Optimal Transmission Rate for MISO Channels with Joint Sum and Per-antenna Power Constraints

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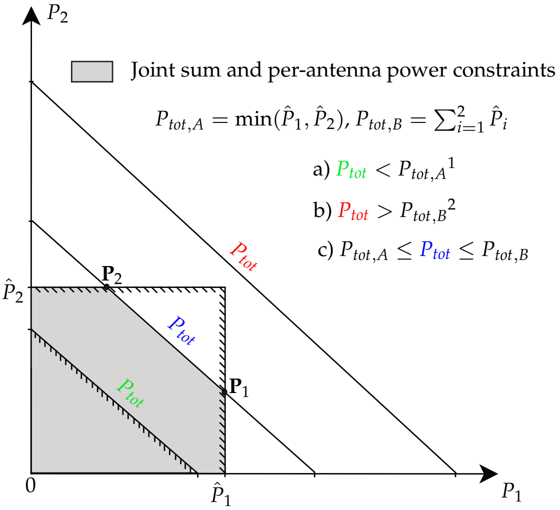
Overview

WHY JOINT SUM AND PER-ANTENNA POWER CONSTRAINTS?

- **Sum power constraints** are imposed e.g., by regulations or to limit the energy consumption,
- **Per-antenna power constraints** are imposed by hardware limitation of each RF chain,
- **Both motivations are simultaneously relevant** for practical systems, thus we consider a system with a **joint sum and per-antenna power constraints**.

FEASIBLE POWER ALLOCATION REGIONS



[1] I. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecommun.*, Nov. 1999.
[2] Mai Vu, "MISO Capacity with Per-antenna power constraint,"*IEEE Trans. on Commun.*, May 2011.

Formal definition of power constraints

- Sum Power Constraint:

$$\mathcal{S}_1 := \{\mathbf{Q} \succcurlyeq 0 : \mathrm{tr}(\mathbf{Q}) \leq P_{tot}\}.$$

- Per-antenna Power Constraints:

$$\mathcal{S}_2 := \{\mathbf{Q} \succcurlyeq 0 : \mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i \leq \hat{P}_i, i = 1, ..., n\}.$$

- Joint Sum and Per-antenna Power Constraints:

$$\begin{aligned}\mathcal{S}_3 &:= \mathcal{S}_1 \cap \mathcal{S}_2 \\ &= \{\mathbf{Q} \succcurlyeq 0 : \mathrm{tr}(\mathbf{Q}) \leq P_{tot}, \mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i \leq \hat{P}_i, i = 1, ..., n\}.\end{aligned}$$

$n \times 1$ MISO SYSTEM MODEL

$$y = \mathbf{x}^T \mathbf{h} + z$$

- Transmit signal $\mathbf{x} = [x_1, ..., x_n]^T \in \mathbb{C}^{n\times 1}$
- Channel $\mathbf{h} = [h_1, ..., h_n]^T \in \mathbb{C}^{n\times 1}$.
- Noise $z \sim \mathcal{CN}(0, \sigma^2)$

GOAL

- Covariance matrix:

$$\mathbf{Q} = \mathbb{E}\left[\mathbf{x}\mathbf{x}^H\right]$$

For $\mathbf{Q} \in \mathcal{S}_3$, Gaussian distributed input maximixed $I(\mathbf{x}; y)$ for the Gaussian MISO channel.

- Achievable transmission rate:

$$R(\mathbf{Q}) = \log\left(1 + \frac{1}{\sigma^2}\mathbf{h}^H\mathbf{Q}\mathbf{h}\right)$$

Find optimal transmit strategy $\mathbf{Q}$

Optimization problem

Convex Optimization Problem (OPi), $i = 1, 2, 3$

$$\begin{array}{ll} \text{maximize} & \log\left(1 + \frac{1}{\sigma^2}\mathbf{h}^H\mathbf{Q}\mathbf{h}\right) \\ \text{subject to} & \mathbf{Q} \in \mathcal{S}_i. \end{array}$$

- OP1 $\rightarrow$ Sum power constraint
- OP2 $\rightarrow$ Per-antenna power constraints
- OP3 $\rightarrow$ Joint sum and per-antenna power constraints

Properties of the optimal transmit strategy

Proposition 1

Beamforming is the optimal transmit strategy.

Proposition 2

The optimal transmit strategy uses full power P_{tot}, i.e., $\mathrm{tr}(\mathbf{Q}^{(3)}) = P_{tot}$.

PROPERTIES OF THE OPTIMAL TRANSMIT STRATEGY

Lemma (Optimal phase)

Let $\mathbf{q}^{(3)}$ be the optimal beamforming vector corresponding to the optimal covariance matrix $\mathbf{Q}^{(3)}$. Then

$$\mathbf{q}^{(3)} \in \mathbb{Q} := \left\{ \mathbf{q} : \mathbf{q} = \left[\frac{\sqrt{P_1} h_1^*}{|h_1|}, \ldots, \frac{\sqrt{P_n} h_n^*}{|h_n|} \right]^T, \mathbf{q}\mathbf{q}^H \in \mathcal{S}_3 \right\}.$$

Optimal power allocation for OP3

Theorem (Problem reduction)

