Optimal Trade-off Between Transmission Rate and Secrecy Rate in Gaussian MISO Wiretap Channels

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***Abstract*—The future wireless networks require both high data transmission rates and secure communications. However, there exists a trade-off between secure and non-secure rates since only a fraction of the maximal achievable reliable rate in a wiretap channel can be considered as secure. This paper studies the optimal transmit strategies that achieve the optimal trade-off between the communication rate and secrecy rate for the MISO wiretap channels with different power constraint settings including sum power constraint only, per-antenna power constraints only, and joint sum and per-antenna power constraints. First, a necessary and sufficient condition to ensure a positive secrecy capacity is shown. After that, closed-form solutions to find an optimal transmit strategy of related problems are derived. This provides a parametrization of the boundary of the maximal achievable rate and secrecy rate without additional time-sharing. The optimal trade-off is characterized by the convex hull of the region. Lastly, the results are illustrated by numerical examples.**

I. Introduction

Security is a critical aspect in wireless communication systems due to the open nature of wireless links. To enhance the security, physical-layer secrecy methods have received much attention recently. One of the pioneer studies is the study of the secrecy capacity of the wiretap channel [1], where Wyner showed that a positive secrecy rate can be achieved when an eavedropper's channel is a degraded version of the main channel. The maximal secrecy rate is given by the largest difference between the rate achievable at the legitimate receiver and the rate achievable at the eavesdropper. Following Wyner's work, researchers in the physical-layer security area have extended and considered the wiretap channel in various aspects. Notable results include the extension to the non-degraded case by Csiszár and Körner [2] and the extension to the single-input single-output Gaussian wiretap channel by Leung-Yan-Cheong and Hellman [3].

The secrecy capacities for MISO and MIMO wiretap channels with a sum power constraint, which can be motivated by ecological aspects to limit the energy consumptions, have been intensively studied in [4]–[6]. In [4], necessary conditions for the optimal input covariance are derived. In particular, a closed-form expression of the MISO secrecy capacity has been shown. For the MIMO case an iterative algorithm is provided. In [5] and [6], iterative optimization algorithms to find the secrecy capacity have been proposed based on the concave-convex procedure (CCCP) and the alternating optimization approach. Alternatively, indirect approaches such as using Sato-like argument and matrix analysis tools are also used to find the secrecy capacity of a MIMO Gaussian wiretap channel [7], [8].

In practice, each antenna has its own power amplifier, which means the power allocation at the transmitter is usually done under per-antenna power constraints instead of a sum power constraint. Particularly, the problem of finding the channel capacity with average per-antenna power constraints has been investigated in both single-user [9], [10] and multi-user setups [11]–[13]. Recently, the capacity of point-to-point channels with joint sum and per-antenna power constraints has been considered [14]–[17]. An interesting aspect of the joint sum and per-antenna power constraints setting is that it can be applied to systems with multiple antenna as well as to distributed systems with separated energy sources. The optimal transmit strategy problem with the joint sum and per-antenna power constraints has been well studied for MISO channel with two transmit antennas in [14] and general case in [15]. In [15], a closed-form characterization of an optimal beam-forming strategy is derived. It is shown that the optimal solution is achieved by allocating the maximal sum power with phases matched to the complex channel coefficients. For the optimization problem with joint sum and per-antenna power constraints, it is shown that whenever the optimal power allocation of the corresponding problem with a sum power constraint only exceeds per-antenna power constraints, it is optimal to allocate the maximal per-antenna power to those antennas. Shortly after [15], similar results have been published in [16]. In [17], the optimal transmit strategy problem for point-to-point MIMO channel with joint sum and per-antenna power constraints has been studied. An iterative algorithm to find the optimal transmit strategy in closed-form using generalized water-filling solution is proposed. However, to our best knowledge, the wiretap channels and in particular the secrecy capacity with per-antenna power constraints and the secrecy capacity with joint sum and per-antenna power constraints have not been studied yet. Further, we are not aware that the optimal trade-off between communication rate and secrecy rate of a wiretap channel has been studied previously.

In this work we look at this problem considering MISO channels with different power constraint settings including sum power constraint only, per-antenna power constraints only, and joint sum and per-antenna power constraints. The optimal trade-off between communication rate and secrecy rate of MISO wiretap channels is motivated by the fact that the

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optimal communication strategy for wiretap channel is using a two-layer codebook where the eavesdropper can effectively decode the message encoded using the public layer codebook. The idea of the coding scheme is that the decoding capability of the eavesdropper is exhausted by the public message, while the legitimate receiver can also decode the message of the second layer. Therefore, instead of sending some useless random message on the public layer, a useful message can be communicated non-securely to the legitimate receivers. Since the maximal transmission rate and secrecy rate are, in general, achieved by different transmit strategies, we face a trade-off between both objectives which we will study in the following.

The rest of this paper is organized as follows. We start by briefly introducing the system model, sets of power constraints including sum power constraint only, per-antenna power constraints only, and joint sum and per-antenna power constraints. After that an equivalent formulation of the weighted rate sum maximization between the transmission rate and the secrecy rate is derived. The weighted rate sum optimal rate pairs provide a characterization of the boundary of the region of the achievable transmission rate and the secrecy rate. It is shown that the optimal transmit strategy of MISO wiretap channels with sum power constraint is a beam-forming strategy. Further, closed-form solutions of the optimal transmit strategy under per-antenna power constraints only and joint sum and per-antenna power constraint considering only two transmit antennas are shown in next sections. These solutions allow us to come up with characterizations of the boundary of the regions between the transmission rate and the secrecy rate. The results are then illustrated and discussed in numerical examples. Finally, we provide some remarks and conclusions.

