Throughput Analysis of ARQ in Interference with Nakagami-m Block Fading Channels

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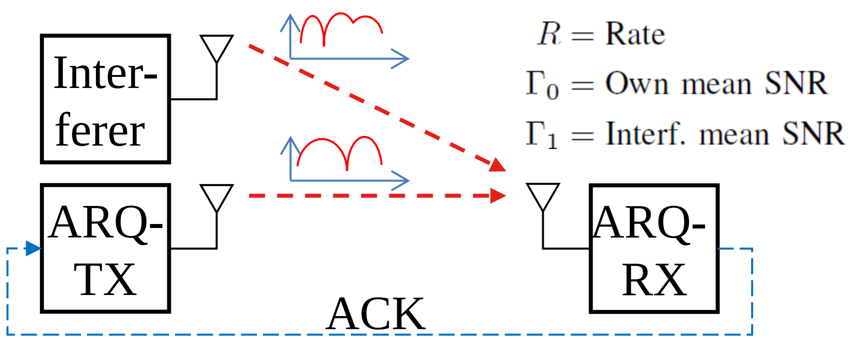
A Simplified Motivating Example

Scenario (for the motivating example)

- ARQ in interference.
- Block Rayleigh fading.



Main problem

- **Optimize ARQ throughput in interference!**
- Closed-form opt. rate point: $R^* = f_R(\Gamma_0, \Gamma_1)$
- Closed-form opt. throughput: $T^* = f_T(\Gamma_0, \Gamma_1)$
- Various usage of the results: **See paper!**

Solution (for the motivating example)

- Decoding probability:

$$P = \frac{e^{-\Theta}}{1+\Theta\Gamma_1}, \quad \Theta \triangleq \frac{e^R - 1}{\Gamma_0}$$

- ARQ Throughput:

$$T = R \cdot P$$

- Optimality condition:
  Try to solve for the optimal rate!

$$\frac{d\ln(T)}{dR} = 0 \Rightarrow$$

$$\frac{1}{R^*} - \frac{e^{R^*}}{\Gamma_0} - \frac{\Gamma_1 e^{R^*}}{\Gamma_0 + \Gamma_1(e^{R^*}-1)} = 0$$

Conclusion (for the motivating example)

- **Opt. rate unsolvable in a closed-form!**
- **But, we want to find closed-form opt. solutions for even more general cases!**

In the Paper We....

Solve the "unsolvable" problem!

- Propose a parametric closed-form optimization framework.
- Find closed-form expressions for
  - Outage (decoding) probability.
  - Throughput.
  - Optimal rate point.
  - Optimal throughput value.

Generalize the problem!

- Include arbitrary # of interferers.
- Use per-user Nakagami-m fading.

Consider also other ARQ problems!

- Scaled-power case. **See paper!**
- Interf.-limited case. **See paper!**

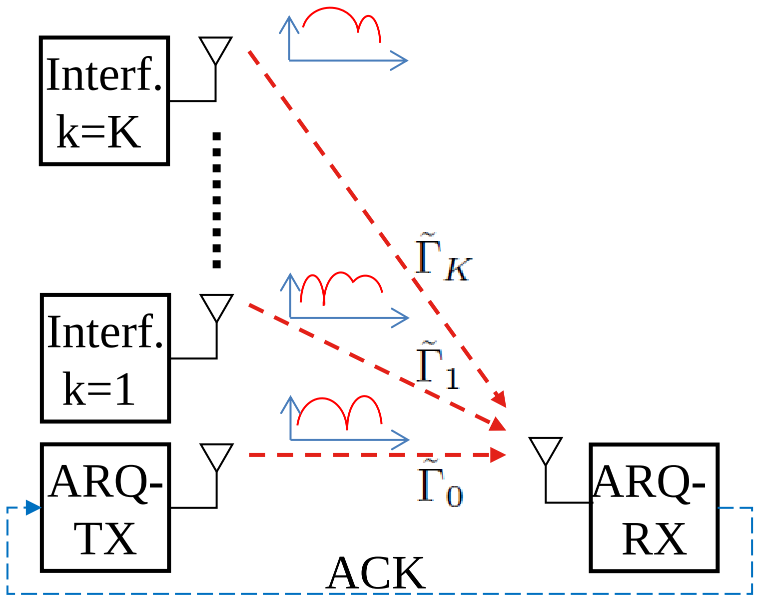
The main contribution is:

A parametric optimization framework (2 methods!) giving closed-form expressions for the optimal throughput value and the optimal rate point for ARQ operating in interference.

