Precoding Design for Massive MIMO Systems with Sub-connected Architecture and Per-antenna Power Constraints

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***Abstract*—This paper provides the necessary conditions to design precoding matrices for massive MIMO systems with a sub-connected architecture, RF power constraints and per-antenna power constraints. The system is configured such that each RF chain serves a group of antennas. The necessary condition to design the digital precoder is established based on a generalized water-filling and joint sum and per-antenna optimal power allocation solution, while the analog precoder is based on a per-antenna power allocation solution only. We study the analytically most interesting case where the power constraint on the RF chain is smaller than the sum of the corresponding per-antenna power constraints. For this, the optimal power is allocated based on two properties: Each RF chain uses full power and if the optimal power allocation of the unconstraint problem violates a per-antenna power constraint then it is optimal to allocate the maximal power for that antenna.**

I. Introduction

In recent years, large-scale multiple-input multiple-output (massive MIMO) for wireless communications has received much attention due to its envisioned application in 5G wireless systems. The motivation of massive MIMO is to use a very large number of antennas to enhance the spectral efficiency significantly [1]–[3]. This becomes necessary and possible since at higher frequency (mmWave) the antennas' sizes reduce and therewith also the radiated energy per-antenna [4], [5].

The massive increase of antennas leads to new technological challenges from a transceiver hardware perspective. We can distinguish between two configurations of large-scale antenna systems namely fully-connected and sub-connected [5]–[7]. In the fully-connected architecture [5], each antenna is connected to all RF chains through analog phase shifters and adders, i.e., each analog precoder output is a combination of all RF signals. One of the biggest drawbacks of this architecture is the requirement of a large number of RF adders and phase shifters, which results in both high hardware costs and power consumption. Different from the fully-connected architecture, a sub-connected architecture has a reduced complexity, where each RF chain is connected to a subset of transmit antennas only. Since this sub-connected architecture requires no adder and fewer phase shifters, it is less expensive to implement than the fully-connected one but results in less freedom for the signalling. Previous studies of transmit strategies for the sub-connected architecture have been done in [7], [8]. However, these works assume a sum power constraint only. Since the

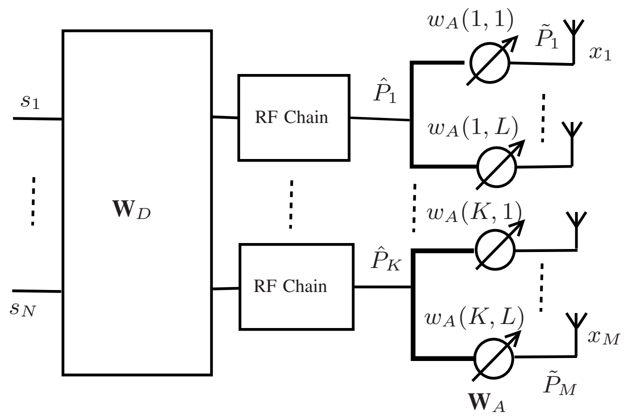

Fig. 1: Sub-connected architecture for Massive MIMO with RF chain and per-antenna power constraints.

RF chain impose physical limitations on the transmitter, it is reasonable to impose a power constraint on each RF chain and possibly also on each antenna to limit the average power. Previous works studied optimal transmit strategies for MISO and MIMO channels with per-antenna power constraints [9]–[17] and joint sum and per-antenna power constraints [18]–[20]. However, the problem has so far not been studied for sub-connected architectures. In this paper, we focus on studying the analog and digital precoders for a massive MIMO system with a sub-connected architecture, RF power constraints and per-antenna power constraints. The single-user large-scale MISO system with a sub-connected architecture, RF power constraints and per-antenna power constraints has been consider in [21].

II. System model

We consider a sub-conneted architecture of a massive MIMO system as depicted in Fig 1. The transmitter is equipped with K RF chains and M_t transmit antennas such that each RF chain is connected to a group of L antennas, i.e., $M_t = KL$. The receiver is equipped with M_r antennas. RF chains are indexed by $k \in \mathcal{K} = \{1, \ldots, K\}$ and antennas connecting to each RF chain are indexed by $l \in \mathcal{L} = \{1, \ldots, L\}$. The transmit data $\mathbf{s} \in \mathbb{C}^{N\times 1}$, where N is number of data stream and $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_N$, is precoded by applying baseband processing (digital precoder) $\mathbf{W}_D \in \mathbb{C}^{K\times N}$

followed by a power allocation matrix $\mathbf{\Lambda} \in \mathbb{C}^{KL\times K}$ and a phase array (analog precoder) $\mathbf{W}_A \in \mathbb{C}^{KL\times KL}$. Due to the architecture, the analog precoder is described by a block diagonal matrix $\mathbf{W}_A = \text{BlockDiag}\{\mathbf{w}_A(1),\ldots,\mathbf{w}_A(K)\}$, $\mathbf{w}_A(k) = \text{diag}\{w_A(k,1),\ldots,w_A(k,L)\}$ with complex phase shift constraints $|w_A(k,l)|^2 = 1$, $\forall k,l$. $\mathbf{\Lambda}$ is a block diagonal matrix to adjust the power allocation and is defined as

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{\Lambda}_1 & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_2 & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{\Lambda}_K \end{bmatrix} \in \mathbb{C}^{KL\times K} \tag{1}$$

with $\mathbf{\Lambda}_k = [\lambda_{k1},\ldots,\lambda_{kL}]^T \in \mathbb{C}^{L\times 1}$, $\forall k,l$.