II. System Model and Power Constraint

We consider a MISO wiretap channel with multiple antennas at the transmitter and single antenna at both legitimate receiver and eavesdropper. Let N_t be the number of transmit antennas. For each channel use, the received signals at the legitimate receiver and the eavesdropper are given as follows

$$y_r = \mathbf{h}_r^H \mathbf{x} + z_r, \tag{1}$$
$$y_e = \mathbf{h}_e^H \mathbf{x} + z_e, \tag{2}$$

where $\mathbf{x} = [x_1, \ldots, x_{N_t}]^T \in \mathbb{C}^{N_t \times 1}$ is the random complex transmit signal vector, $\mathbf{h}_r = [h_{r1}, \ldots, h_{rN_t}]^T \in \mathbb{C}^{N_t \times 1}$ and $\mathbf{h}_e = [h_{e1}, \ldots, h_{eN_t}]^T \in \mathbb{C}^{N_t \times 1}$ are channel coefficient vectors between the transmitter and legitimate receiver, and between the transmitter and eavesdropper. z_r and z_e are the independent additive white complex Gaussian noise terms with powers σ_r^2 and σ_e^2. In this paper, we assume without loss of generality that $\sigma_r^2 = \sigma_e^2 = 1$. The perfect channel state information (CSI) at the transmitter is also assumed. We use a Gaussian distributed codebook with covariance $\mathbf{Q} = \mathbb{E}[\mathbf{x}\mathbf{x}^H]$ which specifies the transmit strategy.

Let P_{tot} denote the maximal average sum transmit power. Let $\hat{P}_k$, $\forall k = 1, \ldots, N_t$, denote the maximal average transmit power at the k-th antenna. Further, let $\mathcal{S}(\hat{\mathbf{p}})$, $\hat{\mathbf{p}} := [P_{tot}, \hat{P}_1, \ldots, \hat{P}_{N_t}]$, denote the set of all transmit strategies satisfying the power constraints $\hat{\mathbf{p}}$, i.e.,

$$\mathcal{S}(\hat{\mathbf{p}}) := \{\mathbf{Q} \succeq 0 : \mathrm{tr}(\mathbf{Q}) \leq P_{tot}, \mathbf{e}_k^T \mathbf{Q} \mathbf{e}_k \leq \hat{P}_k, \forall k \in \mathcal{I}\} \tag{3}$$

where $\mathcal{I} := \{1, \ldots, N_t\}$ and $\mathbf{e}_k$ is the k^{th} Cartesian unit vector. Depending on the per-antenna power constraints $\hat{P}_k$ and the sum power constraint P_{tot}, we can identify three different cases:

- *Sum power constraint only* is considered when the per-antenna power constraints are never active, i.e., $P_{tot} < \min_k(\hat{P}_k)$.
- *Per-antenna power constraints only* is considered when the sum power constraint is never active, i.e., $P_{tot} > \sum_{i=1}^{N_t} \hat{P}_k$.
- *Joint sum and per-antenna power constraints* are considered when the power relations satisfy $\min_k(\hat{P}_k) \leq P_{tot} \leq \sum_{i=1}^{N_t} \hat{P}_k$, i.e., both sum and per-antenna power constraints can be active.

III. Trade-off and Problem Formulation

A. Trade-off Between Transmission Rate and Secrecy Rate

The secrecy capacity of the Gaussian MISO channel [4] with a given set of power constraints $\mathcal{S}(\hat{\mathbf{p}})$ is given by

$$C_s(\hat{\mathbf{p}}) = \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} C_s(\mathbf{Q}), \tag{4}$$

where $C_s(\mathbf{Q}) = C(\mathbf{Q}) - C_e(\mathbf{Q})$, $C(\mathbf{Q}) = \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r)$, $C_e(\mathbf{Q}) = \log(1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e)$. The capacity of MISO point-to-point channel is denoted as

$$C(\hat{\mathbf{p}}) = \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} C(\mathbf{Q}). \tag{5}$$

Proposition 1. *A necessary and sufficient condition for a positive secrecy rate of a Gaussian MISO wiretap channel, i.e., $C_s(\boldsymbol{Q}) > 0$, is that $\boldsymbol{h}_r \boldsymbol{h}_r^H$ - $\boldsymbol{h}_e \boldsymbol{h}_e^H \in \mathbb{C}^{N_t \times N_t}$ has to have a positive eigenvalue.*

Proof. The proof of the Proposition 1 can be found in Appendix A. □

Next, we define the region describing the trade-off between the point-to-point transmission rate and wiretap secrecy rate with a given set of power constraints. Let $\mathcal{R}_{MISO}$ be the rate region defined as

$$\mathcal{R}_{MISO}(\hat{\mathbf{p}}) = \{[R, R_s] : 0 \leq R_s \leq R \leq C(\mathbf{Q}), R_s \leq C_s(\mathbf{Q}), \mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})\}. \tag{6}$$

Note that the secrecy rate is a fraction of the transmission rate and $0 \leq R_s \leq R$ has to be satisfied. Therefore, the boundary of the rate region is also bounded by the straight line for which $R = R_s$. It can be seen from (6) that $\mathcal{R}_{MISO}(\hat{\mathbf{p}})$ is not necessarily a convex set since $C_s(\mathbf{Q})$ is non-convex in $\mathbf{Q}$. However, if we allow time-sharing between rate pairs, we can obtain any achievable points in the convex hull of the rate region which we denoted by

$$\mathcal{C}_{MISO}(\hat{\mathbf{p}}) = \mathrm{Conv}\{[R, R_s] : 0 \leq R_s \leq R \leq C(\mathbf{Q}), R_s \leq C_s(\mathbf{Q}), \mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})\}. \tag{7}$$

The optimal region $\mathcal{C}_{MISO}(\hat{\mathbf{p}})$ is convex and given by the downward comprehensive hull of the weighted optimal rate pairs in $\mathcal{R}_{MISO}(\hat{\mathbf{p}})$ conditioned that $0 \leq R_S \leq R$ holds. Therefore, the region $\mathcal{R}_{MISO}(\hat{\mathbf{p}})$ can be characterized by the set of all weighted rate sum optimal rate pairs. The weighted rate sum for a given weight vector $\mathbf{w} = [w_1, w_2] \in \mathbb{R}_+^2$ with $w_1 + w_2 = 1$ is given by

$$R_{\sum}(\mathbf{Q}, \mathbf{w}) := w_1 C(\mathbf{Q}) + w_2 C_s(\mathbf{Q}), \tag{8}$$

or equivalently

$$R_{\sum}(\mathbf{Q}, \mathbf{w}) = C(\mathbf{Q}) - w_2 C_e(\mathbf{Q}). \tag{9}$$