System Model

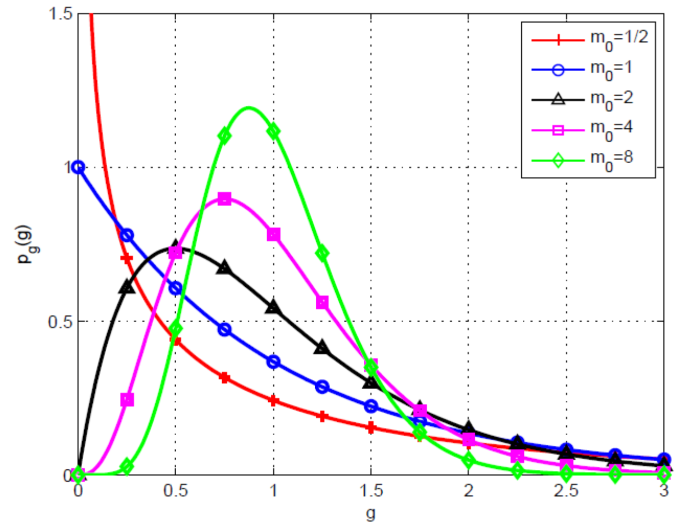
Communication Scenario

- ARQ in interference.



- Assumptions:
  - AWGN & Capacity achieving codes.
  - Inter- and intra-user *i.i.d.* block fading.
  - Error-free ACK & No overhead.
  - Always a packet to send.

Nakagami-*m* block fading channel:



$$\gamma_k = g_k \Gamma_k, \quad \Gamma_k \text{ is the mean SNR}$$

$$f_{g_k}(g_k) = \frac{m_k^{m_k}}{\Gamma(m_k)} g_k^{m_k - 1} \mathrm{e}^{-m_k g_k}, m_k \geqslant \frac{1}{2}$$

- Motivations:
  - Wide range of fading conditions.
  - *m*=1 is Rayleigh fading.
  - *m*=2 is 2-branch MRC/ TX div.
  - Converge to a non-fading ch.
  - Good fit to measurements.

Decoding Probabilities

Generic channel fading - Decoding prob.

- $$P = \mathbb{P}\left\{ \ln\left(1 + \frac{g_0\Gamma_0}{1+\sum_{k=1}^{K} g_k\Gamma_k}\right) > R \right\}$$

Own ch. Rayleigh, other ch. Nakagami-*m*

- $m_0 = 1$

$$P_1 = \frac{e^{-\Theta}}{\prod_{k=1}^{K}(1+\Theta\tilde{\Gamma}_k)^{m_k}} \quad \Theta \triangleq \frac{e^R - 1}{\tilde{\Gamma}_0} \quad \tilde{\Gamma}_k \triangleq \Gamma_k/m_k$$

