Optimal Transmission with Per-antenna Power Constraints for Multiantenna Bidirectional Broadcast Channels

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Overview

Introduction
- Introduction

Problem formulation
- System model
- Problem formulation

Main results
- Optimal stransmit strategy

Numerical examples

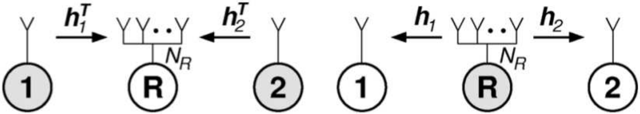
Conclusions

STATE OF THE ART

- Relay communication, relay channels
- Bidirectional communication using the network coding idea can greatly improve the throughput in relaying (P. Larsson *et al.*, AdHoc'05 2005)
- Optimal transmit strategies with sum power constraints for MISO (T. Oechtering *et al.*, TSP 2009), MIMO (T. Oechtering *et al.*, TCOM 2009). MISO bidirectional broadcast channel transmits into the subspace spanned by the channels only.
- Transmitter optimization for the multi-antenna downlink with per-antenna power constraint (W. Yu and T. Lan, TSP 2007)
- MISO capacity (M. Vu, TCOM 2011), MIMO capacity (M. Vu, GLOBECOM 2011; Z. Pi, GLOBECOM 2012) with per-antenna power constraint

The bidirectional broadcast channels with average per-antenna power constraints have not been considered yet.

Bidirectional relay channels



MISO MAC MISO Bi.-BC [1]

[1] T. J. Oechtering, R. F. Wyrembelski, and H. Boche, "Multiantenna bidirectional broadcast channels - Optimal transmit strategies," *IEEE Trans. on Signal Process.*, vol. 57, no. 5, pp. 19481958, May 2009.

WHY PER-ANTENNA POWER CONSTRAINTS?

- **Sum power constraints** are imposed e.g., by regulations or to limit the energy consumption,

$$\mathcal{S}_1 := \{\mathbf{Q} \succcurlyeq 0 : \text{tr}(\mathbf{Q}) \leq P_{tot}\}.$$

- **Per-antenna power constraints** are imposed by hardware limitation of each RF chain,

$$\mathcal{S} := \{\mathbf{Q} \succcurlyeq 0 : \mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i \leq \hat{P}_i, i = 1, ..., n\}.$$

- **Joint sum and per-antenna power constraints**[2].

[2] P. Cao, T. Oechtering, R. Shaefer and M. Skoglund, "Optimal Transmit Strategies for MISO Channels with Joint Sum and Per-antenna power constraints," *Trans. on Signal Processing*., May. 2016.

BIDIRECTIONAL BC CHANNELS - SYSTEM MODEL

$$y_i = \mathbf{x}^T \mathbf{h}_i + z_i,\ i = 1, 2$$

- Transmit signal $\mathbf{x} = [x_1, ..., x_{N_R}]^T \in \mathbb{C}^{N_R \times 1}$
- Channel $\mathbf{h}_i = [h_{i1}, ..., h_{iN_R}]^T \in \mathbb{C}^{N_R \times 1}$, $i = 1, 2$
- Noise $z_i \sim \mathcal{CN}(0, \sigma^2)$, $i = 1, 2$
- Transmit covariance matrix: $\mathbf{Q} = \mathbb{E}\left[\mathbf{x}\mathbf{x}^H\right]$

CAPACITY REGION AND PROBLEM STATEMENT

- Capacity region of the Gaussian MISO bidirectional broadcast channel with per-antenna power constraints:

$$\mathcal{C}_{BC}^{MISO} = \bigcup_{\mathbf{Q} \in \mathcal{S}} \mathrm{dpch}([C_1(\mathbf{Q}), C_2(\mathbf{Q})])$$

with $C_i(\mathbf{Q}) := \log\left(1 + \mathbf{h}_i^H \mathbf{Q} \mathbf{h}_i\right), \; i = 1, 2.$