The remaining question is to find the optimal transmit strategy $\mathbf{Q}_{opt}(\mathbf{w})$ such that the weighted rate sum in (8) is maximized for a given vector $\mathbf{w}$. The optimization problem to find the maximal weighted rate sum for MISO wiretap channels with a given set of power constraints $\mathcal{S}$ can be written as follows

$$\begin{aligned} \max_{\mathbf{Q}} \quad & R_{\sum}(\mathbf{Q}, \mathbf{w}), \\ \text{s.t.} \quad & \mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}}). \end{aligned} \tag{10}$$

In the following, we provide closed-form solutions to find the optimal transmit strategies that characterize the trade-off between the transmission rate of MISO point-to-point channel and the secrecy rate of MISO wiretap channel without time-sharing corresponding to the three different power constraint cases.

B. Equivalent Problem Formulation

Since the objective function of (10) is not concave for $w_2 > 0$, the optimization problem to find the maximal weighted rate sum is a non-convex optimization problem. Therefore, we first reformulate (10) to an equivalent optimization problem using the following lemma.

Lemma 1 ([18, Lemma 2, scalar case])**.** *Consider the function $f(D) = -DE + \log(D) + 1$ where $D, E \in \mathbb{R}, E > 0$. Then,*

$$\max_{D>0} f(D) = \log(E^{-1}), \tag{11}$$

with the optimum value $D^\star = E^{-1}$.

By applying Lemma 1 with $E_i = 1 + \mathbf{h}_i^H \mathbf{Q} \mathbf{h}_i$, $f_i(D_i) = -D_i E_i + \log(D_i) + 1$, $i \in \{r, e\}$, the optimization problem (10) can be expressed as

$$\begin{aligned} R_{\sum}(\hat{\mathbf{p}}) &= \max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r) - w_2 \log(1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e) \\ &= \max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \{ -\max_{D_r>0} f_r(D_r) + w_2 \max_{D_e>0} f_e(D_e) \} \\ &= \max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \{ \min_{D_r>0} (-f_r(D_r)) + w_2 \max_{D_e>0} f_e(D_e) \} \\ &= \max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \{ \min_{D_r>0} (D_r(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r) - \log(D_r) - 1) \\ &\quad + w_2 \max_{D_e>0} (-D_e(1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e) + \log(D_e) + 1) \} \end{aligned} \tag{12}$$

Thus, for given D_r, D_e, and w_2, the optimization problem to obtain the optimal transmit strategy $\mathbf{Q}_{opt}(\hat{\mathbf{p}}, t)$ for a given t can be written as

$$\mathbf{Q}_{opt}(\hat{\mathbf{p}}, t) = \arg \max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - t \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e. \tag{13}$$

where $t = w_2 \frac{D_e}{D_r}$. Note that the objective function in (13) can be written as $\mathrm{tr}(\mathbf{AQ})$ with $\mathbf{A} = \mathbf{h}_r \mathbf{h}_r^H - t \mathbf{h}_e \mathbf{h}_e^H$ for a given t.

In the following, we aim to find the optimal transmit strategy solution with different sets of power constraints. The key idea is that we first fix the value of t and find the optimal transmit strategy for this t. Given the optimal transmit strategy $\mathbf{Q}_{opt}(\hat{\mathbf{p}}, t)$ for a given t we can then compute the corresponding D_e and D_r. Therewith the weight w_2 corresponding to the optimal transmit strategy can be found. Thus, if we compute the optimal transmit strategy for all possible t, we obtain a parametrization of the boundary without time-sharing. This result will be made more clear in Theorem 1. This set can be then also used in a line search algorithm to find the best value of t for a specific weight w_2.

IV. Parametrization of the Boundary of Rate Region

The most interesting part of the boundary is the curved section which corresponds to the set of Pareto optimal rate pairs of $\mathcal{R}_{MISO}(\hat{\mathbf{p}})$. Next, we will provide a parametrization of this section. When $t = 0$, we optimize $C(\mathbf{Q})$ which corresponds to $w_2 = 0$ only, i.e., (10) becomes (5). If $w_2 = 1$, then (10) becomes (4). In the next theorem, it is shown that this corresponds to $t = 2^{C_s(\hat{\mathbf{p}})}$.

Theorem 1. *It is sufficient for the boundary of the rate region $\mathcal{R}_{MISO}(\hat{\boldsymbol{p}})$ to consider*

$$0 \leq t \leq 2^{C_s(\hat{\boldsymbol{p}})}, \tag{14}$$

where $C_s(\hat{\boldsymbol{p}})$ is the secrecy capacity.

Proof. The proof of Theorem 1 can be found in Appendix B. The idea of this proof is to show that with a given weight, the optimization problems (13) and (10) have the same optimal solutions. □

With the corresponding sets of power constraints, the optimal transmit strategy $\mathbf{Q}_{opt}(\hat{\mathbf{p}}, t)$, $t \in [0, 2^{C_s(\hat{\mathbf{p}})}]$ characterizes the curved section of the rate region $\mathcal{R}_{MISO}$ which is given by

$$\begin{aligned} \mathcal{R}_{MISO}(\hat{\mathbf{p}}) = \{ &[R, R_s] : 0 \leq R_s \leq R \leq C(\mathbf{Q}(\hat{\mathbf{p}}, t)), \\ &R_s \leq C_s(\mathbf{Q}(\hat{\mathbf{p}}, t)), t \in [0, 2^{C_s(\hat{\mathbf{p}})}] \}. \end{aligned} \tag{15}$$

V. Optimal Transmit Strategy

In this section, we aim to find closed-form solutions of the optimal transmit strategies for a given t of the weighted rate sum optimization problem for the MISO wiretap channel with a sum power constraint only, per-antenna power constraints only, and with joint sum and per-antenna power constraints respectively. First we discuss the rank of the optimal transmit strategy.