Let $\mathcal{I} \subseteq \{1, \ldots, n\}$ and $\mathcal{P}_V := \{i \in \mathcal{I} : P_i^{(1)} > \hat{P}_i\}$, if $\mathcal{P}_V = \emptyset$ then $P_i^{(3)} = P_i^{(1)}\ \forall i \in \mathcal{I}$, else $P_i^{(3)} = \hat{P}_i\ \forall i \in \mathcal{P}_V$ and the remaining optimal powers can be computed by solving a reduced optimization problem

$$\arg\max_{\mathbf{q}' \in \mathbb{Q}'} |\mathbf{h}'^H \mathbf{q}'|^2$$

where $\mathbf{h}' = [h_i]^T_{i \in \mathcal{P}_V^c} \in \mathbb{C}^{|\mathcal{P}_V^c| \times 1}$, $\mathbb{Q}' := \{\mathbf{q}' : \sum_{i \in \mathcal{P}_V^c} |q_i|^2 \leq P_{tot} - \sum_{i \in \mathcal{P}_V} \hat{P}_i, |q_i|^2 \leq \hat{P}_i, i \in \mathcal{P}_V^c\}$ and $\mathcal{P}_V^c = \mathcal{I} \setminus \mathcal{P}_V$.

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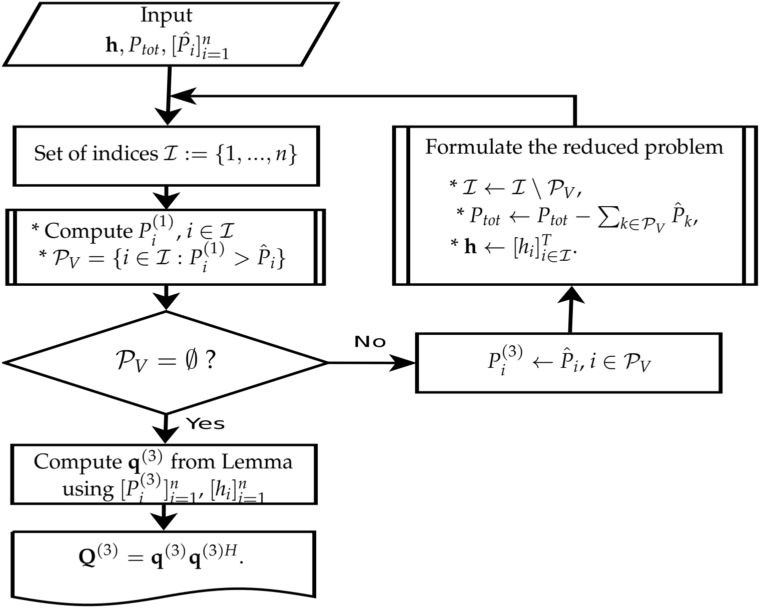
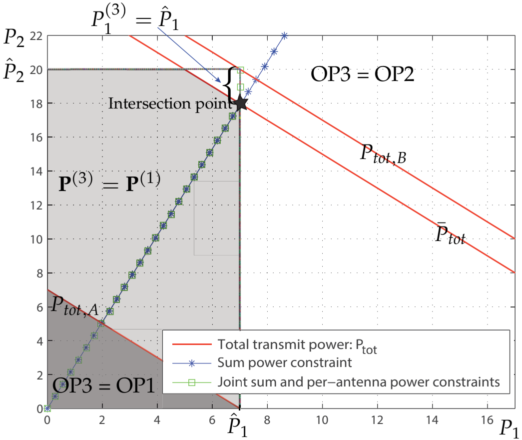
[3] Phuong L. Cao, Tobias J. Oechtering, Rafael F. Schaefer, Mikael Skoglund, "Optimal Transmit Strategy for MISO Channels with Joint Sum and Per-antenna Power Constraints," Submitted to *IEEE Trans. on Signal Processing*., Apr. 2015.

Optimal transmit strategy

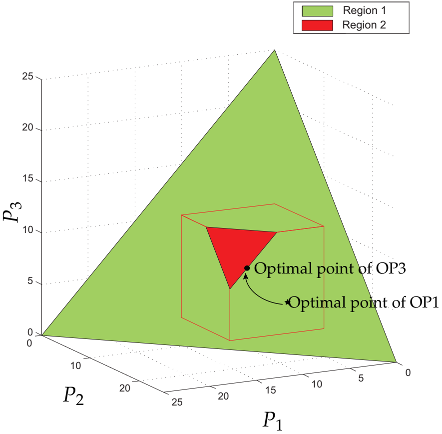


Power constraint domains



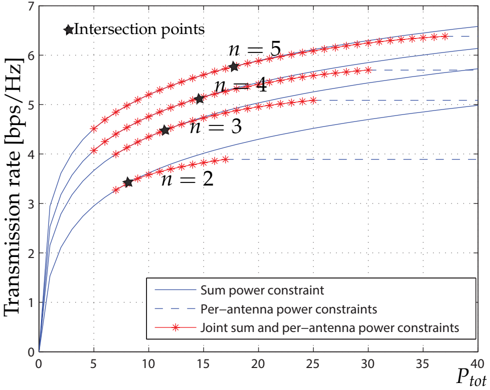
- Crossing the intersection point, the power allocation behavior will change.

POWER CONSTRAINT DOMAINS



- If the optimal power of OP1 violates the per-antenna power constraints, it will reallocated on the boundary of joint sum and per-antenna power constraints region.

Optimal transmission rate examples



- Transmission rate in different power constraint domains and different transmit antenna configurations.
- Keeping a maximum sum power while increase the number of transmit antennas, the probability of power allocation of OP1 violating the per-antenna power constraints reduces.

Conclusions

- Joint sum and per-antenna power constraints are relevant but surprisingly have not studied yet.
- The optimal powers are set equal to the maximum per-antenna powers if their optimal values in sum power constraint only problem violate those per-antenna power constraints.
- The remaining powers can be found by solving a reduced optimization problem.
- Extending to MIMO case.