- The boundary of the capacity region with a given weight vector $\mathbf{w} = [w_1, w_2] \in \mathbb{R}_+^2$ with $w_1 + w_2 = 1$

$$R_{\sum}(\mathbf{Q}, \mathbf{w}) := w_1 \log\left(1 + \mathbf{h}_1^H \mathbf{Q} \mathbf{h}_1\right) + w_2 \log\left(1 + \mathbf{h}_2^H \mathbf{Q} \mathbf{h}_2\right).$$

Boundary of capacity region

- Boundary of capacity region consists of:
  - Two single-user optimal sections
  - Pareto optimal section
- Unidirectional rate (with $\mathbf{w} = [0, 1]$ or $\mathbf{w} = [1, 0]$) can be obtained from the single-user MISO channel with per-antenna power constraints.
- Remaining problem:

Find optimal transmit strategy $\mathbf{Q}$ with weights $w_1, w_2 \neq 0$

Optimization problem

Convex Optimization Problem

$$\begin{aligned} &\text{maximize} && R_{\sum}(\mathbf{Q}, \mathbf{w}) \\ &\text{subject to} && \mathbf{Q} \in \mathcal{S}. \end{aligned} \tag{1}$$

Solution

Our solutions

- Person-by-Person optimality
- Iterative algorithm: Solve optimization problem to obtain $(\mathbf{Q}_{opt}^{[n]}, D_i^{[n]})$ in the n^{th} iteration

ALTERNATIVE OPTIMIZATION APPROACH

Lemma 1 [JPKR'11, Scalar case]

Consider the function $f(D) = -DE + \log(D) + 1$ where $D, E \in \mathbb{R}, E > 0$. Then,

$$\max_{D>0} f(D) = \log(E^{-1}),$$

with the optimum value $D^* = E^{-1}$.

[3] J. Jose, N. Prasad, M. Khojastepour, and S. Rangarajan, "On robust weighted-sum rate maximixation in MIMO interference networks," in *IEEE International Conference on Communications (ICC)*, 2011.

Alternative optimization approach

Alternative optimization problem

$$\max_{\mathbf{Q}\in\mathcal{S}} \sum_{i=1}^{2} w_i \min_{D_i>0} \left(D_i(1+\mathbf{h}_i^H \mathbf{Q}\mathbf{h}_i) - \log(D_i) - 1 \right) \quad (2)$$

Person-by-Person optimality

We propose to solve the following optimization problems to obtain $(\mathbf{Q}_{opt}^{[n]}, D_i^{[n]}), i = 1, 2$ in the n^{th} iteration

$$D_i^{[n]} = \arg\min_{D_i>0} \left(D_i(1+\mathbf{h}_i^H \mathbf{Q}^{[n-1]}\mathbf{h}_i) - \log(D_i) \right) \quad (3)$$

$$\mathbf{Q}_{opt}^{[n]} = \arg\max_{\mathbf{Q}\in\mathcal{S}} \sum_{i=1}^{2} w_i D_i^{[n]} \mathbf{h}_i^H \mathbf{Q}\mathbf{h}_i. \quad (4)$$

Closed-form solutions

Problem

$$D_i^{[n]} = \arg\min_{D_i > 0} \Big(D_i (1 + \mathbf{h}_i^H \mathbf{Q}^{[n-1]} \mathbf{h}_i) - \log(D_i) \Big)$$