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Lemma 2. *The rank of the optimal transmit strategy* (13) *for a given t is at most two, i.e.,* $\mathrm{rank}(\boldsymbol{Q}) \leq 2$.

Proof. The proof of Lemma 2 can be found in Appendix C. □

Since a transmit strategy of rank one actually denotes a beam-forming strategy, it will be interesting to see where beam-forming is sufficient for optimality. This is the case in the sum power constraint only setup as well as for the per-antenna power constraint cases if one considers only two transmit antennas.

A. Optimal Transmit Strategy with Sum Power Constraint

In the sum power constraint only case, the per-antenna power constraints are never active, e.g., if we have $P_{tot} < P_k$ $\forall k$. Let $\mathcal{S}_{SPC}$ denote the set of all power allocations which satisfy the sum power constraint P_{tot} only. Then we can obtain the set as $\mathcal{S}_{SPC} = \{\mathbf{Q} \succeq 0 : \mathrm{tr}(\mathbf{Q}) \leq P_{tot}\}$. The equivalent problem of finding the weighted rate sum optimal transmit strategy for the MISO wiretap channel with sum power constraint only for a given t can be written as

$$\mathbf{Q}_{SPC}(t) = \arg \max_{\mathbf{Q} \in \mathcal{S}_{SPC}} \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - t \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e. \qquad (16)$$

Theorem 2. *The closed-form expression for the optimal transmit strategy of* (16) *is given by*

$$\boldsymbol{Q}_{SPC}(t) = P_{tot} \boldsymbol{v} \boldsymbol{v}^H \qquad (17)$$

where $\boldsymbol{v}$ is the eigenvector associated with the largest eigenvalue of $\boldsymbol{h}_r \boldsymbol{h}_r^H - t \boldsymbol{h}_e \boldsymbol{h}_e^H$ for a given t.

Proof. The proof of Theorem 2 can be found in the Appendix D. □

Remark 1. *The rank of the optimal transmit strategy of* (17) *for a given t is one, i.e., a beam-forming strategy is optimal.*

In the next sections, we aim to find the optimal solution for the two remaining cases with per-antenna power constraints only and with joint sum and per-antenna power constraints.

B. Optimal Transmit Strategy with Per-antenna Power Constraints

In contrast to the sum power constraint only case, the per-antenna power constraints only case is considered when the sum power constraint is never active, e.g., $P_{tot} > \sum_{k \in \mathcal{I}} \hat{P}_k$. Let $\mathcal{S}_{PAPC}$ denote the set of all power allocations which satisfy the per-antenna power constraints $\hat{P}_K$, $\forall k \in \mathcal{I}$, only. Then $\mathcal{S}_{PAPC} = \{\mathbf{Q} \succeq 0 : \mathbf{e}_k^T \mathbf{Q} \mathbf{e}_k \leq \hat{P}_k, k \in \mathcal{I}\}$. The equivalent problem of finding the weighted rate sum optimal transmit strategy for the MISO wiretap channel with per-antenna power constraints only for a given t can be written as

$$\mathbf{Q}_{PAPC}(t) = \arg \max_{\mathbf{Q} \in \mathcal{S}_{PAPC}} \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - t \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e. \qquad (18)$$

In general, the diagonal elements of the optimal transmit strategy can be obtained by the proposition below.

Proposition 2. *The optimal transmit strategy $\boldsymbol{Q}_{PAPC}(t)$ of problem* (18) *has diagonal elements $q_{kk} = \hat{P}_k$, $\forall k \in \mathcal{I}$.*

Proof. The proof of Proposition 2 can be found in Appendix E. □

Proposition 2 shows that for the per-antenna power constraint only problem, it is optimal to allocate maximal individual power on the transmit antennas. The remaining problem is to find off-diagonal elements of $\mathbf{Q}_{PAPC}(t)$ for a given t.

The main difficulty here is the positive semi-definite constraint. To overcome this, we consider a relaxed optimization problem involving a 2×2 principal minors of $\mathbf{Q}_{PAPC}(t)$ similarly as done in [19]. Let $\mathbf{X}_{k,l}(t)$ is a principal minor matrix which is obtained by removing $N_t - 2$ columns, except k and l, and the corresponding transposed $N_t - 2$ rows of $\mathbf{Q}$. Then, $\mathbf{X}_{k,l}(t)$ is given as

$$\mathbf{X}_{k,l}(t) = \begin{bmatrix} \hat{P}_k & q_{kl}^*(t) \\ q_{kl}(t) & \hat{P}_l \end{bmatrix} \qquad (19)$$

where $k, l \in \mathcal{I}$, $k \neq l$. Therewith, we can formulate a relaxed optimization problem as follows

$$\begin{aligned} &\max_{\mathbf{Q}} \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - t \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e, \qquad (20) \\ &\text{s. t. } q_{kk} = \hat{P}_k, \forall k \in \mathcal{I} \\ &\qquad \mathbf{X}_{k,l}(t) \succeq 0, k, l \in \mathcal{I}, k \neq l. \end{aligned}$$

The off-diagonal elements of the covariance matrix in (20) then can be obtained using the following theorem.

Theorem 3. *The optimal transmit strategy $\boldsymbol{Q}_{PAPC-R}(t)$ of the relaxed optimization problem* (20) *has off-diagonal elements*

$$q_{kl}(t) = \frac{h_{rk}^* h_{rl} - t h_{ek}^* h_{el}}{|h_{rk}^* h_{rl} - t h_{ek}^* h_{el}|} \sqrt{\hat{P}_k \hat{P}_l}. \qquad (21)$$

with $k, l \in \mathcal{I}$; $k \neq l$.

