Optimal Transmit Strategies for Gaussian MISO Wiretap Channels

Phuong Le Cao, *Student Member, IEEE* and Tobias J. Oechtering, *Senior Member, IEEE*

***Abstract*—This paper studies the characterization of the trade-off between secrecy and non-secrecy capacity of the MISO wiretap channels with different power constraint settings, in particular, sum power constraint only, per-antenna power constraints only and joint sum and per-antenna power constraints. The problem is motivated by the fact that the capacity of a channel is usually larger than the secrecy capacity. First, a necessary and sufficient condition to ensure a positive secrecy capacity is shown. After that, related problems to find optimal transmit strategies that maximize the weighted rate sum for wiretap channels with given sets of power constraints are derived. Since these problems are not necessarily convex, equivalent problem formulations are used to derive optimal transmit strategies using both closed-form solutions and iterative algorithms. This provides the boundary of the rate region describing the optimal trade-off between transmission and secrecy rate. Lastly, the theoretical results are illustrated by numerical examples.**

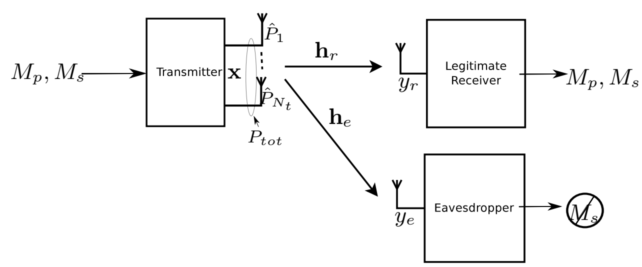

Fig. 1: MISO wiretap channel with joint sum and per-antenna power constraints, private message M_p and confidential message M_s

I. Introduction

Security is a critical aspect in wireless communication systems due to the open nature of wireless links. To enhance the security, physical-layer secrecy methods have received much attention recently. One of the pioneer studies is the study of the secrecy capacity of the wiretap channel [1], where Wyner showed that a positive secrecy rate can be achieved when an eavedropper's channel is a degraded version of the main channel. The maximal secrecy rate is given by the largest difference between the mutual information to the legitimate receiver and the mutual information to the eavesdropper. Following Wyner's work, researchers in the physical-layer security area have extended and considered the wiretap channel in various aspects. Notable results include the extension to the non-degraded case by Csiszár and Körner [2] and the extension to the single-input single-output Gaussian wiretap channels by Leung-Yan-Cheong and Hellman [3].

The secrecy capacities for Gaussian multiple-input single-output (MISO) and multiple-input multiple-output (MIMO) wiretap channels with a sum power constraint have been studied in [4]–[10]. In [4] and [5], the authors developed upper bounds that enable to characterize the secrecy capacities for MISO and MIMO wiretap channels. The proposed solutions are to reduce the wiretap system into a set of parallel channels based on the generalized singular value decomposition and using an independent Gaussian wiretap code books on those resulting channels. In [7], necessary conditions for the optimal input covariance matrix are derived. In particular, a closed-form expression of the MISO secrecy capacity has been shown. For the MIMO case an iterative algorithm is provided. In [9] and [10], iterative optimization algorithms to find the secrecy capacity have been proposed based on the concave-convex alternating optimization procedure. Alternatively, indirect approaches such as using Sato-like arguments or matrix analysis tools are also used to find bounds on the secrecy capacity of a MIMO Gaussian wiretap channel in [4]–[6].

In practice, each antenna has its own power amplifier, which means the power allocation at the transmitter is usually done under per-antenna power constraints instead of a sum power constraint. Particularly, the problem of finding the channel capacity with average per-antenna power constraints has been investigated in both single-user [11]–[14] and multi-user setups [15]–[17]. Recently, the capacity of point-to-point channels with joint sum and per-antenna power constraints has been considered [18]–[21]. An interesting aspect of the joint sum and per-antenna power constraints setting is that it can be applied to systems with multiple antenna as well as to distributed systems with separated energy sources. The optimal transmit strategy problem with the joint sum and per-antenna power constraints has been studied first for MISO channel with two transmit antennas in [18] and the general case in [19]. In [19], a closed-form characterization of an optimal beam-forming strategy is derived. It is shown that the optimal solution is achieved by allocating the maximal sum power with phases matched to the complex channel coefficients. For the optimization problem with joint sum and per-antenna power constraints, it is shown that whenever the optimal power allocation of the corresponding problem with a sum power constraint only exceeds per-antenna power constraints, it is optimal to allocate the maximal per-antenna power to those

Authors are with School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology, Stockholm 10044, Sweden (email: {plcao,oech}@kth.se).

antennas. In [21], the optimal transmit strategy problem for point-to-point MIMO channel with joint sum and per-antenna power constraints has been studied. An iterative algorithm to find the optimal transmit strategy in closed-form using generalized water-filling solution is proposed.

In this work we study MISO wiretap channels with different power constraint settings including sum power constraint only, per-antenna power constraints only, and joint sum and per-antenna power constraints. The optimal trade-off between communication rate and secrecy rate of MISO wiretap channels is motivated by the fact that the optimal coding strategy for the wiretap channel is using a two-layer codebook. The layers are combined using a coding scheme with superposition coding, random binning and rate splitting [22]. The idea of the coding scheme is that the decoding capability of the eavesdropper is exhausted by a part of the private message on the public layer codebook, while the legitimate receiver can decode both the private and confidential messages using both public and private layer codebook. Therefore, instead of sending some useless random messages on the public layer, a useful message can be communicated non-securely to the legitimate receiver [22]–[24] (see Fig. 1). Since the maximal transmission rate and secrecy rate are, in general, achieved by different transmit strategies, we face a trade-off between both objectives which we will study in the following. Some initial results of this paper have been presented in [25]. The contributions of the paper can be summarized as follows:

- We characterize the optimal trade-off between the transmission rate and the secrecy rate of the Gaussian MISO wiretap channel under three different power constraint configurations including: the sum power constraint only, the per-antenna power constraints only, and the joint sum and per-antenna power constraints.
- The closed-form solution of the optimal transmit strategies for a general MISO wiretap channel with the sum power constraints and a 2×2 MISO wiretap channel with per-antenna power constraints only and joint sum and per-antenna power constraints are derived.
- An iterative algorithm to find the optimal solution for general MISO wiretap channels with different power constraint settings are developed.
- A parametrization of the boundary of the rate region of the transmission rate and the secrecy rate of the wiretap channel based on the weighted rate sum optimal rate pairs is derived.

This paper is organized as follows. We start by briefly introducing the system model, sets of power constraints including sum power constraint only, per-antenna power constraints only, and joint sum and per-antenna power constraints. After that an equivalent formulation of the weighted rate sum maximization between the transmission rate and the secrecy rate is derived. The weighted rate sum optimal rate pairs provide a characterization of the boundary of the region of the achievable transmission rate and the secrecy rate. Optimal transmit strategies for MISO wiretap channels with different antenna settings using closed-form solutions and iterative algorithms are then shown in next sections. These solutions allow us to come up with a characterization of the boundary of the region of feasible transmission and secrecy rates. The results are then illustrated and discussed in numerical examples. Finally, we provide some remarks and conclusions.

Notation

We use bold lower-case letters for vectors, bold capital letters for matrices. The superscripts $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$ stand for transpose, conjugate, and conjugate transpose. We use $\succcurlyeq$ for positive semi-definite relation, $\mathrm{tr}(\cdot)$ for trace, $\mathrm{rank}(\cdot)$ for rank and $\mathrm{diag}\{\cdot\}$ for diagonal matrix. The expectation operator of a random variable is given by $\mathbb{E}[\cdot]$. $\mathbb{R}_+$ and $\mathbb{C}$ are sets of non-negative real and complex numbers.