Own ch. Rayleigh+MRC, other ch. Nakagami-*m*

- $m_0 = 2$

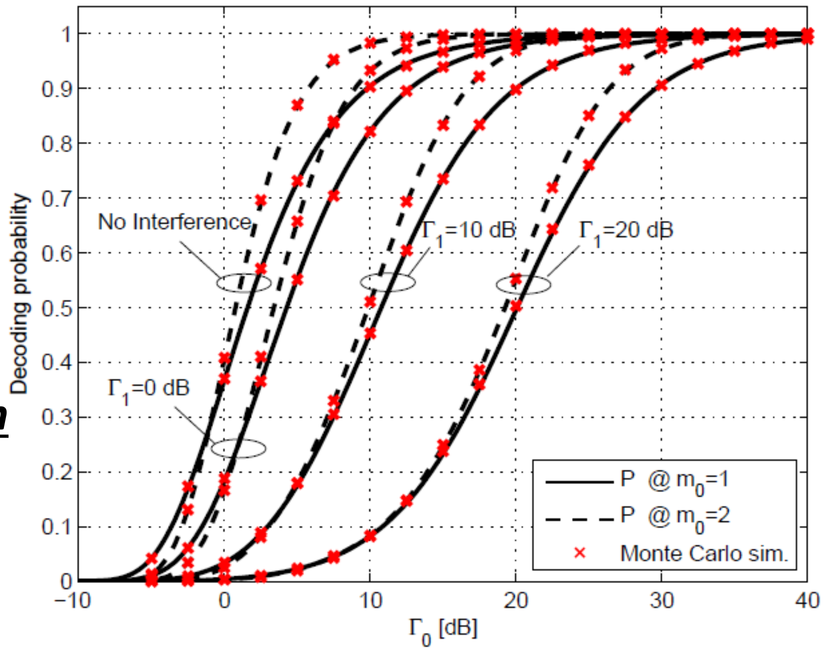
$$P_2 = P_1\left(1 + \Theta + \sum_{k=1}^{K}\frac{m_k\Theta\tilde{\Gamma}_k}{1+\Theta\tilde{\Gamma}_k}\right)$$

All fading channels are Nakagami-*m*

- **See paper!**

Analytical and simulation results

- A check against Monte Carlo simulation results. Ok!



The Key Idea: Parametric Optimization

Parameterization method (1) in R

- Throughput expressed as
  $T = R \cdot P, \quad P = f(R, \tilde{\Gamma}_0, \tilde{\Gamma}_1, ..., \tilde{\Gamma}_K)$
- Optimality condition

$$\frac{d\ln(T)}{dR} = 0$$

$$\Rightarrow \frac{1}{R} + \frac{P'_R}{P} = 0$$

- Idea: Parameterize in R^*. Solve for Γ_0. (if at all possible). Insert in T.
- Solution

$$\Gamma_0 = f_\Gamma(R^*, \tilde{\Gamma}_1, ..., \tilde{\Gamma}_K)$$

$$T^* = f_T(R^*, \tilde{\Gamma}_1, ..., \tilde{\Gamma}_K)$$

$$R^*$$

- Insight: It is hard to solve for R^* vs. Γ_0, but generally easy to solve Γ_0 vs. R^*.

Parameterization method (2) in Θ

- Throughput expressed as
  $T = \ln(1 + \tilde{\Gamma}_0\Theta)P, \ P = f(\Theta, \tilde{\Gamma}_1, ..., \tilde{\Gamma}_K)$
- Optimality condition

$$\Theta\frac{d\ln(T)}{d\Theta} = 0$$

$$\Rightarrow (1 + \Theta\tilde{\Gamma}_0)\ln(1 + \Theta\tilde{\Gamma}_0)/\Theta\tilde{\Gamma}_0 = t$$

$$t(\Theta, \tilde{\Gamma}_1, ..., \tilde{\Gamma}_K) \triangleq -\Theta P'_\Theta / P$$

- Idea: Parameterize in Θ. Solve for Γ_0. Insert in R and T.
- Solution

$$\Gamma_0(\Theta) = m_0 \frac{\frac{-t}{W_0(-te^{-t})} - 1}{\Theta}$$

$$T^*(\Theta) = R^* \cdot P$$

$$R^*(\Theta) = (t + W_0(-te^{-t}))$$

Lambert's W-function

- Insight: It is easier to solve for Γ_0 if only R, but not P, is expressed in Γ_0.

Example: Parametric Optimization

Parameterization method (1):
Own ch. Rayleigh, 1 Rayleigh Interf.