Proof: The proof of Theorem 3 can be found in Appendix F. ■

From Proposition 2 and Theorem 3, we have the following conclusions and remarks.

Remark 2. *If the solution* (21) *leads to a positive semi-definite solution, then it is also an optimal solution of* (18).

Corollary 1. *If there are only two transmit antennas, i.e.,$N_t = 2$, then* (21) *always leads to positive semi-definite solution with eigenvalues zero and $\hat{P}_1 + \hat{P}_2$, i.e., the optimal solution* (21) *of the relaxed optimization problem* (20) *is actually the optimal solution of* (18).

Remark 3. *The optimal transmit strategy $\boldsymbol{Q}_{PAPC-R}(t)$ of the relaxed optimization problem* (20) *has rank one solution, i.e., beam-forming is optimal.*

The numerical experiments show that the results in Proposition 2 and Theorem 3 also hold for $N_t > 2$, i.e., Proposition 2 and Theorem 3 specifies an optimal solution.

C. Optimal Transmit Strategy with Joint Sum and Per-antenna Power Constraints

In this section, we discuss the optimization for the wiretap channel for the interesting case when both sum and per-antenna power constraints can be active, i.e., $\min_k(\hat{P}_k) \leq P_{tot} \leq \sum_{i=k}^{N_t} \hat{P}_k$ [15]. Let $\mathcal{S}_{PAPC}$ denote the set of all power allocations which satisfy the joint sum and per-antenna power constraints such that $\min_k(\hat{P}_k) \leq P_{tot} \leq \sum_{i=k}^{N_t} \hat{P}_k$. Then we can obtain the set $\mathcal{S}_{JSPC} = \{\mathbf{Q} \succeq 0 : \mathrm{tr}(\mathbf{Q}) \leq P_{tot}, \mathbf{e}_k^T \mathbf{Q} \mathbf{e}_k \leq \hat{P}_k, \forall k \in \mathcal{I}\}$. Similar to the optimization problem with sum power constraint only and per-antenna power constraints only, by applying Lemma 1, an equivalent optimization problem for finding the optimal transmit strategy for the wiretap channel with joint sum and per-antenna power constraints can be stated as

$$\mathbf{Q}_{JSPC}(t) = \arg\max_{\mathbf{Q} \in \mathcal{S}_{JSPC}} \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - t \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e \tag{22}$$

for a given t.

Proposition 3. *The optimal solution for the MISO wiretap channel with joint sum and per-antenna power constraints problem can be achieved when the transmit strategy uses full power P_{tot}, i.e.,* $\mathrm{tr}(\boldsymbol{Q}_{JSPC}(t)) = P_{tot}$.

Proof. The proof of Proposition 3 can be found in Appendix G. □

This proposition permits us to consider only transmit strategies which allocate full power P_{tot}. If the solution of the sum power constraint only problem does not violate the per-antenna power constraints, then it is also the solution of the joint sum and per-antenna power constraints problem. However, this is not always the case. In such cases, the maximum power will be allocated to those antennas where the sum power constraint only optimal solution violates the per-antenna power constraints. In the next theorem we show how this leads to the optimal transmit strategy considering two transmit antennas.

Theorem 4. *Let $\boldsymbol{Q}_{SPC}(t)$ be the optimal transmit strategy under the sum power constraint only. Let $\mathcal{P} := \{k \in \mathcal{I} : \boldsymbol{e}_k^T \boldsymbol{Q}_{SPC}(t) \boldsymbol{e}_k > \hat{P}_k\}$ where $\mathcal{I} := \{1, 2\}$. Then, for the optimization problem with joint sum and per-antenna power constraints, we have*

- *If $\mathcal{P} = \emptyset$, $\boldsymbol{Q}_{JSPC}(t) = \boldsymbol{Q}_{SPC}(t)$*
- *Otherwise $\boldsymbol{Q}_{JSPC}(t)$ has diagonal elements*

$$\begin{cases} \boldsymbol{e}_k^T \boldsymbol{Q}_{JSPC}(t) \boldsymbol{e}_k = \hat{P}_k, \\ \boldsymbol{e}_l^T \boldsymbol{Q}_{JSPC}(t) \boldsymbol{e}_l = P_{tot} - \hat{P}_k, \end{cases} \tag{23}$$

and off-diagonal elements

$$q_{kl}^{\star}(t) = \frac{h_{rk}^* h_{rl} - t h_{ek}^* h_{el}}{|h_{rk}^* h_{rl} - t h_{ek}^* h_{el}|} \sqrt{\hat{P}_k (P_{tot} - \hat{P}_k)}, \tag{24}$$

with $k \in \mathcal{P}$, $l \neq k$.

Proof. Since the case with $\mathcal{P} = \emptyset$ is obvious, we focus to prove the remaining case. Consider a scalar function $f(\mathbf{Q}) := \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - t \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e$. From [20, Lemma 3.10], we know that the scalar function $f(\mathbf{Q}) : \mathbf{Q} \to \mathbb{R}_+$ is monotone. Therefore, if any k-th optimal power of the sum power constraint only solution violated the per-antenna power constraints, it has to set equal to the maximal individual power $\hat{P}_k$, $k = \{1, 2\}$. Consequently, the remaining optimal transmit power has to set equal to $P_{tot} - \hat{P}_k$ and the off-diagonal elements of the optimal transmit strategy for the wiretap channel with joint sum and per-antenna power constraints are then calculated using Theorem 3 with the corresponding optimal transmit powers $\mathbf{e}_k^T \mathbf{Q}_{JSPC}(t) \mathbf{e}_k = \hat{P}_k$ and $\mathbf{e}_l^T \mathbf{Q}_{JSPC}(t) \mathbf{e}_l = P_{tot} - \hat{P}_k$, $k \in \mathcal{P}$, $l \neq k$. □

Remark 4. *Since the solution for the two antennas case is either the solution of the sum power constraint only problem or the per-antenna power constraints only problem, beamforming is again optimal.*