The channel coefficient matrix is denoted as $\mathbf{H} \in \mathbb{C}^{M_r\times M_t}$ and is known at both transmitter and receiver. Then, the received signal can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{W}_A\tilde{\mathbf{W}}_D\mathbf{s} + \mathbf{z}, \tag{2}$$

where $\tilde{\mathbf{W}}_D = \mathbf{\Lambda}\mathbf{W}_D \in \mathbb{C}^{M_t\times N}$ is the digital precoder matrix and $\mathbf{z} \sim \mathcal{CN}(0,\mathbf{I})$ is additive white Gaussian noise.

We consider individual power constraints at each transmit antenna $\tilde{P}_{kl}$ and at each RF chain $\hat{P}_k$, $\forall k \in \mathcal{K}$, $\forall l \in \mathcal{L}$. If $\hat{P}_k \geq \sum_{l=1}^{L}\tilde{P}_{kl}, \forall k \in \mathcal{K}$, then we face the per-antenna power constraints only problem. If $\hat{P}_k \leq \sum_{l=1}^{L}\tilde{P}_{kl}, \forall k \in \mathcal{K}$, i.e., the transmit power on a certain RF chain is more restricted than the total transmit power on antennas connecting to that RF chain, we face the optimization problem where both sum and per-antenna power constraints are active. In this work, we focus on the later case only. Solutions to the other one follow straight forwardly. We are interested in finding the optimal precoding matrices $\mathbf{W}_A$ and $\tilde{\mathbf{W}}_D$ that achieve the capacity of the point-to-point MIMO channel (2). This is the standard problem of finding the optimal covariance matrix of the zero mean Gaussian distributed input but here with covariance matrix structure $\mathbf{W}_A\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\mathbf{W}_A^H$ reflecting the hardware design. Thus, the optimization problem is given as follows

$$\max_{\tilde{\mathbf{W}}_D,\mathbf{W}_A} \quad f(\mathbf{W}_A,\tilde{\mathbf{W}}_D) \tag{3}$$

$$\text{s.t.} \quad \mathbf{e}_{kl}^T\mathbf{W}_A\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\mathbf{W}_A^H\mathbf{e}_{kl} \leq \tilde{P}_{kl}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}, \tag{2a}$$

$$\sum_{l=1}^{L}\mathbf{e}_{kl}^T\mathbf{W}_A\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\mathbf{W}_A^H\mathbf{e}_{kl} \leq \hat{P}_k, \forall k \in \mathcal{K}, \tag{2b}$$

$$|w_A(k,l)|^2 = 1, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}, \tag{2c}$$

where $f(\mathbf{W}_A,\tilde{\mathbf{W}}_D) = \log|\mathbf{I} + \mathbf{H}\mathbf{W}_A\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\mathbf{W}_A^H\mathbf{H}^H|$. (2a), (2b), and (2c) are the per-antenna, RF chain, and phase shifter constraints. $\mathbf{e}_{kl}$ is the Cartesian unit vector with elements at $((k-1)L+l)$-th position equal to 1 and the rest is 0.

III. Precoding Design

If $L = 1$, then we have fully digital precoding where every antenna has its own RF chain. This case has been studied in [22] and [10]. In this paper we focus on the hybrid precoding for the case where the number of RF chains is strictly smaller than the number of transmit antennas only, i.e., $K < M_t$. We first provide the necessary condition in designing the analog precoder. After that the digital precoder and the power allocation are considered.

A. Analog precoder

In this part, we provide the necessary condition to design the optimal $\mathbf{W}_A$ assuming that the optimal $\tilde{\mathbf{W}}_D^\star$ is given. This gives a necessary condition for the optimal design. Under this assumption, power constraints (2a) and (2b) are already satisfied since $\mathbf{W}_A$ contains phase shifts only. Then the optimization problem to find $\mathbf{W}_A$ can be formed as

$$\max_{\mathbf{W}_A} f(\mathbf{W}_A,\tilde{\mathbf{W}}_D^\star), \text{ s.t. (2c)}. \tag{4}$$