II. Problem Formulation

A. System Model and Power Constraint

We consider a MISO wiretap channel with multiple antennas at the transmitter and single antenna at both legitimate receiver and eavesdropper. Let N_t be the number of transmit antennas. For each channel use, the received signals at the legitimate receiver and the eavesdropper are given as follows

$$\begin{aligned} y_r &= \mathbf{h}_r^H \mathbf{x} + z_r, \\ y_e &= \mathbf{h}_e^H \mathbf{x} + z_e, \end{aligned} \tag{1}$$

where $\mathbf{x} = [x_1, \ldots, x_{N_t}]^T \in \mathbb{C}^{N_t \times 1}$ is the random complex transmit signal vector, $\mathbf{h}_r = [h_{r1}, \ldots, h_{rN_t}]^T \in \mathbb{C}^{N_t \times 1}$, $h_{ri} \neq 0 \; \forall i = 1, \ldots, N_t$, and $\mathbf{h}_e = [h_{e1}, \ldots, h_{eN_t}]^T \in \mathbb{C}^{N_t \times 1}$ are channel coefficient vectors between the transmitter and legitimate receiver, and between the transmitter and eavesdropper, which are perfectly know at the transmitter. z_r and z_e are independent additive white complex Gaussian noise terms with $\sigma_r^2 = \sigma_e^2 = 1$. We will use a Gaussian distributed codebook generated with covariance $\mathbf{Q} = \mathbb{E}[\mathbf{x}\mathbf{x}^H]$, which also specifies the transmit strategy.

Let P_{tot} denote the maximal average sum transmit power and $\hat{P}_k$, $1 \leq k \leq N_t$, denotes the maximal average transmit power at the k-th antenna. Further, let $\mathcal{S}(\hat{\mathbf{p}})$, $\hat{\mathbf{p}} = [P_{tot}, \hat{P}_1, \ldots, \hat{P}_{N_t}]$, denote the set of all transmit strategies satisfying the power constraints $\hat{\mathbf{p}}$, i.e.,

$$\mathcal{S}(\hat{\mathbf{p}}) := \{\mathbf{Q} \succeq 0 : \mathrm{tr}(\mathbf{Q}) \leq P_{tot}, \mathbf{e}_k^T \mathbf{Q} \mathbf{e}_k \leq \hat{P}_k, \forall k \in \mathcal{I}\} \tag{2}$$

where $\mathcal{I} := \{1, \ldots, N_t\}$ and $\mathbf{e}_k$ is the k-th Cartesian unit vector. Depending on the per-antenna power constraints $\hat{P}_k$ and the sum power constraint P_{tot}, we can identify three different cases: (i) *Sum power constraint only* case is considered when the per-antenna power constraints are never active, i.e., $P_{tot} < \min_k(\hat{P}_k)$, (ii) *Per-antenna power constraints only* case is considered when the sum power constraint is never active, i.e., $P_{tot} > \sum_{k=1}^{N_t} \hat{P}_k$, and (iii) *Joint sum and per-antenna power constraints* case are considered when the power constraints relations satisfy $\min_k(\hat{P}_k) \leq P_{tot} \leq \sum_{k=1}^{N_t} \hat{P}_k$, i.e., both sum and per-antenna power constraints can be active.

B. Trade-off Between Transmission Rate and Secrecy Rate

A wiretap channel consists of a legitimate receiver who wishes to receive messages of high rate from a transmitter in the presence of an eavesdropper. In general, for the general wiretap channel, Csiszár and Körner considered transmitting a message M, which is uniformly distributed over $\{1, \dots, 2^{nR}\}$ where n and R are the block-length and the transmission rate of communication, to the legitimate receiver under the observation of the eavesdropper [2]. They showed that for a discrete memoryless wiretap channel, there exists a weakly-secure coding scheme with transmission rate R and equivocation rate $R_{eq} \leq \frac{1}{n} H(M|Y_e^n) - \epsilon$, for some $\epsilon > 0$, corresponding to input-output variables that satisfy $M \to X^n \to Y_r^n Y_e^n$. Let $\mathcal{R}$ denote the set consisting of all non-negative achievable rate pairs of transmission rate R and equivocation rate R_{eq}. Then $(R, R_{eq}) \in \mathcal{R}$ if and only if there exist random variables $U \to V \to X \to Y_r Y_e$ such that

$$0 \leq R_{eq} \leq I(V; Y_r|U) - I(V; Y_e|U) \quad (3)$$

$$R_{eq} \leq R \leq I(V; Y_r|U) + \min[I(U; Y_r), I(U; Y_e)]. \quad (4)$$

The secrecy capacity, which is defined as the maximum rate at which the message can be securely sent to the legitimate receiver, is then given as

$$C_s = \max_{p(u)p(v|u)p(x|v)} I(V; Y_r|U) - I(V; Y_e|U). \quad (5)$$

It is known from [3]–[5] that for Gaussian channels, the secrecy capacity can be achieved with Gaussian distributed inputs. It can be obtained by solving the following optimization problem

$$C_s(\hat{\mathbf{p}}) = \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} R_s(\mathbf{Q}), \quad (6)$$

where $R_s(\mathbf{Q}) = \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r) - \log(1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e)$.

In the following proposition, we provide a condition for a positive secrecy capacity.

Proposition 1. *A necessary and sufficient condition for a positive secrecy capacity of a Gaussian MISO wiretap channel, i.e., $C_s(\hat{\boldsymbol{p}}) > 0$, is that $\boldsymbol{h}_r \boldsymbol{h}_r^H$ - $\boldsymbol{h}_e \boldsymbol{h}_e^H \in \mathbb{C}^{N_t \times N_t}$ has to have a positive eigenvalue.*

Proof: The proof of Proposition 1 is in Appendix A. ■

In [22] and [24], authors extended the problem in [2] to a problem of simultaneously transmitting private and confidential message. The coding schemes are designed such that the legitimate receiver can decode the message of both public and private layers while the eavesdropper might be able to decode the message on the public layer only. Therefore, it is reasonable to use the public layer to transmit useful messages non-securely to the legitimate receiver instead of broadcasting useless random messages in order to exhaust the capacity of the eavesdropper only. In this setting, $M = (M_p, M_s)$ where a private message M_p and a confidential message M_s are uniformly distributed over $\{1, \dots, 2^{nR_p}\}$ and $\{1, \dots, 2^{nR_s}\}$, is transmitted from the transmitter to the legitimate receiver (see Fig. 1). The confidential message M_s needs to be kept secret from the eavesdropper while there is no secrecy constraint is applied on M_p.

Following [22, Theorem 1], the region of the transmission rate and equivocation of the Gaussian wiretap channel under the power constraint (2) is given by the set of pairs of transmission rate and equivocation rate satisfying

$$R_e = R_s \leq \min\{R, C_s(\hat{\mathbf{p}})\} \quad (7)$$

$$R = R_p + R_s \leq C(\hat{\mathbf{p}}), \quad (8)$$

where

$$C(\hat{\mathbf{p}}) = \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} R(\mathbf{Q}), \quad (9)$$

with $R(\mathbf{Q}) = \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r)$.