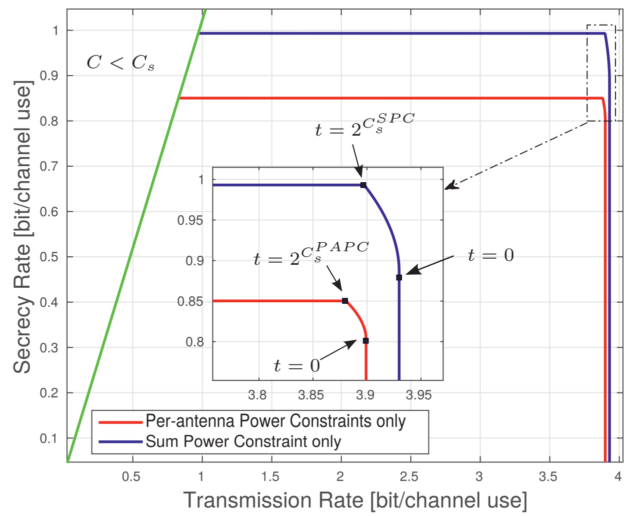

Fig. 1: The optimal regions between the transmission rates and the secrecy rate with sum power constraint only $P_{tot} = 14$ and per-antenna power constraints only $\hat{P}_1 = 6$ and $\hat{P}_2 = 8$

VI. Numerical Examples

In this section, numerical examples for the optimization problems with sum power constraint only and per-antenna power constraint only with two antennas at the transmitter, and one antenna at legitimate receiver and eavesdropper each are shown. We first provide a MISO wiretap channel with two transmit antennas. The complex channel coefficients corresponds to legitimate receiver and eavesdropper are given as $\mathbf{h}_r = [0.3737 + 0.8912i, 0.9795 + 1.2926i]^T$ and $\mathbf{h}_e = [0.4387 + 0.7655i, 0.3816 + 0.7952i]^T$. The powers on maximum transmit power on antennas are set as $\hat{P}_1 = 6$ and $\hat{P}_2 = 8$. The sum power constraint $P_{tot} = 14$.

Fig. 1 depicts optimal regions between the transmission rate and the secrecy rate of the wiretap channel with two different set of power constraints: sum power constraint only and per-antenna power constraints only. The figure shows that the regions are fully characterized by the curved sections which can be obtained from the parametrized optimal solution in Theorem 1. It also shows the trade-off between the transmission rate and the secrecy rate. For instance, we can see that the strategies that maximize the secrecy rates, $t = 2^{C_s^{SPC}}$ with $C_s^{SPC} = 0.995$ for the case with sum power constraint only and $t = 2^{C_s^{PAPC}}$ with $C_s^{PAPC} = 0.85$ for the case with per-antenna power constraints only.

VII. Conclusions

In this paper, we investigated the trade-off between the MISO point-to-point transmission rate and the wiretap secrecy rate considering different power constraint settings. We derived an equivalent formulation of the optimization problem and characterized capacity region from the optimal rate pairs using a parametrization of the rate region. We showed that beamforming is the optimal solution for the optimization problem with a sum power constraint only. It is also the optimal solution for the remaining optimization problems with two transmit antennas only. Under per-antenna power constraints only, the diagonal elements of the covariance matrix are set to be equals maximal individual transmit power on every antennas. The optimal transmit strategy with joint sum and per-antenna power constraints is achieved when full sum transmit power is used. In addition, the transmit power is set equal to the maximal per-antenna transmit power if an optimal power allocation of the sum power constraint only solution exceeds a per-antenna power constraint.

Appendix

A. Proof of Proposition 1

The proof of the proposition includes two following steps:

Step 1: For the necessary part, we need to show that for $C_s(\mathbf{Q}) > 0$, $\mathbf{h}_r\mathbf{h}_r^H - \mathbf{h}_e\mathbf{h}_e^H$ has at least one positive eigenvalue. The secrecy capacity can be written as

$$\begin{aligned} C_s(\mathbf{Q}) &= \log(1 + \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r) - \log(1 + \mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e) \\ &= \log\left(1 + \frac{\mathrm{tr}\{(\mathbf{h}_r\mathbf{h}_r^H - \mathbf{h}_e\mathbf{h}_e^H)\mathbf{Q}\}}{1 + \mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e}\right) > 0. \end{aligned} \tag{25}$$

Since $1+\mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e > 0$, it suffices to show that for $\mathrm{tr}\{(\mathbf{h}_r\mathbf{h}_r^H - \mathbf{h}_e\mathbf{h}_e^H)\mathbf{Q}\} > 0$ the matrix $\mathbf{A} = \mathbf{h}_r\mathbf{h}_r^H - \mathbf{h}_e\mathbf{h}_e^H$ has to have at least one positive eigenvalue. We have $\mathrm{tr}(\mathbf{A}) = \sum_{i\in\mathcal{I}} \lambda_i(\mathbf{Q})\mathbf{v}_i^H\mathbf{A}\mathbf{v}_i > 0$. Since $\lambda_i(\mathbf{Q}) \geq 0$, $\forall i$, for a largest eigenvalue $\lambda_{\max}(\mathbf{A})$, we have

$$\lambda_{\max}(\mathbf{A}) = \max_{\|\mathbf{w}_i\|} \mathbf{w}_i^H\mathbf{A}\mathbf{w}_i \geq \mathbf{v}_i^H\mathbf{A}\mathbf{v}_i > 0. \tag{26}$$

Step 2: For the sufficient part, we need to show that if $\mathbf{A} = \mathbf{h}_r\mathbf{h}_r^H - \mathbf{h}_e\mathbf{h}_e^H$ has a positive eigenvalue, then there exists $\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})$ such that $C_s(\mathbf{Q}) > 0$.