- Decoding probability $m_0 = 1$

$$P = \frac{e^{-\Theta}}{1+\Theta\Gamma_1}, \quad \Theta \triangleq \frac{e^R - 1}{\Gamma_0}$$

- Optimality condition

$$\frac{1}{R^*} - \frac{e^{R^*}}{\Gamma_0} - \frac{\Gamma_1 e^{R^*}}{\Gamma_0 + \Gamma_1(e^{R^*}-1)} = 0$$

can be rewritten as

$$\frac{a}{1+x} + \frac{b}{x} = 1$$ where $x \triangleq \Gamma_0/\Gamma_1(e^{R^*}-1)$, $a \triangleq R^*/(1-e^{-R^*})$, $b \triangleq a/\Gamma_1$

- Solving for a positive x

$$x_+ = \frac{a+b-1+\sqrt{(a+b-1)^2+4b}}{2}$$

- Back substitution yields

$$T^* = \frac{R^* e^{-\frac{1}{\Gamma_1 x_+}}}{1+x_+^{-1}}$$

$$\Gamma_0 = x_+\Gamma_1(e^{R^*}-1)$$

Parametric closed-form for optimal throughput!

Parameterization method (2):
Own ch. Rayleigh, K Nakagami-m Interf.

- Decoding probability $m_0 = 1$

$$P = \frac{e^{-\Theta}}{\prod_{k=1}^{K}(1+\Theta\tilde{\Gamma}_k)^{m_k}},$$

- Optimality condition

$$(1+\Theta\tilde{\Gamma}_0)\ln(1+\Theta\tilde{\Gamma}_0)/\Theta\tilde{\Gamma}_0 = t,$$

where

$$t(\Theta) \triangleq \frac{1}{\Theta + \sum_{k=1}^{K}\frac{m_k\Theta\tilde{\Gamma}_k}{1+\Theta\tilde{\Gamma}_k}}$$

- Insert in

$$\Gamma_0(\Theta) = m_0\frac{\frac{-t}{W_0(-te^{-t})} - 1}{\Theta}$$

$$T^*(\Theta) = R^* \cdot P$$

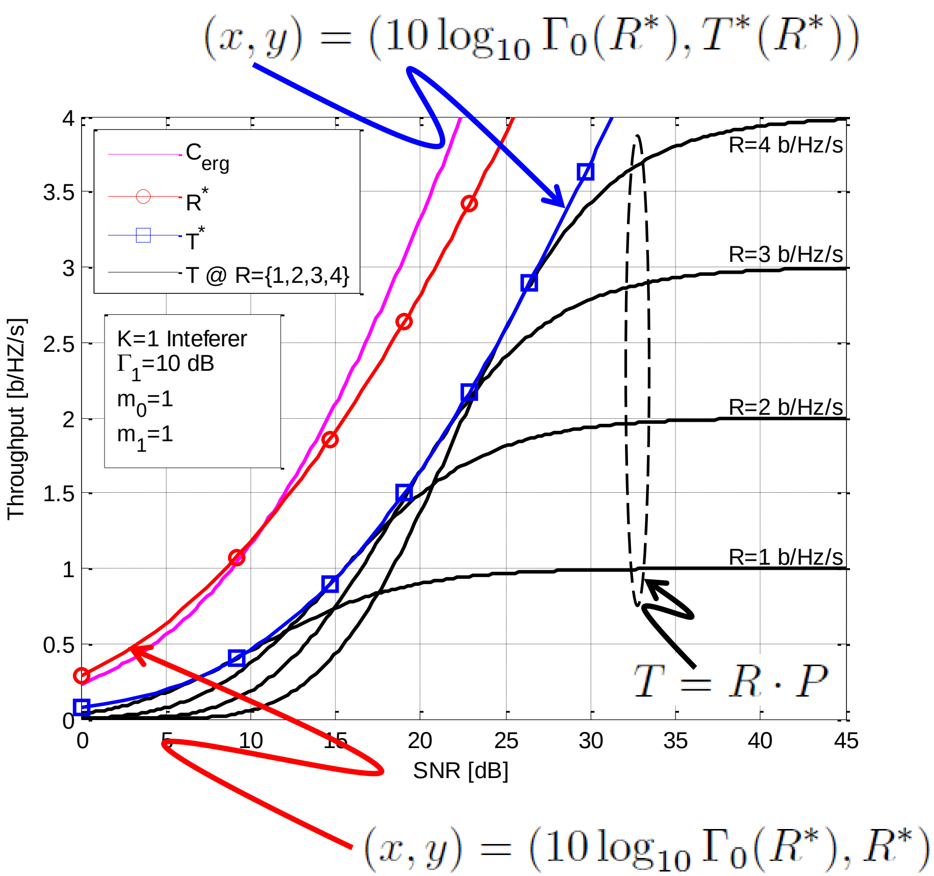
$$R^*(\Theta) = t + W_0(-te^{-t})$$

Parametric closed-form for optimal throughput!