Due to the hardware design, the MIMO precoding matrix can be constructed as $\mathbf{V}^H = \mathbf{W}_A\tilde{\mathbf{W}}_D^\star$. By letting $\mathbf{V} = [\mathbf{v}(1),\ldots,\mathbf{v}(M_t)]$ where $\mathbf{v}(m) = [v_{m1},\ldots,v_{mN}]^T$ with v_{mn}, $m \in \{1,\ldots,M_t\}$, $n \in \{1,\ldots,N\}$, is the precoding coefficient for the i-th antenna and the j-th data stream, and $\tilde{\mathbf{W}}_D^\star = [\tilde{\mathbf{w}}_D^\star(1),\ldots,\tilde{\mathbf{w}}_D^\star(M_t)]^T$ with $\tilde{\mathbf{w}}_D^\star(m) = [\tilde{w}_D^\star(m1),\ldots,\tilde{w}_D^\star(mN)]^T$, we can express the MIMO precoding matrix as follows.

$$\begin{bmatrix} \mathbf{v}^H(1) \\ \vdots \\ \mathbf{v}^H(M_t) \end{bmatrix} = \begin{bmatrix} w_A(1,1) & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & w_A(K,L) \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{w}}_D^{\star T}(1) \\ \vdots \\ \tilde{\mathbf{w}}_D^{\star T}(M_t) \end{bmatrix}. \tag{5}$$

This implies that

$$\begin{bmatrix} v_{m1}^* \\ \vdots \\ v_{mN}^* \end{bmatrix} = \begin{bmatrix} w_A(k,l)\tilde{w}_D^{\star *}(m1) \\ \vdots \\ w_A(k,l)\tilde{w}_D^{\star *}(mN) \end{bmatrix}, \tag{6}$$

$\forall k \in \mathcal{K}$, $\forall l \in \mathcal{L}$, and $m = (k-1)L + l$.

As a result, if we let γ_{mn} and θ_{mn} be optimal phases of $v_{mn}^\star$ and $\tilde{w}_D^\star(mn)$, $\forall n \in \{1,\ldots,N\}$, then at the optimum, the elements of the optimal analog precoder have to satisfy the following conditions

$$w_A^\star(k,l) = e^{i(\theta_{m1}-\gamma_{m1})} = \cdots = e^{i(\theta_{mN}-\gamma_{mN})}, \tag{7}$$

for all $m \in \{1,\ldots,M_t\}$.

Since we assume that the optimal $\tilde{\mathbf{W}}_D^\star$ is given, $\tilde{w}_D^\star(mn)$ is known. The remaining problem is to find the optimal $v_{mn}^\star$ and its equivalent $\mathbf{v}^\star(m)$.

Let $P_m^\star$ denotes the optimal power allocated for antenna $m \in \{1,\ldots,M_t\}$, then the optimal value of $\mathbf{V}$ can be obtained by solving the following problem

$$\mathbf{V}^\star = \arg\max_{\mathbf{V}} \log|\mathbf{I} + \mathbf{V}\mathbf{F}\mathbf{V}^H|, \tag{8}$$

$$\text{s.t.} \ \|\mathbf{v}(m)\|^2 \leq P_m^\star \ \forall m = 1,\ldots,M_t,$$

where $\mathbf{F} = \mathbf{H}^H\mathbf{H}$. Note that (8) is a MIMO channel with per-antenna power constraints only problem in which the per-antenna power constraints $P_m^\star$, $\forall m = 1,\ldots,M_t$ are equal to the optimal power allocation given by $\tilde{\mathbf{W}}_D^\star$. [10] showed that there always exists an optimal solution for (8) that allocates full power on all antennas. However there is no closed-form solution for (8). The optimal value of $\mathbf{V}$ can be obtained by using techniques in [22] which will be discussed in more detail in the following.

Instead solving (8) for $\mathbf{V}$ with all $\mathbf{v}(m)$ at once, we derive a necessary condition for each $\mathbf{v}(m)$, $m \in \{1,\dots,M_t\}$, separately. In order to extract the contribution of $\mathbf{v}(m)$ of the m-th antenna to the objective function of (8), we can reorder $\mathbf{F}$ as

$$\mathbf{F}_m = \begin{bmatrix} f_m & \mathbf{f}_m^H \\ \mathbf{f}_m & \tilde{\mathbf{F}}_m \end{bmatrix}, \tag{9}$$

where $\tilde{\mathbf{F}}_m \in \mathbb{C}^{(M_t-1)\times(M_t-1)}$ obtained by removing m-th row and columns from $\mathbf{F}$, $\mathbf{f}_m$ is the m-th column of $\mathbf{F}$ without diagonal item f_m. Then, following [22], the objective function of (8) can be written as