Alternatively, in [22], authors also pointed out that a non-negative pair $(R, R_e) = (R_p + R_s, R_s)$ is achievable for a discrete memoryless wiretap channel if and only if (R_p, R_s) is an achievable private-confidential rate pair for the same channel. Rates in the rate pair above is actually the transmission rate R and secrecy rate R_s, i.e., $(R, R_e) = (R_p + R_s, R_s) = (R, R_s)$. Since the optimal choices of the optimal transmit strategies of the transmission capacity in (9) and the secrecy capacity in (5) do not have to be the same, a trade-off between two objectives appears.

For a Gaussian MISO wiretap channel, the rate region $\mathcal{R}$ that describes the trade-off, which is controlled by optimal transmit strategy $\mathbf{Q}$, between the transmission rate and secrecy rate with a given set of power constraints can be described as

$$\mathcal{R}_{MISO}(\hat{\mathbf{p}}) = \{[R, R_s] : 0 \leq R_s \leq R \leq R(\mathbf{Q}), \\ R_s \leq R_s(\mathbf{Q}), \mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})\}. \quad (10)$$

The boundary of the rate region $\mathcal{R}_{MISO}(\hat{\mathbf{p}})$ can be determined from the bounds on the secrecy rate (6), transmission rate (9) and the Pareto optimal section for $w \in (0, 1)$. This implies that with weight $0 \leq w \leq 1$ the optimal trade-off between transmission and secrecy rates for a Gaussian MISO wiretap channel with power constraints $\hat{\mathbf{p}}$, can be obtain by solving the following optimization problem

$$R_{\sum}(\hat{\mathbf{p}}, w) = \max_{\mathbf{Q}} R_{\sum}(\mathbf{Q}, w), \text{ s.t.} \, \mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}}), \quad (11)$$

where

$$R_{\sum}(\mathbf{Q}, w) = (1 - w) R(\mathbf{Q}) + w R_s(\mathbf{Q}) = R(\mathbf{Q}) - w R_e(\mathbf{Q}) \quad (12)$$

with $R_s(\mathbf{Q}) = R(\mathbf{Q}) - R_e(\mathbf{Q})$, $R(\mathbf{Q}) = \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r)$, and $R_e(\mathbf{Q}) = \log(1 + \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e)$.

Note that the secrecy rate is a fraction of the transmission rate, i.e., $0 \leq R_s \leq R$ has to be satisfied. Therefore, the boundary of the rate region is also bounded by a line for which $R = R_s$. If this region is convex (see Fig. 2) then the set of weighted rate sum optimal rate pairs characterize the boundary of the rate region. If this region is non-convex, then the set of all weighted rate sum optimal rate pairs can be used to characterize the boundary of the convex hull of the rate region, i.e., $\mathcal{C}_{MISO}(\hat{\mathbf{p}}) = \text{ConvexHull}(\mathcal{R}_{MISO}(\hat{\mathbf{p}}))$. In this case, we need to allow time-sharing between two rate pairs.

In the following, we provide solutions for the optimization problem (11). These also provide us a characterization of the rate region (10) that describes the trade-off between the transmission rate and the secrecy rate of the Gaussian MISO wiretap channel.

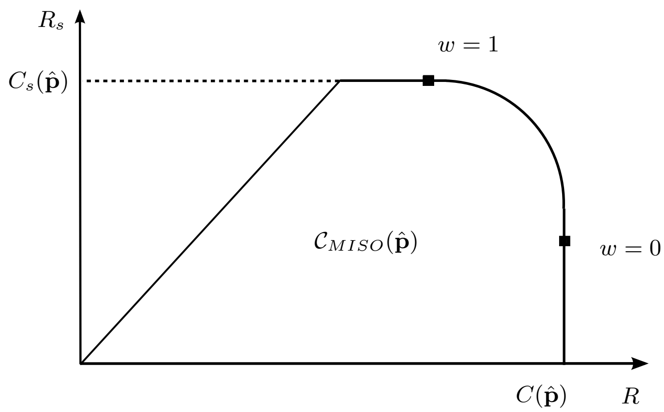

Fig. 2: Capacity region illustrating the trade-off between transmission rate R and secrecy rate R_s

III. Equivalent Problem Formulations and Parametrizations of the boundary of rate region

Since (11) is not a convex optimization problem because $R_s(\mathbf{Q})$ is non-convex in $\mathbf{Q}$, we first reformulate (11) using the following lemma to an equivalent convex optimization problem that allows further analysis.

Lemma 1 ([26, Lemma 2, scalar case])**.** *Consider the function $f(D) = -DE + \log(D) + 1$ where $D, E \in \mathbb{R}, E > 0$. Then,*

$$\max_{D>0} f(D) = \log(E^{-1}), \tag{13}$$

with the optimum value $D^\star = E^{-1}$.

In the following, we provide two reformulations of (11) using Lemma 1. Lemma 1 is first applied to both $R(\mathbf{Q})$ and $R_e(\mathbf{Q})$ of the weighted rate sum function (12). After that, Lemma 1 is applied to $R_e(\mathbf{Q})$ of the weighted rate sum function (12) only. The first reformulation is later used to derive a closed-form solution of the optimal transmit strategy for certain cases. The second reformulation is then used for the derivation of an efficient iterative optimization algorithm.

A. Equivalent problem formulation for closed-form solution

By applying Lemma 1 with $\log E_i^{-1} = \max_{D_i>0} f_i(D_i)$ where $E_i = 1 + \mathbf{h}_i^H\mathbf{Q}\mathbf{h}_i$ and $f_i(D_i) = -D_iE_i + \log(D_i) + 1$ for $i \in \{r, e\}$, the optimization problem (11) can be expressed as

$$\begin{aligned} R_{\sum}(\hat{\mathbf{p}}, w) &= \max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \log(1 + \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r) - w\log(1 + \mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e) \\ &= \max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} (-\max_{D_r>0} f_r(D_r) + w\max_{D_e>0} f_e(D_e)) \\ &= \max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \min_{D_r>0}\max_{D_e>0} (-f_r(D_r) + wf_e(D_e)) \\ &= \max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \min_{D_r>0}\max_{D_e>0} D_r(1 + \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r) - \log(D_r) \\ &\quad - 1 + w(-D_e(1 + \mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e) + \log(D_e) + 1) \\ &= \max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \min_{D_r>0}\max_{D_e>0} D_r(\mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r + w\frac{D_e}{D_r}\mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e) \\ &\quad + D_r - \log(D_r) - 1 + w(-D_e + \log(D_e) + 1). \end{aligned} \tag{14}$$

For a given w, let us define $t := w\frac{D_e}{D_r}$ and $\phi^{(1)}(\mathbf{Q}, t) = \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r - t\mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e = \mathrm{tr}(\mathbf{A}\mathbf{Q})$ with $\mathbf{A} := \mathbf{h}_r\mathbf{h}_r^H - t\mathbf{h}_e\mathbf{h}_e^H$, then (14) can be written as

$$\begin{aligned} R_{\sum}(\hat{\mathbf{p}}, w) = \max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \min_{D_r>0}\max_{D_e>0} D_r\phi^{(1)}(\mathbf{Q}, t) + D_r - \log(D_r) \\ + w(-D_e + \log(D_e) + 1) - 1. \end{aligned} \tag{15}$$

Although t is dependent on D_e, D_r and w, the optimization with respect to $\mathbf{Q}$ only depends on t. Thus, we first find the optimal transmit strategy $\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t)$ by solving

$$\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t) = \arg\max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \phi^{(1)}(\mathbf{Q}, t) \tag{16}$$

for a given t and power constraints $\hat{\mathbf{p}}$. After having the optimal $\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t)$ for a given t, we can obtain the corresponding optimal $D_e^\star$ and $D_r^\star$ following Lemma 1. The corresponding w is then given by $t\frac{D_r^\star}{D_e^\star}$. The following theorem shows that the previous procedure can be used to compute the optimal weighted rate sum $R_{\sum}(\hat{\mathbf{p}}, w)$.