Since $\mathbf{A}$ has a positive eigenvalue, there exist a vector $\mathbf{v} : \|\mathbf{v}\| = 1$ such that $\mathbf{v}^H\mathbf{A}\mathbf{v} > 0$. This implies that we can construct $\mathbf{Q} = \xi\mathbf{v}\mathbf{v}^H$, $\xi > 0$, such that $\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})$ and $\mathrm{tr}(\mathbf{A}\mathbf{Q}) > 0$. Then we have

$$\begin{aligned} C_s(\mathbf{Q}) &= \log(1 + \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r) - \log(1 + \mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e) \\ &= \log\left(1 + \frac{\mathrm{tr}(\mathbf{A}\mathbf{Q})}{1 + \mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e}\right) > 0. \end{aligned} \tag{27}$$

■

B. Proof of Theorem 1

In (13), we defined $t = w_2\frac{D_e}{D_r}$. Since $w_2 \in [0,1]$, $D_e, D_r > 0$, it suffices to consider the value of t as $t \in [0, t_{\max}]$, where $t_{\max}$ is the largest value of t. In the following, we show that the optimization problem (13) and (10) have the same solutions. The optimization problem in (13) can be equivalently rewritten as

$$\mathbf{Q}_{opt}(\hat{\mathbf{p}}) = \arg\max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} D_r\mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r - w_2D_e\mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e. \tag{28}$$

Let $\phi(\mathbf{Q}, \mathbf{w}, D_r, D_e)$ denote the objective function of (28). Suppose that $\mathbf{Q}^\star$ is an optimal solution of (13) and (28), then following [21, Section 4.2] and Lemma 1 we know that for a given $\mathbf{Q}^\star$ the corresponding values $D_r^\star$ and $D_e^\star$ are given by

$$D_r^\star = (1 + \mathbf{h}_r^H\mathbf{Q}^\star\mathbf{h}_r)^{-1}, \tag{29}$$

$$D_e^\star = (1 + \mathbf{h}_e^H\mathbf{Q}^\star\mathbf{h}_e)^{-1}. \tag{30}$$

Thus,

$$\begin{aligned} &\frac{\partial\phi(\mathbf{Q}^\star, \mathbf{w}, D_r^\star, D_e^\star)}{\partial\mathbf{Q}^\star} = D_r^\star\mathbf{h}_r\mathbf{h}_r^H - w_2D_e^\star\mathbf{h}_e\mathbf{h}_e^H \\ &= \mathbf{h}_r(1 + \mathbf{h}_r^H\mathbf{Q}^\star\mathbf{h}_r)^{-1}\mathbf{h}_r^H - w_2\mathbf{h}_e(1 + \mathbf{h}_e^H\mathbf{Q}^\star\mathbf{h}_e)^{-1}\mathbf{h}_e^H \\ &= \frac{\partial R_{\sum}(\mathbf{Q}^\star, \mathbf{w})}{\partial\mathbf{Q}^\star}. \end{aligned} \tag{31}$$

This implies that $\mathbf{Q}^\star$ is also a stationary point of (10). Note that with growing w_2, $D_e^\star$ becomes larger and $D_r^\star$ becomes smaller so that t_{max} is obtained at $w_2 = 1$ when (9) corresponds to the secrecy optimization problem (4). Accordingly, for $w_2 = 1$, we can write

$$\begin{aligned} C_s(\hat{\mathbf{p}}) &= \log(1 + \mathbf{h}_r^H\mathbf{Q}^\star\mathbf{h}_r) - \log(1 + \mathbf{h}_e^H\mathbf{Q}^\star\mathbf{h}_e) \\ &= \log\left(\frac{1 + \mathbf{h}_r^H\mathbf{Q}^\star\mathbf{h}_r}{1 + \mathbf{h}_e^H\mathbf{Q}^\star\mathbf{h}_e}\right) \\ &= \log(\frac{D_e^\star}{D_r^\star}) \\ &= \log(t_{\max}). \end{aligned} \tag{32}$$

Therefore, $t_{\max} = 2^{C_s(\hat{\mathbf{p}})}$, i.e., it is sufficient for the boundary of the rate region to consider $t \in [0, 2^{C_s(\hat{\mathbf{p}})}]$. This proves Theorem 1. ■

C. Proof of Lemma 2

Consider the optimization problem

$$\max_{\mathbf{Q}} \ \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r - t\mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e, \ \mathrm{s.t.}\, \mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}}). \tag{33}$$

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The Lagrangian for problem (33) is given by

$$\mathcal{L} = \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r - t\mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e - \mathrm{tr}(\mathbf{D}(\mathbf{Q}-\hat{\mathbf{P}})) - \mu(\mathrm{tr}(\mathbf{Q}) - P_{tot}) + \mathrm{tr}(\mathbf{M}\mathbf{Q}), \quad (34)$$

where $\mathbf{D} = \mathrm{diag}\{\nu_i\}$ is a diagonal matrix of Lagrangian multiplier for the per-antenna power constraints, μ is the Lagrangian multiplier for the sum power constraint, $\mathbf{M}$ is the Lagrangian multiplier for the positive semi-definite constraint, and $\hat{\mathbf{P}} = \mathrm{diag}\{\hat{P}_i\}$, $\forall i = \mathcal{I}$ is a diagonal matrix of the per-antenna power constraints.

Taking the first derivative of the Lagrangian above and set equal to zero, we have

$$\frac{\partial \mathcal{L}}{\partial \mathbf{Q}} = \mathbf{h}_r\mathbf{h}_r^H - t\mathbf{h}_e\mathbf{h}_e^H - \mathbf{D} - \mu\mathbf{I} + \mathbf{M} \stackrel{!}{=} 0. \quad (35)$$

Using the slackness condition $\mathbf{M}\mathbf{Q} = 0$, we obtain

$$(\mathbf{h}_r\mathbf{h}_r^H - t\mathbf{h}_e\mathbf{h}_e^H)\mathbf{Q} = \mathbf{K}\mathbf{Q}, \quad (36)$$

where $\mathbf{K} = \mathbf{D} + \mu\mathbf{I}$. Since $\mathbf{K}$ has full rank, at the optimum, we have

$$\mathrm{rank}(\mathbf{Q}^\star) \leq \mathrm{rank}(\mathbf{h}_r\mathbf{h}_r^H) + \mathrm{rank}(\mathbf{h}_e\mathbf{h}_e^H) = 2. \quad (37)$$