$$\begin{aligned}
&\log|\mathbf{I}+\mathbf{V}\mathbf{F}\mathbf{V}^H| \\
&= \log\left|\mathbf{I}+[\mathbf{v}(m)\ \tilde{\mathbf{V}}_m]\begin{bmatrix} f_m & \mathbf{f}_m^H \\ \mathbf{f}_m & \tilde{\mathbf{F}}_m \end{bmatrix}\begin{bmatrix} \mathbf{v}(m)^H \\ \tilde{\mathbf{V}}_m^H \end{bmatrix}\right| \\
&= \log\Big|\mathbf{I}+\tilde{\mathbf{V}}_m\tilde{\mathbf{F}}_m\tilde{\mathbf{V}}_m^H + f_m(\mathbf{v}(m)\mathbf{v}^H(m) \\
&\qquad\qquad +\mathbf{v}(m)\mathbf{w}_m^H+\mathbf{w}_m\mathbf{v}^H(m))\Big| \\
&= \log\Big|\mathbf{I}+\tilde{\mathbf{V}}_m\tilde{\mathbf{F}}_m\tilde{\mathbf{V}}_m^H - f_m\mathbf{w}_m\mathbf{w}_m^H + f_m\mathbf{w}_m\mathbf{w}_m^H \\
&\qquad\qquad + f_m\left(\mathbf{v}(m)\mathbf{v}^H(m)+\mathbf{v}(m)\mathbf{w}_m^H+\mathbf{w}_m\mathbf{v}^H(m)\right)\Big| \\
&= \log\left|\mathbf{D}_m + f_m(\mathbf{v}(m)+\mathbf{w}_m)(\mathbf{v}(m)+\mathbf{w}_m)^H - f_m\mathbf{w}_m\mathbf{w}_m^H\right| \\
&= \log|\mathbf{D}_m| + \log\big(1+f_m(\mathbf{v}(m)+\mathbf{w}_m)^H\mathbf{D}_m^{-1}(\mathbf{v}(m)+\mathbf{w}_m) \\
&\qquad\qquad - f_m\mathbf{w}_m^H\mathbf{D}_m^{-1}\mathbf{w}_m\big),
\end{aligned} \tag{10}$$

with $\mathbf{D}_m = \mathbf{I}+\tilde{\mathbf{V}}_m\tilde{\mathbf{F}}_m\tilde{\mathbf{V}}_m^H$, $\mathbf{w}_m = \frac{1}{f_m}\tilde{\mathbf{V}}_m\mathbf{f}_m$, and $\tilde{\mathbf{V}}_m$ is the sub-matrix of $\mathbf{V}$ with $\mathbf{v}(m)$ removed.

Since $\log(\cdot)$ is a monotonically increasing function, and $\mathbf{D}_m$, $\mathbf{w}_m$, and f_m are independent of $\mathbf{v}(m)$ for a fixed matrix $\tilde{\mathbf{V}}_m$, we can reformulate the optimization problem (8) into

$$\tilde{\mathbf{v}}^\star(m) = \underset{\tilde{\mathbf{v}}(m):\|\tilde{\mathbf{v}}(m)\|^2\le P_m^\star}{\arg\max} (\tilde{\mathbf{v}}(m)+\tilde{\mathbf{w}}_m)^H\boldsymbol{\Sigma}_m(\tilde{\mathbf{v}}(m)+\tilde{\mathbf{w}}_m), \tag{11}$$

where $\mathbf{Z}_m$ and $\boldsymbol{\Sigma}_m$ are left singular and diagonal matrices obtained from the singular value decomposition of $\mathbf{D}_m$, i.e., $\mathbf{D}_m = \mathbf{Z}_m\boldsymbol{\Sigma}_m\mathbf{Z}_m^H$, as well as $\tilde{\mathbf{v}}(m) = \mathbf{Z}_m^H\mathbf{v}(m)$ and $\tilde{\mathbf{w}}_m = \mathbf{Z}_m^H\mathbf{w}_m$.

Following [22], the objective function is maximized when the phases of complex numbers $\tilde{v}_{mn}$ and $\tilde{w}_{mn}$, $\forall n \in \{1,\dots,N\}$ are the same. Let a_{mn} and b_{mn} be the amplitudes of $\tilde{v}_{mn}$ and $\tilde{w}_{mn}$. Then (11) can be simplified to

$$\mathbf{a}_m^\star = \arg\max_{\mathbf{a}_m}\sum_{n=1}^{N^\star}\frac{(a_{mn}+b_{mn})^2}{\sigma_{mn}}, \text{s.t.}\sum_{n=1}^{N}a_{mn}^2 \le P_m^\star, \tag{12}$$

where σ_{mn}, $n\in\{1,\dots,N^\star\}$ are the diagonal elements of $\boldsymbol{\Sigma}_m$, $\mathbf{a}_m = [a_{m1},\dots,a_{mN}]$, $\mathbf{b}_m = [b_{m1},\dots,b_{mN}]$, and $N^\star = \text{rank}(\tilde{\mathbf{W}}_D) \le N$. Following [22], we know that the contours of the objective function in (12) are $N^\star$-dimension ellipsoids centered at $(-b_{1n},\dots,-b_{M_tn})$, and the per-antenna power allocation must satisfy all the per-antenna power constraints with equality. Therefore, the optimal solution of (12) has to satisfy the following condition

