|$$

$$+ \left|T\left(\underbrace{\psi\circ p\, d\phi}_{\leq 1}\wedge \ldots\right)\right| \leq$$

$$\leq M\left(\|\partial T\|(A) + \|T\|(A)\right).$$

What we want for $g = (0, \ldots, g^i, \ldots 0)$

$$g^i = \psi$$

Next time Monday Normal

Schedule.

$\underbrace{f \in BV}$ $\Longrightarrow$ graph(f) is

countably rectifiable

$$\boxed{\underbrace{u \in MBV} \Longrightarrow \bigcup_x \operatorname{spt}(u[x]) \text{ is cont. Rect.}}$$

$\Longrightarrow$ $\underline{\operatorname{spt}(T)}$ is rectifiable.