Theorem 1. *Let $t_{\max} = 2^{C_s(\hat{\mathbf{p}})}$, the optimal solution of* (11) *can be determined as follows.*

(i) *For every $w \in [0, 1]$ there exists a $t \in [0, t_{\max}]$ such that $\boldsymbol{Q}_{opt}^{(1)}(\hat{\boldsymbol{p}}, t)$ is an optimal transmit strategy, i.e., $R_{\sum}(\hat{\boldsymbol{p}}, w) = R_{\sum}(\boldsymbol{Q}_{opt}^{(1)}(\hat{\boldsymbol{p}}, t), w)$.*

(ii) *For every $t \in [0, t_{\max}]$ there exists a $w \in [0, 1]$ such that $\boldsymbol{Q}_{opt}^{(1)}(\hat{\boldsymbol{p}}, t)$ is an optimal transmit strategy, i.e., $R_{\sum}(\hat{\boldsymbol{p}}, w) = R_{\sum}(\boldsymbol{Q}_{opt}^{(1)}(\hat{\boldsymbol{p}}, t), w)$.*

Proof: The proof of Theorem 1 is in Appendix B ■

As a result, the optimal region that describes the trade-off between the point-to-point transmission rate and the wire-tap secrecy rate is equivalently described as $\mathcal{R}_{MISO}(\hat{\mathbf{p}}) = \{[R, R_s] : 0 \le R_s \le R \le R(\mathbf{Q}(\hat{\mathbf{p}}, t)), R_s \le R_s(\mathbf{Q}(\hat{\mathbf{p}}, t)), t \in [0, t_{\max}]\}$.

B. Equivalent problem formulation for iterative algorithm

By applying Lemma 1 with $E_e(\mathbf{Q}) = 1 + \mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e$ and $f_e(D_e, \mathbf{Q}) = -D_eE_e(\mathbf{Q}) + \log(D_e) + 1$ only, the optimization problem (11) can be alternatively expressed as

$$\begin{aligned} R_{\sum}(\hat{\mathbf{p}}, w) &= \max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \log(1 + \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r) - w\log(1 + \mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e) \\ &= \max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \max_{D_e>0} \log(1 + \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r) + wf_e(D_e, \mathbf{Q}) \\ &= \max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \max_{D_e>0} \log(1 + \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r) \\ &\quad + w(-D_e(1 + \mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e) + \log(D_e) + 1). \end{aligned} \tag{17}$$

Similarly, if we define $s = wD_e$ and $\phi^{(2)}(\mathbf{Q}, s) = \log(1 + \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r) - s\mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e$, then (17) can be written as

$$R_{\sum}(\hat{\mathbf{p}}, w) = \max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \max_{D_e>0} \phi^{(2)}(\mathbf{Q}, s) - s + w\log(D_e) + w.$$

Then the problem of finding the optimal transmit strategy $\mathbf{Q}_{opt}^{(2)}(\hat{\mathbf{p}}, s)$ depends only on s can be expressed as

$$\mathbf{Q}_{opt}^{(2)}(\hat{\mathbf{p}}, s) = \arg\max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \phi^{(2)}(\mathbf{Q}, s). \tag{18}$$

The approach is similar as for the previous reformulation in Section III.A. Although s is dependent on D_e and w, the optimization with respect to $\mathbf{Q}$ only depends on s. Thus, we first solve (18) for a given s and power constraints $\hat{\mathbf{p}}$. Once $\mathbf{Q}_{opt}^{(2)}(\hat{\mathbf{p}}, s)$ is obtained, the corresponding $D_e^\star$ and weight w can be found using Lemma 1 and formula $w = s/D_e^\star = s(1 + \mathbf{h}_e^H \mathbf{Q}_{opt}^{(2)} s(\hat{\mathbf{p}}, s)\mathbf{h}_e)$.

Theorem 2. *Let $s_{\max} = (1 + \boldsymbol{h}_e^H \boldsymbol{Q}^s \boldsymbol{h}_e)^{-1}$ with $\boldsymbol{Q}^s = \arg\max_{\boldsymbol{Q}\in\mathcal{S}(\hat{\boldsymbol{p}})} R_{\sum}(\boldsymbol{Q}, w = 1) = \arg\max_{\boldsymbol{Q}\in\mathcal{S}(\hat{\boldsymbol{p}})} R_s(\boldsymbol{Q})$, the optimal solution of (11) can be determined as follows.*

(i) *For every $w \in [0, 1]$ there exists a $s \in [0, s_{\max}]$ such that $\boldsymbol{Q}_{opt}^{(2)}(\hat{\boldsymbol{p}}, s)$ is an optimal transmit strategy, i.e., $R_{\sum}(\hat{\boldsymbol{p}}, w) = R_{\sum}(\boldsymbol{Q}_{opt}^{(2)}(\hat{\boldsymbol{p}}, s), w)$.*

(ii) *For every $s \in [0, s_{\max}]$ there exists a $w \in [0, 1]$ such that $\boldsymbol{Q}_{opt}^{(2)}(\hat{\boldsymbol{p}}, s)$ is an optimal transmit strategy, i.e., $R_{\sum}(\hat{\boldsymbol{p}}, w) = R_{\sum}(\boldsymbol{Q}_{opt}^{(2)}(\hat{\boldsymbol{p}}, s), w)$.*

Proof: The proof of Theorem 2 can be done by simply replace $t \in [0, t_{\max}]$ in the proof of Theorem 1 by $s \in [0, s_{\max}]$. ■

As a result, the optimal region that describes the trade-off between the transmission rate and the secrecy rate is equivalently described as $\mathcal{R}_{MISO}(\hat{\mathbf{p}}) = \{[R, R_s] : 0 \leq R_s \leq R \leq R(\mathbf{Q}(\hat{\mathbf{p}}, s)), R_s \leq R_s(\mathbf{Q}(\hat{\mathbf{p}}, s)), s \in [0, s_{\max}]\}$.

In the following, we propose solutions to find the optimal transmit strategy and characterize the optimal trade-off between the transmission rate and secrecy rate of the Gaussian wiretap channels using the reformulations above.

C. Beamforming Optimality

The rank of the optimal transmit strategy for a given set of power constraints $\hat{\mathbf{p}}$ is given in the following theorem.

Theorem 3. *For an optimal transmit strategy, it is sufficient to consider beam-forming strategies, i.e., there exists always an optimal rank one solution.*

Proof: The proof of Theorem 3 is in Appendix C. ■

IV. Finding Optimal Transmit Strategies using Closed-form Solution

In this section, we present closed-form solutions of the optimal transmit strategies for a given t of the weighted rate sum optimization problem for the MISO wiretap channel for three different power constraint cases: (i) with a sum power constraint only; (ii) per-antenna power constraints only; and (iii) with joint sum and per-antenna power constraints.