$$(\rho_m^\star\boldsymbol{\Sigma}_m - \mathbf{I})\mathbf{a}_m^\star = \mathbf{b}_m, \tag{13}$$

where $\rho_m^\star > 0$ is the Lagrange multiplier chosen such that $\mathbf{a}_m\mathbf{a}_m^H = \sum_{n=1}^N a_{mn}^2 = P_m^\star$. The value of $\rho_m^\star$ can be found by solving the following equation derived from (13)

$$\sum_{n=1}^{N^\star}\frac{b_{mn}^2}{(\rho_m^\star\sigma_{mn}-1)^2} = P_m^\star \tag{14}$$

using, e.g., Newton's method [23] (see Appendix A).

From (13) and the fact that $\tilde{v}_{mn}^\star$ and $\tilde{w}_{mn}$ are in-phase for all $n\in\{1,\dots,N\}$, we can obtain the solution of (11) as

$$\tilde{\mathbf{v}}^\star(m) = (\rho_m^\star\boldsymbol{\Sigma}_m - \mathbf{I})^{-1}\tilde{\mathbf{w}}_m. \tag{15}$$

Thus in the optimum, it is necessary that

$$\mathbf{v}^\star(m) = \frac{1}{f_m}\mathbf{Z}_m(\rho_m^\star\boldsymbol{\Sigma}_m-\mathbf{I})^{-1}\mathbf{Z}_m^H\tilde{\mathbf{V}}_m^\star\mathbf{f}_m, \tag{16}$$

with $\tilde{\mathbf{V}}_m^\star$ is the optimal sub-matrix of $\mathbf{V}$. This necessary condition can be used in an person-by-person optimality algorithm to find $\mathbf{v}^\star(m)$, $\forall m\in\{1,\dots,M_t\}$. From (16) and the assumption that the optimal digital precoder is given, the necessary condition for the optimal analog precoder is provided. The elements of the optimal analog precoder have to satisfy the condition in (7).

B. Digital precoder

Next, we derive the conditions for the design of the digital precoder $\tilde{\mathbf{W}}_D$ with a given optimal $\mathbf{W}_A^\star$ above. Note that in the hybrid precoding design, the analog precoder contains phases only, while the power adjustment is performed by the digital precoder. The convex optimization problem to find the digital precoder $\tilde{\mathbf{W}}_D$ can be formed as

$$\begin{aligned}
\max_{\tilde{\mathbf{W}}_D}\quad & f(\mathbf{W}_A^\star,\tilde{\mathbf{W}}_D) \\
\text{s.t.}\quad & \forall k,l: \mathbf{e}_{kl}^T\mathbf{W}_A^\star\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\mathbf{W}_A^{\star H}\mathbf{e}_{kl} \le \tilde{P}_{kl}, \\
& \forall k: \sum_{l=1}^{L}\mathbf{e}_{kl}^T\mathbf{W}_A^\star\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\mathbf{W}_A^{\star H}\mathbf{e}_{kl} \le \hat{P}_k.
\end{aligned} \tag{17}$$

Proposition 1. *With $\hat{P}_k \le \sum_{l=1}^L \tilde{P}_{kl}, \forall k$, there exist always an optimal solution of* (17) *which allocates full power on each RF chain.*

Proof. Suppose that there exists no optimal transmit strategy with full power allocation on each RF chain. Let $\mathbf{Q}^\star$ denotes the optimal solution of (17), then there exists at least one k, $1\le k\le K$ such that $\sum_{l=1}^L \mathbf{e}_{kl}^T\mathbf{Q}^\star\mathbf{e}_{kl} = \sum_{l=1}^L P_{kl}^\star = P_k^\star < \hat{P}_k$. Since for those k, $\sum_{l=1}^L P_{kl}^\star = P_k^\star < \hat{P}_k \le \sum_{l=1}^L\tilde{P}_{kl}$ and $P_{kl}^\star \le \tilde{P}_{kl}$, there exists a positive semi-definite Hermitian matrix $\mathbf{Q} \succcurlyeq \mathbf{Q}^\star$ such that by increasing the per-antenna power allocation from $\mathbf{Q}^\star$, full power on each RF chain is allocated in $\mathbf{Q}$. Let $R^\star$ denotes the maximum transmission rate $R^\star = f(\mathbf{Q}^\star)$, then we have $f(\mathbf{Q}) \geq R^\star = f(\mathbf{Q}^\star)$ since $f(\mathbf{Q})$ is matrix-monotone in $\mathbf{Q}$ [24]. This contradicts with the assumption that there does not always exit an optimal solution which allocates full power on each RF chain. □