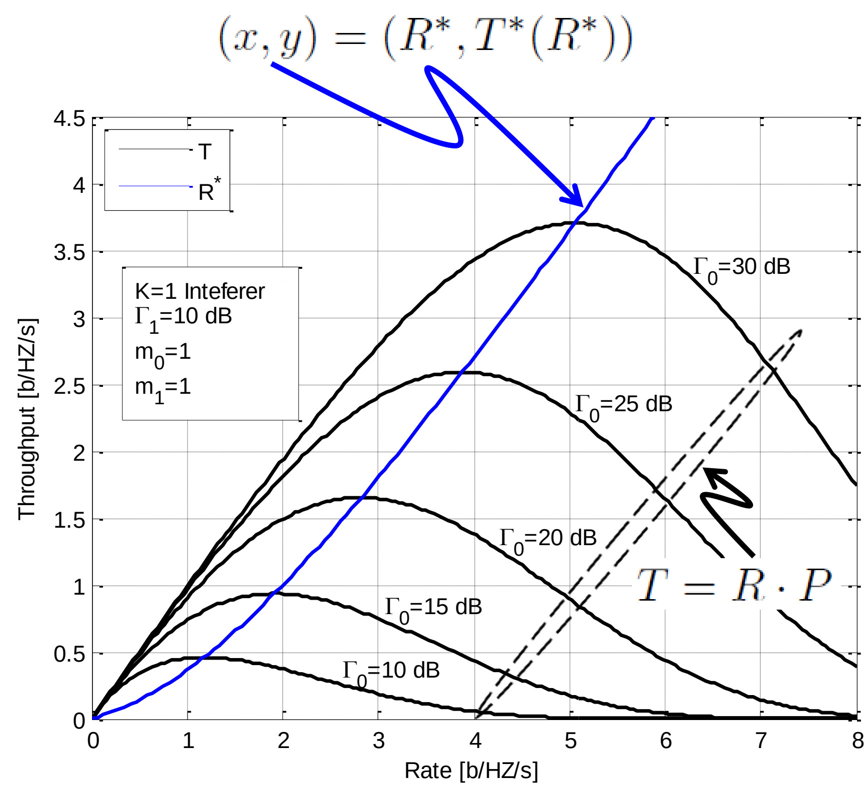
Note that method 2 handles any arbitrary interferences case!

Selected Results

Optimal throughput and optimal rate vs. SNR



Optimal throughput vs. optimal rate



Apart from plotting curves, we can e.g. also study asymptotes parametrically. **See paper!**

Summary and Conclusions

Summary

- Studied ARQ in interference.
- Proposed parameterized optimization for closed-form results.
  - Method 1: Parameterized in R. Handles some interference cases.
  - Method 2: Parameterized in Θ. Handles any interference case.
- Derived some closed-form expressions.
- Studied other problems in the paper
  - Scaled-power case
  - Interf.-limited case

Conclusions

- Solved an “unsolvable” opt. problem.
- Noted large losses with interference.
- Use as reference cases for bench-marking new ARQ+MCS schemes.

The material was adapted for the interactive presentation format. Please see the paper for more general assumptions and analysis, as well as other studied problems.

Related papers

- Please also consider the following related papers:
  - P. Larsson, L.K. Rasmussen, M. Skoglund, "**Throughput Analysis of ARQ Schemes in Gaussian Block Fading Channels**," *IEEE Trans. Commun*., vol.62, no.6, pp. 2569-2588, Jul. 2014.
  - P. Larsson, L. K. Rasmussen, M. Skoglund, "**Multi-layer Network Coded ARQ for multiple unicast flows,**" in *Proc. of SWE-CTW*, Lund, Oct. 2012.