The closed-form solution can be computed using Lemma 1 as

$$D_i^{[n]} = (1 + \mathbf{h}_i^H \mathbf{Q}_{opt}^{[n-1]} \mathbf{h}_i)^{-1}.$$

Closed-form solutions

Problem

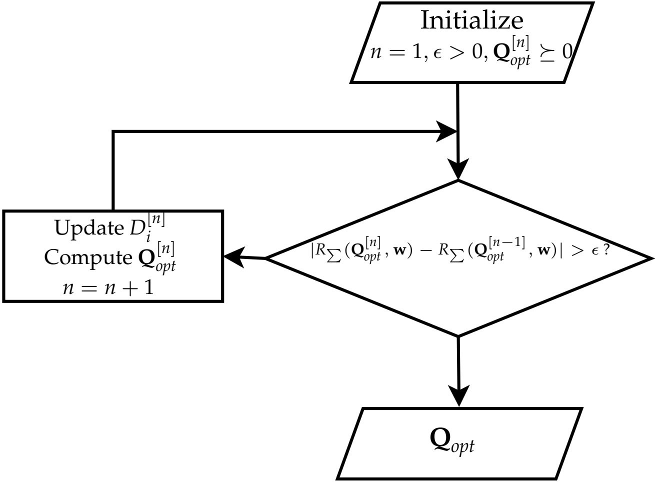
$$\mathbf{Q}_{opt}^{[n]} = \arg\max_{\mathbf{Q}\in\mathcal{S}} \sum_{i=1}^{2} w_i D_i^{[n]} \mathbf{h}_i^H \mathbf{Q} \mathbf{h}_i.$$

- Diagonal elements: $q_{kk}^{[n]} = \hat{P}_k, \forall k = 1, \ldots, N_R$
- Off-diagonal elements:

$$q_{kl}^{[n]} = \frac{\sum_{i=1}^{2} w_i D_i^{[n]} h_{ik}^* h_{il}}{\left|\sum_{i=1}^{2} w_i D_i^{[n]} h_{ik}^* h_{il}\right|} \sqrt{\hat{P}_k \hat{P}_l}.$$

$\forall k, l = 1, \ldots, N_R, k \neq l.$

ALGORITHM



Generated $\{\mathbf{Q}_{opt}\}$ converges to the global optimum.

Algorithm performance

Algorithm	Number of antennas at the relay			
	2	4	6	8
IA	0.014	0.016	0.057	0.142
CVX	1.425	1.899	3.243	4.827

Average running time (in secs.)

[4] The experiment is performed under: Processor - Intel Core i7-3740QM CPU @ 2.70GHz $\times$ 8; Memory - 8GB; OS: Ubuntu 14.04; Matlab R2015a.

Capacity region

Characterization of boundary of capacity region:

- Two single-user optimal sections
- Pareto optimal section (curved part)

The optimal transmit strategies $\mathbf{Q}_{opt}$ of the curved part can be parametrized as follows

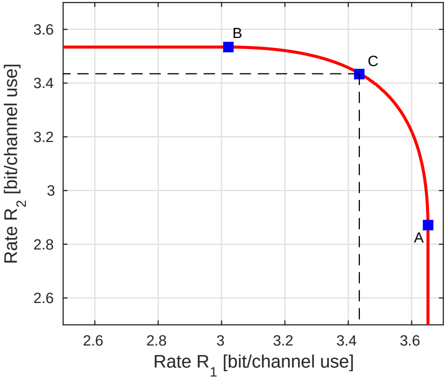
$$\mathbf{Q}_{opt}(t) = \arg \max_{\mathbf{Q} \in \mathcal{S}} t \mathbf{h}_1^H \mathbf{Q} \mathbf{h}_1 + (1-t) \mathbf{h}_2^H \mathbf{Q} \mathbf{h}_2, \qquad (5)$$

with $t \in [0,1]$.

The whole capacity region

$$\mathcal{C}_{BC}^{MISO} = \bigcup_{\mathbf{Q}(t) \in \mathcal{S}} \text{dpch}([R_1(t), R_2(t)] : R_i(t) := \log(1 + \mathbf{h}_i^H \mathbf{Q}(t) \mathbf{h}_i), \quad i = 1, 2, t \in [0,1]).$$

CAPACITY REGION



Capacity region for MISO bidirectional broadcast channel with per-antenna power constraints $\hat{P}_1 = 6, \hat{P}_2 = 8$.

- A and B correspond to the cases $\mathbf{w} = [1, 0]$ and $\mathbf{w} = [0, 1]$,
- C denotes the egalitarian solution at $\mathbf{w} = [0.405, 0.595]$.

SUMMARY AND CONCLUSIONS

- With the help of Lemma 1, the optimization problem can be decomposed in to an equivalent formulation which can be solved by an efficient algorithm,
- The capacity region can be characterized by the parametrization,
- Practical applications.

Q & A