Accordingly, it is sufficient for the optimization to consider only transmit strategies which allocate full power on all RF chains, i.e., the RF chain power constraints are always active. Let $\tilde{\mathbf{H}} = \mathbf{H}\mathbf{W}_A^\star$, then (17) can be equivalently expressed as

$$\begin{aligned} \max_{\tilde{\mathbf{W}}_D} \quad & \log|\mathbf{I} + \tilde{\mathbf{H}}\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\tilde{\mathbf{H}}^H| \qquad (18) \\ \text{s.t.} \quad & \forall k, l : \mathbf{e}_{kl}^T\mathbf{W}_A^\star\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\mathbf{W}_A^{\star H}\mathbf{e}_{kl} \leq \tilde{P}_{kl}, \\ & \forall k : \sum_{l=1}^{L}\mathbf{e}_{kl}^T\mathbf{W}_A^\star\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\mathbf{W}_A^{\star H}\mathbf{e}_{kl} \leq \hat{P}_k. \end{aligned}$$

The optimal solution of (18) subject to the RF chain power constraints only is given by the following generalized water-filling solution from [20], [25].

Lemma 1 ([20], [25]). *Let $\boldsymbol{A} = \tilde{\boldsymbol{W}}_D\tilde{\boldsymbol{W}}_D^H$, then the optimal $\boldsymbol{A}^\star$ of (18) subjects to the RF chain power constraints only is given by*

$$\boldsymbol{A}^\star = \boldsymbol{D}^{-\frac{1}{2}}[\boldsymbol{U}]_{:,1:R}[\boldsymbol{U}]_{:,1:R}^H\boldsymbol{D}^{-\frac{1}{2}} - [\boldsymbol{U}]_{:,1:R}\boldsymbol{\Lambda}^{-1}[\boldsymbol{U}]_{:,1:R}^H \qquad (19)$$

where diagonal matrix $\boldsymbol{D}$ is a Lagrange multiplier; diagonal matrix $\boldsymbol{\Lambda}$ and the first $R = \min(M_t, M_r)$ columns of a unitary matrix $[\boldsymbol{U}]_{:,1:R}$ are obtained from eigenvalue decomposition $\tilde{\boldsymbol{H}}^H\tilde{\boldsymbol{H}} = \boldsymbol{U}\begin{bmatrix}\boldsymbol{\Lambda} & 0\\ 0 & 0\end{bmatrix}\boldsymbol{U}^H$. The diagonal elements of $R \times R$ diagonal matrix $\boldsymbol{\Lambda}$ are positive real values in decreasing order.

The elements of the diagonal matrix $\mathbf{D}$ for the optimal solution of (18) can be computed as follows. For all $k \in \mathcal{K}$, we have

$$[\mathbf{D}]_{k,l} = \frac{\sum_l[[\mathbf{U}]_{:,1:R}[\mathbf{U}]_{:,1:R}^H]_{kl}}{\hat{P}_k + \sum_l[[\mathbf{U}]_{:,1:R}\mathbf{\Lambda}^{-1}[\mathbf{U}]_{:,1:R}^H]_{kl}} \ \forall l \in \mathcal{L}. \qquad (20)$$

However it may happen that the optimal powers allocated under RF chain power constraints only problem may exceed the maximum allocated per-antenna powers. Following [20] and [18], if an antenna has an optimal power allocation that violates the per-antenna power constraint, then it is optimal to allocate the maximal per-antenna power on that antenna. This behaviour is explained in the following lemma

Lemma 2 (Lemma 1 in [20] and Lemma 2 in [18]). *For all $k \in \mathcal{K}$, let $\mathcal{P}_k := \{l \in \mathcal{L} : P_{kl}^{WF} > \tilde{P}_{kl}\}$ where P_{kl}^{WF} are the corresponding diagonal elements of $\boldsymbol{A}^\star$ in (19). Then, the optimal power can be allocated as*

$$\begin{cases} \forall k, \forall l \in \mathcal{L} : P_{kl}^\star = P_{kl}^{WF}, & \text{if } \mathcal{P}_k = \emptyset, \\ \forall k, \forall l \in \mathcal{P}_k : P_{kl}^\star = \tilde{P}_{kl}, & \text{otherwise.} \end{cases} \qquad (21)$$

We observe from Lemma 2 that if there exists any optimal power which violates the per-antenna power constraint, then full per-antenna power is allocated on that antenna. The remaining optimal power allocation can be found by considering a reduced optimization problem as follows

$$\begin{aligned} \max_{\tilde{\mathbf{W}}_D} \quad & \log|\mathbf{I} + \tilde{\mathbf{H}}\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\tilde{\mathbf{H}}^H| \qquad (22) \\ \text{s.t.} \quad & \forall k, \forall l \in \mathcal{P}_k : \mathbf{e}_{kl}^T\mathbf{W}_A^\star\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\mathbf{W}_A^{\star H}\mathbf{e}_{kl} = \tilde{P}_{kl}, \\ & \forall k, \forall l \in \mathcal{L}_k : \mathbf{e}_{kl}^T\mathbf{W}_A^\star\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\mathbf{W}_A^{\star H}\mathbf{e}_{kl} \leq \tilde{P}_{kl}, \\ & \forall k, \forall l \in \mathcal{L}_k : \sum_l\mathbf{e}_{kl}^T\mathbf{W}_A^\star\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\mathbf{W}_A^{\star H}\mathbf{e}_{kl} \leq \hat{P}_k'. \end{aligned}$$