A. Sum Power Constraint Only

In the sum power constraint only case, the per-antenna power constraints are never active, e.g., if we have $P_{tot} < \hat{P}_k$ $\forall k$. Let $\mathcal{S}_{SPC}$ denote the set of all allocated powers which satisfy the sum power constraint P_{tot} only, i.e., $\mathcal{S}_{SPC} = \{\mathbf{Q} \succeq 0 : \mathrm{tr}(\mathbf{Q}) \leq P_{tot}\}$. The equivalent problem of finding the weighted rate sum optimal transmit strategy for the MISO wiretap channel with sum power constraint only for a given t can be written as

$$\mathbf{Q}_{SPC}(t) = \arg\max_{\mathbf{Q}\in\mathcal{S}_{SPC}} \phi^{(1)}(\mathbf{Q}, t), \tag{19}$$

where $\phi^{(1)}(\mathbf{Q}, t) = \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - t\mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e$.

Theorem 4. *The closed-form expression for the optimal transmit strategy of (19) is given by*

$$\boldsymbol{Q}_{SPC}^{(1)}(t) = P_{tot} \boldsymbol{v}\boldsymbol{v}^H \tag{20}$$

where $\boldsymbol{v}$ is the eigenvector associated with the largest eigenvalue of $\boldsymbol{h}_r \boldsymbol{h}_r^H - t\boldsymbol{h}_e \boldsymbol{h}_e^H$ for a given t.

Proof: The proof of Theorem 4 is in Appendix D. ■

In the next sections, we aim to find optimal solutions for the two remaining cases with per-antenna power constraints only and with joint sum and per-antenna power constraints.

B. Per-antenna Power Constraints Only

The per-antenna power constraints only case is considered when the sum power constraint is never active, e.g., $P_{tot} > \sum_{k\in\mathcal{I}} \hat{P}_k$ with $\mathcal{I} = \{1, \ldots, N_t\}$. Let $\mathcal{S}_{PAPC}$ denote the set of all power allocations which satisfy the per-antenna power constraints $\hat{P}_k$, $\forall k \in \mathcal{I}$, only, i.e., $\mathcal{S}_{PAPC} = \{\mathbf{Q} \succeq 0 : \mathbf{e}_k^T \mathbf{Q} \mathbf{e}_k \leq \hat{P}_k, \forall k \in \mathcal{I}\}$. The equivalent problem of finding the weighted rate sum optimal transmit strategy for the MISO wiretap channel with per-antenna power constraints only for a given t can be written as

$$\mathbf{Q}_{PAPC}^{(1)}(t) = \arg\max_{\mathbf{Q}\in\mathcal{S}_{PAPC}} \phi^{(1)}(\mathbf{Q}, t), \tag{21}$$

where $\phi^{(1)}(\mathbf{Q}, t) = \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - t\mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e$.

In general, the diagonal elements of the optimal transmit strategy can be obtained by the proposition below.

Proposition 2. *The optimal transmit strategy $\boldsymbol{Q}_{PAPC}(t)$ of problem (21) has diagonal elements $q_{kk} = \hat{P}_k$, $\forall k \in \mathcal{I}$.*

Proof: The proof of Proposition 2 is in Appendix E. ■

Proposition 2 shows that for the per-antenna power constraint only problem, it is optimal to allocate maximal individual power on the transmit antennas. The remaining problem is to find off-diagonal elements of $\mathbf{Q}_{PAPC}(t)$ for a given t.

The main difficulty here is the positive semi-definite constraint. To overcome this, we consider a relaxed optimization problem involving the 2×2 principal minors of $\mathbf{Q}_{PAPC}(t)$ similarly as done in [27]. Let $\mathbf{X}_{k,l}(t)$ be a principal minor matrix which is obtained from $\mathbf{Q}$ by removing $N_t - 2$ columns, except columns k and l, and the corresponding $N_t - 2$ rows except rows k and l. Then, $\mathbf{X}_{k,l}(t)$ is given as

$$\mathbf{X}_{k,l}(t) = \begin{bmatrix} \hat{P}_k & q_{kl}^*(t) \\ q_{kl}(t) & \hat{P}_l \end{bmatrix} \tag{22}$$

where $k, l \in \mathcal{I}$, $k \neq l$. Therewith, we can formulate a relaxed optimization problem as follows

$$\max_{\mathbf{Q}} \ \phi^{(1)}(\mathbf{Q}, t), \qquad \mathrm{s.t.}\ q_{kk} = \hat{P}_k, \forall k \in \mathcal{I} \tag{23}$$
$$\mathbf{X}_{k,l}(t) \succeq 0, \forall k, l \in \mathcal{I}, k \neq l.$$

The off-diagonal elements of the covariance matrix in (23) then can be obtained using the following theorem.

Theorem 5. *The optimal transmit strategy $\boldsymbol{Q}_{PAPC-R}(t)$ of the relaxed optimization problem* (23) *has off-diagonal elements*

$$q_{kl}(t) = \frac{h_{rk}^* h_{rl} - t h_{ek}^* h_{el}}{|h_{rk}^* h_{rl} - t h_{ek}^* h_{el}|} \sqrt{\hat{P}_k \hat{P}_l}, \quad k, l \in \mathcal{I}, k \neq l. \quad (24)$$

Proof: The proof of Theorem 5 is in Appendix F. ∎

From Proposition 2 and Theorem 5, we have the following conclusion and remarks.

Corollary 1. *If there are only two transmit antennas, i.e.,$N_t = 2$, then* (24) *always leads to a positive semi-definite solution with eigenvalues zero and $\hat{P}_1 + \hat{P}_2$, i.e., the optimal solution* (24) *of the relaxed optimization problem* (23) *is actually the optimal solution of* (21).

For $n > 2$, it is not clear if (24) always results in a positive semi-definite solution. Numerical experiments suggest this assumption, but a proof is missing. So that, we have only the following remark.

Remark 1. *If the solution* (24) *leads to a positive semi-definite solution, then it is also an optimal solution of* (21).

Thus it is a good strategy to first compute the solution according to (24) and then test if it is positive semi-definite.

C. Joint Sum and Per-antenna Power Constraints

In this section, we discuss the optimization for the wiretap channel for the interesting case when both sum and per-antenna power constraints can be active, i.e., $\min_k(\hat{P}_k) \leq P_{tot} \leq \sum_{i=1}^{N_t} \hat{P}_k$ [19]. Let $\mathcal{S}_{JSPC} = \{\mathbf{Q} \succeq 0 : \operatorname{tr}(\mathbf{Q}) \leq P_{tot}, \mathbf{e}_k^T \mathbf{Q} \mathbf{e}_k \leq \hat{P}_k, \forall k \in \mathcal{I}\}$ denote the set of all power allocations which satisfy the joint sum and per-antenna power constraints. Then similar to the optimization problem with sum power constraint only and per-antenna power constraints only, an equivalent optimization problem for finding the optimal transmit strategy for the Gaussian wiretap channel with joint sum and per-antenna power constraints can be stated as

$$\mathbf{Q}_{JSPC}^{(1)}(t) = \underset{\mathbf{Q} \in \mathcal{S}_{JSPC}}{\arg\max}\, \phi^{(1)}(\mathbf{Q}, t) \quad (25)$$

for a given t, where $\phi^{(1)}(\mathbf{Q}, t) = \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - t \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e$.