■

D. Proof of Theorem 2

By using singular value decomposition, for a given t, we have $\mathbf{h}_r\mathbf{h}_r^H - t\mathbf{h}_e\mathbf{h}_e^H = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H$. Let $\tilde{\mathbf{Q}} = \mathbf{V}^H\mathbf{Q}\mathbf{V}$, we obtain $\tilde{\mathbf{Q}} \succeq 0$. Then

$$\begin{aligned}\mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r - t\mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e &= \mathrm{tr}\{(\mathbf{h}_r\mathbf{h}_r^H - t\mathbf{h}_e\mathbf{h}_e^H)\mathbf{Q}\} \\ &= \mathrm{tr}\{\mathbf{\Lambda}\tilde{\mathbf{Q}}\} \\ &= \mathrm{tr}\{\mathbf{\Lambda}\,\mathrm{diag}(\tilde{\mathbf{Q}})\} \\ &\leq \lambda_{max}P_{tot} \quad (38)\end{aligned}$$

with λ_{max} is the largest entry in $\mathbf{\Lambda}$ and $\mathrm{tr}(\tilde{\mathbf{Q}}) = \mathrm{tr}(\mathbf{Q}) = P_{tot}$.

Equation (38) holds with equality if $\tilde{\mathbf{Q}}$ is diagonal and has a unique nonzero entry equal to P_{tot} corresponding to the largest entry of $\mathbf{\Lambda}$. This implies that $\mathbf{Q}$ and $\mathbf{h}_r\mathbf{h}_r^H - t\mathbf{h}_e\mathbf{h}_e^H$ share the same eigenvectors and $\mathbf{Q}$ has rank one [22]. Therefore, we have $\mathbf{Q}_{SPC}(t) = P_{tot}\mathbf{v}\mathbf{v}^H$ where $\mathbf{v}$ is the eigenvector associated with the largest eigenvalue of $\mathbf{h}_r\mathbf{h}_r^H - t\mathbf{h}_e\mathbf{h}_e^H$ for a given t. This proves the Theorem 2. ■

E. Proof of Proposition 2

Consider the optimization problem

$$\max_{\mathbf{Q}} \ \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r - t\mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e, \ \mathrm{s.t.}\ \mathbf{Q} \in \mathcal{S}_{PAPC}. \quad (39)$$

The Lagrangian for the problem (39) is given by

$$\mathcal{L} = \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r - t\mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e - \mathrm{tr}(\mathbf{D}(\mathbf{Q}-\hat{\mathbf{P}})) + \mathrm{tr}(\mathbf{M}\mathbf{Q}), \quad (40)$$

where $\mathbf{D} = \mathrm{diag}\{\nu_i\}$ is a diagonal matrix of Lagrangian multiplier for the per-antenna power constraints, $\mathbf{M}$ is the Lagrangian multiplier for the positive semi-definite constraint, and $\hat{\mathbf{P}} = \mathrm{diag}\{\hat{P}_i\}$, $\forall i \in \mathcal{I}$ is a diagonal matrix of the per-antenna power constraints. Based on the KKT conditions, we then obtain a set of optimality conditions as follows

$$\mathbf{h}_r\mathbf{h}_r^H - t\mathbf{h}_e\mathbf{h}_e^H = \mathbf{D} + \mathbf{M} \quad (41)$$

$$\mathbf{M}\mathbf{Q} = 0 \quad (42)$$

$$\mathbf{D} \succ 0 \quad (43)$$

$$\text{Hermitian } \mathbf{Q}, \mathbf{M} \succeq 0. \quad (44)$$

Since $\mathbf{D} \succ 0$ has full-rank, at the optimum the power constraint must be met with equality, i.e., $q_{kk} = \hat{P}_k$, $\forall k \in \mathcal{I}$, otherwise we can always increase the power and get higher rate. This proves Proposition 2. ■

F. Proof of Theorem 3

Consider an optimization problem (20). The Lagrangian for problem (20) is given by

$$\mathcal{L} = \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r - t\mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e - \sum_{k\neq l}\lambda_{kl}(|q_{kl}(t)|^2 - \hat{P}_k\hat{P}_l) - \sum_k \mu_k(q_{kk} - \hat{P}_k), \quad (45)$$

where λ_{kl} and μ_k are the Lagrange multipliers, and $k, l \in \mathcal{I}$. Taking the first derivative of (45) and set it equal to zero, we have

$$\frac{\partial \mathcal{L}}{\partial q_{kl}} = h_{rk}^*h_{rl} - th_{ek}^*h_{el} - \lambda_{kl}q_{kl}(t) \stackrel{!}{=} 0, \quad (46)$$

or equivalently

$$q_{kl}(t) = \frac{h_{rk}^*h_{rl} - th_{ek}^*h_{el}}{\lambda_{kl}}. \quad (47)$$

Similar to [13], the optimal value of q_{kl} in (47) is obtained when its constraint is satisfied with equality, i.e., $|q_{kl}(t)|^2 = \hat{P}_k\hat{P}_l$. By combining this condition with (47), we have the value of $q_{kl}(t)$ as in (21). ■

G. Proof of Proposition 3

Given function $f(\mathbf{Q}) : \mathbf{Q} \rightarrow \mathbb{R}_+$, $f(\mathbf{Q}) := \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r - t\mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e = \mathrm{tr}(\mathbf{A}\mathbf{Q})$. From [20], it follows that for any positive semi-definite Hermitian matrices $\mathbf{Q}_1 \succeq \mathbf{Q}_2$, we have $f(\mathbf{Q}_1) \geq f(\mathbf{Q}_2)$. This implies that for (22) the optimal solution is achieved when the optimal transmit strategy allocates the maximal sum power P_{tot}, i.e., $\mathrm{tr}(\mathbf{Q}_{JSPC}(t)) = P_{tot}$.

■

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