where $\hat{P}_k' = \hat{P}_k - \sum_{l\in\mathcal{L}_k}\tilde{P}_{kl}$, $\mathcal{L}_k = \mathcal{L}\setminus\mathcal{P}_k$ is a set of indices of antennas connecting RF chain k-th that have not been fixed to full per-antenna powers, and $\mathcal{P}_k$ is defined in Lemma 2.

The optimal solution of the reduced optimization problem (22) can be solved by using Lemma 1 again. However, the diagonal matrix $\mathbf{D}$ corresponding to the water level is different since some optimal powers have been fixed to maximal per-antenna powers following Lemma 2. The elements of the diagonal matrix $\mathbf{D}$ for the optimal solution of (22) therefore can be computed as follows. For all $k \in \mathcal{K}$, we have

$$[\mathbf{D}]_{k,l} = \frac{[[\mathbf{U}]_{:,1:R}[\mathbf{U}]_{:,1:R}^H]_{kl}}{\tilde{P}_{kl} + [[\mathbf{U}]_{:,1:R}\mathbf{\Lambda}^{-1}[\mathbf{U}]_{:,1:R}^H]_{kl}} \text{ if } l \in \mathcal{P}_k \qquad (23)$$

and

$$[\mathbf{D}]_{k,l} = \frac{\sum_l[[\mathbf{U}]_{:,1:R}[\mathbf{U}]_{:,1:R}^H]_{kl}}{\hat{P}_k' + \sum_l[[\mathbf{U}]_{:,1:R}\mathbf{\Lambda}^{-1}[\mathbf{U}]_{:,1:R}^H]_{kl}} \text{ if } l \in \mathcal{L}_k. \qquad (24)$$

However it may still happen that optimal power powers in the optimal solution of the (22) may exceed the remaining maximum per-antenna powers. Therefore, an iteration process starting from Lemma 2 can be used to allocate optimal powers on the remaining antennas. Note that with a reduced optimization problem, in Lemma 2, $\mathcal{L}$ will be replaced by $\mathcal{L}_k$ in every iteration. This process stops when all power constraints are satisfied.

Based on analysis above, an iterative algorithm is proposed to compute the optimal value of $\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H$. To see this, we consider the following sequence of optimization problems

$$\begin{aligned} \max_{\tilde{\mathbf{W}}_D\in\mathcal{S}(\emptyset)} f(\tilde{\mathbf{W}}_D) &= \max_{\tilde{\mathbf{W}}_D\in\mathcal{S}(\emptyset)\cap\mathcal{R}(1)} f(\mathbf{W}_A^\star, \tilde{\mathbf{W}}_D) \\ &\geq \max_{\tilde{\mathbf{W}}_D\in\mathcal{S}(\emptyset)\cap\mathcal{R}(2)} f(\tilde{\mathbf{W}}_D) \\ &\dots \\ &\geq \max_{\tilde{\mathbf{W}}_D\in\mathcal{S}(\emptyset)\cap\mathcal{R}(K(L-1))} f(\tilde{\mathbf{W}}_D) = (18). \end{aligned} \qquad (25)$$

where $\mathcal{S}(\emptyset) := \{\tilde{\mathbf{W}}_D \in \mathbb{C}^{M_t\times N} : \sum_{l=1}^{L}\mathbf{e}_{kl}^T\mathbf{W}_A^\star\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\mathbf{W}_A^{\star H}\mathbf{e}_{kl} \leq \hat{P}_k \ \forall k\}$; $\mathcal{R}(i) = \{\tilde{\mathbf{W}}_D \in \mathbb{C}^{M_t\times N} : \mathbf{e}_{kl}^T\mathbf{W}_A^\star\tilde{\mathbf{W}}_D\tilde{\mathbf{W}}_D^H\mathbf{W}_A^{\star H}\mathbf{e}_{kl} \leq \tilde{P}_{kl}, \forall k, \forall l \in \mathcal{P}_k(i)\}$ with $\mathcal{P}_k(i)$ is the set of indices of powers which violates the per-antenna power constraints connecting to k-th antenna at the i-th iteration with $\mathcal{R}(1) = \emptyset$.