Proposition 3. *For $\min_k(\hat{P}_k) \leq P_{tot} \leq \sum_{k=1}^{N_t} \hat{P}_k$, the optimal solution for the MISO wiretap channel with joint sum and per-antenna power constraints problem can be achieved when the transmit strategy uses full power P_{tot}, i.e., $\operatorname{tr}(\boldsymbol{Q}_{JSPC}^{(1)}(t)) = P_{tot}$.*

Proof: The proof of Proposition 3 is in Appendix G. ∎

This proposition permits us to consider only transmit strategies which allocate full power P_{tot}. If the solution of the sum power constraint only problem does not violate the per-antenna power constraints, then it is also the solution of the joint sum and per-antenna power constraints problem. However, this is not always the case. In such cases, the maximum per-antenna power will be allocated to those antennas for which the sum power constraint only optimal solution violates the per-antenna power constraints. For arbitrary number of transmit antennas, the following theorem will show that if there exists any antenna for which the optimal power allocation of the optimization problem with sum power constraint only exceeds the per-antenna power constraints, then for those antennas the optimal power allocation is equal the per-antenna power constraints and (25) reduces to a new optimization problem with a smaller total transmit power and a reduced number of channel coefficients.

Theorem 6. *For $\min_k(\hat{P}_k) \leq P_{tot} \leq \sum_{k=1}^{N_t} \hat{P}_k$, let $\boldsymbol{Q}_{SPC}^{(1)}(t)$ be the optimal transmit strategy under the sum power constraint only, $\boldsymbol{A} = \boldsymbol{h}_r \boldsymbol{h}_r^H - t \boldsymbol{h}_e \boldsymbol{h}_e^H = \sum_{l=1}^2 \lambda_l \boldsymbol{u}_l \boldsymbol{u}_l^H$ and $\mathcal{P} := \{k \in \mathcal{I} : \boldsymbol{e}_k^T \boldsymbol{Q}_{SPC}^{(1)}(t) \boldsymbol{e}_k > \hat{P}_k\}$ where $\mathcal{I} := \{1, \ldots, N_t\}$. If $\mathcal{P} = \emptyset$ then $\boldsymbol{Q}_{JSPC}^{(1)}(t) = \boldsymbol{Q}_{SPC}^{(1)}(t)$, else $\boldsymbol{e}_k^T \boldsymbol{Q}_{JSPC}^{(1)}(t) \boldsymbol{e}_k = \hat{P}_k$, $\forall k \in \mathcal{P}$ and the remaining optimal powers can be computed by solving a reduced optimization problem*

$$\underset{\boldsymbol{q}' \in \mathbb{Q}'}{\arg\max} \sum_{l=1}^{2} \lambda_l |\boldsymbol{q}'^H \boldsymbol{u}'_l|^2 \quad (26)$$

where $\mathcal{P}^c = \mathcal{I} \setminus \mathcal{P}$ and $\mathbb{Q}' := \{\boldsymbol{q}' : \sum_{k \in \mathcal{P}^c} |q_k|^2 \leq P_{tot} - \sum_{k \in \mathcal{P}} \hat{P}_k, |q_k|^2 \leq \hat{P}_k, \forall k \in \mathcal{P}^c\}$ and $\boldsymbol{u}'_l = [u_{lk}]_{k \in \mathcal{P}^c}$.

Proof: The proof of Theorem 6 is in Appendix H ∎

For a special case where the transmitter has two transmit antennas only, a closed-form solution of the optimal transmit strategy can be shown in the following theorem.

Theorem 7. *For $\min_k(\hat{P}_k) \leq P_{tot} \leq \sum_{k=1}^{2} \hat{P}_k$, let $\boldsymbol{Q}_{SPC}(t)$ be the optimal transmit strategy under the sum power constraint only. Let $\mathcal{P} := \{k \in \{1, 2\} : \boldsymbol{e}_k^T \boldsymbol{Q}_{SPC}(t) \boldsymbol{e}_k > \hat{P}_k\}$. Then, for the optimization problem with joint sum and per-antenna power constraints, we have*

- *If $\mathcal{P} = \emptyset$, $\boldsymbol{Q}_{JSPC}^{(1)}(t) = \boldsymbol{Q}_{SPC}^{(1)}(t)$*
- *Otherwise $\boldsymbol{Q}_{JSPC}(t)$ has diagonal elements*

$$\begin{cases} \boldsymbol{e}_k^T \boldsymbol{Q}_{JSPC}^{(1)}(t) \boldsymbol{e}_k = \hat{P}_k, \\ \boldsymbol{e}_l^T \boldsymbol{Q}_{JSPC}^{(1)}(t) \boldsymbol{e}_l = P_{tot} - \hat{P}_k, \end{cases} \quad (27)$$

and off-diagonal elements

$$q_{kl}(t) = \frac{h_{rk}^* h_{rl} - t h_{ek}^* h_{el}}{|h_{rk}^* h_{rl} - t h_{ek}^* h_{el}|} \sqrt{\hat{P}_k (P_{tot} - \hat{P}_k)}, \quad (28)$$

for $k \in \mathcal{P}$, $l \neq k$.

Proof: Since the case with $\mathcal{P} = \emptyset$ is obvious, we focus to prove the remaining case. Consider a scalar function $\phi^{(1)}(\mathbf{Q}, t) := \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - t \mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e$. From [28, Lemma 3.10], we know that for a given t the scalar function $\phi^{(1)}(\mathbf{Q}, t) : \mathbf{Q} \to \mathbb{R}_+$ is matrix-monotone in $\mathbf{Q}$. Therefore, if any k-th optimal power of the sum power constraint only solution violates a per-antenna power constraint, then it has to be set equal to the maximal individual power $\hat{P}_k$, $k \in \{1, 2\}$. Due to Proposition 3, the remaining optimal transmit power has to set equal to $P_{tot} - \hat{P}_k$ and the off-diagonal elements of the optimal transmit strategy for the wiretap channel with joint sum and per-antenna power constraints are then calculated using Theorem 5 with the corresponding optimal transmit powers $\mathbf{e}_k^T \mathbf{Q}_{JSPC}(t) \mathbf{e}_k = \hat{P}_k$ and $\mathbf{e}_l^T \mathbf{Q}_{JSPC}(t) \mathbf{e}_l = P_{tot} - \hat{P}_k$, $k \in \mathcal{P}$, $l \neq k$. ∎

V. Finding Optimal Transmit Strategy using Iterative Algorithms

In this section, we aim to find the solutions of the optimal transmit strategy for a given s of the weighted rate sum optimization problem for the MISO wiretap channel under a given set of power constraints. Since the equivalent problem in (18) is convex, it allows optimal solutions to be found numerically using convex optimization tools [29]. However more tailored approaches have lower computational complexity and are therefore more interesting for practical systems. We provide in the following an algorithmic solution based on the semi-definite programming (SDP) framework with significantly lower complexity.

Let us consider the optimization problem for a given s

$$\max_{\mathbf{Q}} \phi^{(2)}(\mathbf{Q}, s) \quad \text{s. t.}\, \mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}}), \tag{29}$$

where $\phi^{(2)}(\mathbf{Q}, s) = \log(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r) - s\mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e$.

The Lagrangian for the problem (29) is given by

$$\mathcal{L} = \phi^{(2)}(\mathbf{Q}, s) - \mathrm{tr}(\mathbf{D}(\mathbf{Q} - \hat{\mathbf{P}})) - \mu(\mathrm{tr}(\mathbf{Q}) - P_{tot}) + \mathrm{tr}(\mathbf{M}\mathbf{Q}),$$

where dual variable $\mathbf{D} = \mathrm{diag}\{\nu_i\}$ is a diagonal matrix of Lagrangian multipliers for the per-antenna power constraints, dual variable μ is the Lagrangian multiplier for the sum power constraint, dual variable $\mathbf{M}$ is the Lagrangian multiplier for the positive semi-definite constraint, and $\hat{\mathbf{P}} = \mathrm{diag}\{\hat{P}_i\}$, $\forall i \in \mathcal{I}$ is a diagonal matrix of the per-antenna power constraints.

