Fractal Geometry
Homework 1
Due on February 8th

Liviana Palmisano

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Question 1. Prove that if μ is a measure on D and $f : D \to \mathbb{R}$ satisfies $f(x) \geq 0$ for all $x \in D$ and $\int_D f d\mu = 0$ then $f(x) = 0$ for μ-almost all x.

Question 2. Show that the middle third Cantor set, is compact and totally disconnected. What is its interior, closure and boundary?

Question 3. What are the sets of real numbers that are both open and closed?

Question 4. Use the equivalent definition of $N_\delta(F)$ in (i) to check that the upper box dimension of the von Koch curve is at most $\log 4/\log 3$ and the equivalent definition of $N_\delta(F)$ in (v) to check that the lower box dimension is at least this value.

Question 5. (More challenging exercise. Not mandatory) Construct a set F for which $\underline{\dim}_B F < \overline{\dim}_B F$. (Hint : let $k_n = 10^n$, and adapt the Cantor set construction by deleting, at the kth stage, the middle $1/3$ of intervals if $k_{2n} < k \leq k_{2n+1}$, but the middle $3/5$ of intervals if $k_{2n-1} < k \leq k_{2n}$).