Algorithm 1: Optimal power allocation in digital precoder

```
Solve (18) subject to RF chain power constraints only to
  find P_kl^{WF} using Lemma 1 and (20).
for k = 1 : K do
    Check the optimal power allocation P_kl^{WF} with
    per-antenna power constraints P̃_kl and form
    𝒫_k := {l ∈ ℒ : P_kl^{WF} > P̃_kl}.
    while 𝒫_k ≠ ∅ do
        P*_kl ← P̃_kl, ∀l ∈ 𝒫_k.
        ℒ_k = ℒ \ 𝒫_k.
        Form the reduced optimization problem (22).
        Solve (22) using Lemma 1, (23) and (24).
        ℒ ← ℒ_k.
        Return 3.
    end while
    P*_kl ← P_kl^{WF}, ∀l ∈ ℒ.
end for
```

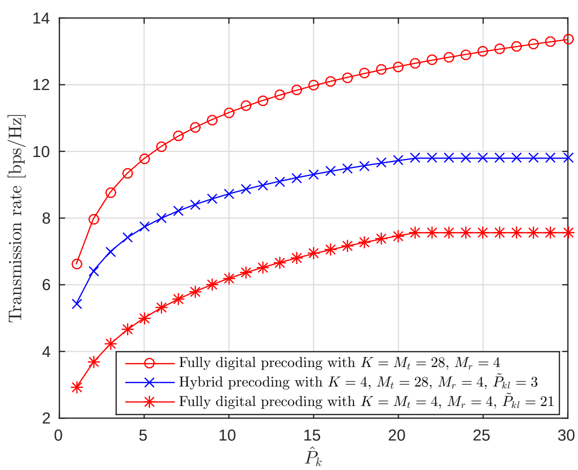

Fig. 2: Transmission rate of the sub-connected architecture massive MIMO system with different RF chain and antenna configurations.

Every reduced optimization problem in (25) is convex. The global convergence of the proposed iteration method is guaranteed after solving at most $K(L-1)$ convex optimization problems.

IV. Numerical Example

In this section, we consider a massive MIMO system with different transmit antenna configurations. We first evaluate the transmission rate for the case that the number of RF chains and the number of antennas are the same, i.e., fully digital beamforming is used. The considered system has either $K = 4$ or $K = 28$ RF chains and $M_t = 28$ antennas with per-antenna power constraints $\tilde{P}_{kl} = 3$. Further, we includes the case with $K = 4$ and $M_t = 4$ with per-antenna power constraints $\tilde{P}_{kl} = 21$. At the receiver, we set the number of received antennas as $M_r = 4$. For the plotted curves in Fig. 2, we have the same transmit power constraint $\hat{P}_k$ on all RF chains that we gradually increase from $\hat{P}_k = 1$ to $\hat{P}_k = 30$.

We observe from the figure that for the hybrid beamforming, the transmission rate increases together with the increasing of the RF chain power if the RF chain power constraint is more restrictive than the sum of all individual powers of a group of antennas connected to that RF chain, i.e., in the example $\hat{P}_k \leq 21$. Beyond that, i.e., $\hat{P}_k \leq 21$, the transmission rate remains constant with increasing RF chain power constraints since the allocated RF chain powers remain constant due to the per-antenna power constraints. It means the RF power constraint is never active for any $\hat{P}_k > 21$, and it is optimal to transmit with the maximal individual power $\tilde{P}_{kl} = 3$ on all antennas.

In comparison to the fully digital precoding, we observe that the sub-connected architecture provides significant performance gains if a hardware with a few RF chains should be used, but performs worse if a fully connected system with many antennas is available. The gap between the fully connected and the sub-connected architecture is due to the per-antenna power constraints.

V. Conclusion

In this paper, we provide necessary conditions to design the optimal analog and digital precoders for a massive MIMO system with sub-connected architecture. It is shown that precoders can be found by applying similar techniques from the optimization problems of the point-to-point MIMO system with per-antenna power constraints only and with joint sum and per-antenna power constraints. If the sum of the per-antenna power constraints of antennas is larger than the RF chain power constraint of which they are connecting to, then there exists always an optimal policy that allocates full RF power. The power allocation on antennas follows the allocated behaviour of one with joint sum and per-antenna power constraints. The overall phase is controlled by the analog precoder while the power allocation is adjusted by the digital precoder only.

Appendix

A. Newton's method

Let $f(\rho_m)$ be a function of ρ_m,

$$f(\rho_m) = \sum_{n=1}^{N^\star} \frac{b_{mn}^2}{(\rho_m \sigma_{mn} - 1)^2} - P_m^\star. \tag{26}$$

Then, $\rho_m^\star$ is a root to the equation $f(\rho_m) = 0$. Following the Newton's method, at the i-th iteration, the value of ρ_m is updated as follows

$$\rho_m(i) = \rho_m(i-1) + \frac{f(\rho_m(i-1))}{f(\rho_m(i-1))'}, \tag{27}$$

where $f(\rho_m(i-1))'$ is the first derivative of $f(\rho_m)$ with respect to ρ_m at iteration $(i-1)$. The initial value $f(\rho_m(0))$ can be set as $\max_n \frac{1}{\sigma_{mn}}(1 + \sqrt{\frac{b_{mn}^2}{P_m^\star}})$. ■

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