Taking the first derivative of the Lagrangian above and setting it equal to zero, we obtain

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \mathbf{Q}} &= \mathbf{h}_r(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r)^{-1} \mathbf{h}_r^H - s\mathbf{h}_e \mathbf{h}_e^H - \mathbf{D} - \mu \mathbf{I} + \mathbf{M} \\ &= \mathbf{h}_r(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r)^{-1} \mathbf{h}_r^H - \mathbf{K} + \mathbf{M} \stackrel{!}{=} 0,\end{aligned} \tag{30}$$

with $\mathbf{K} = s\mathbf{h}_e \mathbf{h}_e^H + \mathbf{D} + \mu \mathbf{I}$.

The KKT conditions then can be derived as

$$\begin{aligned}&\mathbf{h}_r(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r)^{-1} \mathbf{h}_r^H = \mathbf{K} - \mathbf{M} && && (31)\\ &\mathrm{tr}(\mathbf{Q}) \le P_{tot} && \mathbf{e}_k^T \mathbf{Q} \mathbf{e}_k \le \hat{P}_k, \forall k \in \mathcal{I} && \mathbf{Q} \succeq 0 \\ &\mu \ge 0 && \mathbf{D} \succeq 0 && \mathbf{M} \succeq 0 \\ &\mu(\mathrm{tr}(\mathbf{Q}) - P_{tot}) = 0 && \mathrm{tr}(\mathbf{D}(\mathbf{Q} - \hat{\mathbf{P}})) = 0 && \mathbf{M}\mathbf{Q} = 0.\end{aligned}$$

Since (29) is a convex optimization problem, the solution for the optimal transmit strategy $\mathbf{Q}$ corresponding to a given set of power constraints can be found from above equations. Based on the KKT conditions, we obtain a set of optimality conditions corresponding to a given s and a set of power constraints as well as establish the optimal value of $\mathbf{Q}$ as an explicit function of the dual variables. As a result, in the following we can design iterative algorithms to find the optimal dual variables and optimal transmit strategies corresponding to sets of power constraints above. Since $\mathbf{K}$ is invertible, we have the following lemma.

Lemma 2. *The optimality condition for the optimization problem* (18) *with a given set of power constraints is*

$$\boldsymbol{h}_r^H \check{\mathbf{K}} \boldsymbol{h}_r - \boldsymbol{h}_r^H \boldsymbol{Q} \boldsymbol{h}_r = 1, \tag{32}$$

with $\check{\mathbf{K}} = \boldsymbol{K}^{-1}$.

Proof: The proof of Lemma 2 is in Appendix I ■

Next, we establish the optimal transmit strategy $\mathbf{Q}$ in terms of dual variables $\mathbf{D}$ and μ. Similarly as in [11], to approach the solution, we first need to decompose channel $\mathbf{h}_r^H$ as follows.

$$\mathbf{h}_r^H = [\sigma \ \mathbf{0}_{1,n-1}][\mathbf{v}_1 \ \mathbf{V}_2]^H = \sigma \mathbf{v}_1^H, \tag{33}$$

where $n = N_t$, σ is a singular value, $\mathbf{v}_1$ is the first column which contains the basis for the row space of $\mathbf{h}_r^H$ and $\mathbf{V}_2$ is the last $n-1$ columns of the right singular vector which contains the basis for the null space of $\mathbf{h}_r^H$.

Theorem 8. *Let* $\check{\boldsymbol{h}}_r^H = \boldsymbol{v}_1 \sigma^{-1}$. *For a given dual variable* $\boldsymbol{R} = \boldsymbol{D} + \mu \boldsymbol{I} \succ 0$, *the optimal transmit strategy* $\boldsymbol{Q}$ *for a given* s *satisfying the optimality condition* (32) *is given by*

$$\boldsymbol{Q}_{opt}^{(2)}(\hat{\boldsymbol{p}}, s) = \boldsymbol{R}^{-1} - \check{\boldsymbol{h}}_r^H \check{\boldsymbol{h}}_r - \boldsymbol{X} - \boldsymbol{Y}, \tag{34}$$

with Hermitian matrices

$$\boldsymbol{X} = \boldsymbol{V}_2 \boldsymbol{A} \boldsymbol{V}_2^H + \boldsymbol{v}_1 \boldsymbol{B} \boldsymbol{V}_2^H + \boldsymbol{V}_2 \boldsymbol{B}^H \boldsymbol{v}_1^H, \tag{35}$$

$$\boldsymbol{A} = \Big(\boldsymbol{I}_{n-1} - \boldsymbol{B}^H \boldsymbol{v}_1^H (\boldsymbol{R} + s\boldsymbol{h}_e \boldsymbol{h}_e^H) \boldsymbol{V}_2\Big)\Big(\boldsymbol{V}_2^H (\boldsymbol{R} + s\boldsymbol{h}_e \boldsymbol{h}_e^H) \boldsymbol{V}_2\Big)^{-1}, \tag{36}$$

$$\boldsymbol{B} = \left(\boldsymbol{v}_1^H \check{\boldsymbol{h}}_r^H \check{\boldsymbol{h}}_r (\boldsymbol{R} + s\boldsymbol{h}_e \boldsymbol{h}_e^H) \boldsymbol{V}_2\right)\left(\boldsymbol{V}_2^H (\boldsymbol{R} + s\boldsymbol{h}_e \boldsymbol{h}_e^H) \boldsymbol{V}_2\right)^{-1}, \tag{37}$$

$$\boldsymbol{Y} = \boldsymbol{R}^{-1} s\boldsymbol{h}_e \boldsymbol{h}_e^H (\boldsymbol{I}_n + \boldsymbol{R} s\boldsymbol{h}_e \boldsymbol{h}_e^H)^{-1} \boldsymbol{R}^{-1}. \tag{38}$$

Proof: The proof of Theorem 8 is in Appendix J ■

Theorem 8 suggests an iterative algorithm to find the dual variables $\mathbf{D}$ and μ for the optimization problem (18), which correlate to the optimal transmit strategy $\mathbf{Q}_{opt}^{(2)}(\hat{\mathbf{p}}, s)$. The algorithm is initialised with an arbitrary starting point $\mathbf{D} \succeq 0$ and $\mu \ge 0$ such that $\mathbf{R} = \mathbf{D} + \mu \mathbf{I} \succ 0$. At the i-th iteration, we obtain $\mathbf{R}_i$, and can compute

$$\mathbf{X}_i = \mathbf{V}_2 \mathbf{A}_i \mathbf{V}_2^H + \mathbf{v}_1 \mathbf{B}_i \mathbf{V}_2^H + \mathbf{V}_2 \mathbf{B}_i^H \mathbf{v}_1^H \tag{39}$$

$$\mathbf{Y}_i = \mathbf{R}_i^{-1} s\mathbf{h}_e \mathbf{h}_e^H (\mathbf{I}_n + \mathbf{R}_i s\mathbf{h}_e \mathbf{h}_e^H)^{-1} \mathbf{R}_i^{-1} \tag{40}$$

$$\mathbf{Q}_i^{(2)}(\hat{\mathbf{p}}, s) = \mathbf{R}_i^{-1} - \check{\mathbf{h}}_r^H \check{\mathbf{h}}_r - \mathbf{X}_i - \mathbf{Y}_i. \tag{41}$$

The covariance matrix $\mathbf{Q}_i^{(2)}(\hat{\mathbf{p}}, s)$ as computed in (41) has rank one, but it is not guaranteed that $\mathbf{Q}_i^{(2)}(\hat{\mathbf{p}}, s)$ will satisfy all power constraints. To satisfy all power constraints, the dual variables in (31) have to be updated such that duality gaps between them and primal variables, i.e., power constraints, approach to zero. Since the optimization problem to find the optimal transmit strategy with sum power constraint only can be done using closed-form solution in Theorem 4, in the following, it is only interesting for us to show how the dual variables are updated in two remaining power constraint settings: per-antenna power constraints only and joint sum and per-antenna power constraints.

A. Per-antenna Power Constraints Only

For the per-antenna power constraints only problem, i.e., $P_{tot} > \sum_{k=1}^{N_t} \hat{P}_k$, we have the dual variable of the sum power constraint $\mu = 0$ while the dual variable of per-antenna power constraints $\mathbf{D} \succ 0$. This implies that at the i-th iteration we have $\mathbf{R}_i = \mathbf{D}_i$, so that the optimal transmit strategy of (29) with per-antenna power constraints only can be written as

$$\mathbf{Q}^{(2)}_{PAPC,i}(s) = \mathbf{D}_i^{-1} - \check{\mathbf{h}}_r^H \check{\mathbf{h}}_r - \mathbf{X}_{PAPC,i} - \mathbf{Y}_{PAPC,i}. \quad (42)$$

However, for $\mathbf{Q}^{(2)}_{PAPC,i}(s)$ as computed in (42), it is not guaranteed that $\mathrm{diag}\{\mathbf{Q}^{(2)}_{PAPC,i}(s)\} = \hat{\mathbf{P}}$. Therefore, the dual variable has to be updated to $\mathbf{D}_{i+1}$ as follows

$$\mathbf{D}_{i+1}^{-1} = \mathbf{D}_i^{-1} + \hat{\mathbf{P}} - \mathrm{diag}(\mathbf{Q}^{(2)}_{PAPC,i}(s)). \quad (43)$$

The algorithm stops when $\mathrm{diag}\{\mathbf{Q}^{(2)}_{PAPC,i}(s)\}$ is close to $\hat{\mathbf{P}}$ within an acceptable tolerance, i.e., the duality gap is sufficiently small. For a given $\mathbf{D}$, the duality gap for the equivalent optimization problem with per-antenna power constraints only is defined as

$$\mathcal{G}(\mathbf{D}) = -\,\mathrm{tr}(\mathbf{D}(\mathbf{Q}^{(2)}_{PAPC}(s) - \hat{\mathbf{P}})). \quad (44)$$

Since (18) is convex and Slater's condition holds, it is guaranteed that the duality gap will converge to zero [29], i.e., the algorithm will converge to the optimum. The detail analysis of the duality gap is shown in Proposition 4, which ensures that algorithm converges.

B. Joint Sum and Per-antenna Power Constraints

From Proposition 3, we know that the optimal transmit strategy can be achieved when full power P_{tot} is used. This allows us to consider only transmit strategies which allocate full power P_{tot}. Furthermore, from Theorem 6, we know that if the solution of the sum power constraint only problem does not violate the per-antenna power constraints, then it is also the solution of the joint sum and per-antenna power constraints problem. Otherwise, the maximum power will be allocated to those antennas for which the sum power constraint only optimal solution violates the per-antenna power constraints. Therefore, for $\min_k(\hat{P}_k) \leq P_{tot} \leq \sum_{k=1}^{N_t} \hat{P}_k$, we have the dual variable of the sum power constraint $\mu > 0$ and the dual variable of per-antenna power constraints $\mathbf{D} \succeq 0$. Then, $\mathbf{Q}^{(2)}_{JSPC,i}(s)$ can be computed as

$$\mathbf{Q}^{(2)}_{SJPC,i}(s) = \mathbf{R}_i^{-1} - \check{\mathbf{h}}_r^H \check{\mathbf{h}}_r - \mathbf{X}_{SJPC,i} - \mathbf{Y}_{SJPC,i}. \quad (45)$$

The dual variables are updated as

$$\mu_{i+1}^{-1} = \mu_i^{-1} + P_{tot} - \mathrm{tr}(\mathbf{Q}^{(2)}_{JSPC,i}(s)), \quad (46)$$

$$\mathbf{D}_{i+1}^{-1} = \mathbf{D}_i^{-1} + \hat{\mathbf{P}} - \mathrm{diag}(\mathbf{Q}^{(2)}_{JSPC,i}(s)), \quad (47)$$

$$\mathbf{R}_{i+1}^{-1} = (\mu_{i+1}\mathbf{I} + \mathbf{D}_{i+1})^{-1}. \quad (48)$$

The algorithm stops when $\mathrm{tr}(\mathbf{Q}^{(2)}_{SJPC,i}(s)) = P_{tot}$ and the diagonal elements of $\mathbf{Q}^{(2)}_{JSPC,i}(s)$ are smaller or equal to $\hat{P}_k$ $\forall k$, i.e., $\hat{\mathbf{P}} - \mathrm{diag}\{\mathbf{Q}^{(2)}_{JSPC,i}(s)\} \succeq 0$.

The details of the algorithm to find the optimal transmit strategy with two different power constraint settings, per-antenna power constraints only and joint sum and per-antenna power constraints only, are summarized in Algorithm 1. Next, we analyse the convergence properties of the algorithm. The convergence of the algorithm is guaranteed due to the following proposition.

Algorithm 1: $\mathbf{Q}_{opt}(s) = \mathit{OptimalQ}(s, \mathbf{h}_r, \mathbf{h}_e, \hat{\mathbf{P}}, P_{tot})$

Initialize $i = 1$, $\epsilon > 0$, $\mu_1 > 0$, $\mathbf{D}_1 \succ 0$.
Compute $\mathbf{v}_1$, $\mathbf{V}_2$ by decomposing $\mathbf{h}_r^H$ as in (33).
`// Per-antenna Power Constraints Only`
if $P_{tot} > \sum_{k=1}^{N_t} \hat{P}_k$ **then**
  repeat
    Compute $\mathbf{B}_{PAPC,i}$, $\mathbf{A}_{PAPC,i}$, $\mathbf{X}_{PAPC,i}$, $\mathbf{Y}_{PAPC,i}$, and $\mathbf{Q}_{PAPC,i}(s)$ from (37), (36), (39), (40), and (42) with $\mathbf{R}_i = \mathbf{D}_i$
    Update $\mathbf{D}_{i+1}$ using (43)
    $i \leftarrow i + 1$.
  until $|\,\mathrm{tr}(\boldsymbol{D}_i(\boldsymbol{Q}_{PAPC,i}(s) - \hat{\boldsymbol{P}})| < \epsilon$;
  Return $\mathbf{Q}_{opt}(s) = \mathbf{Q}_{PAPC}(s)$
end if
`// Joint Sum and Per-antenna Power Constraints`
if $\min_k(\hat{P}_k) \leq P_{tot} \leq \sum_{k=1}^{N_t} \hat{P}_k$ **then**
  repeat
    Set $\mathbf{R}_i = \mathbf{D}_i + \mu_i \mathbf{I}$
    Compute $\mathbf{B}_{SJPC,i}$, $\mathbf{A}_{SJPC,i}$, $\mathbf{X}_{SJPC,i}$, $\mathbf{Y}_{SJPC,i}$, and $\mathbf{Q}_{SJPC,i}(s)$ from (37), (36), (39), (40), and (45)
    Update $\mathbf{R}_{i+1}$ using (46), (47) and (48)
    $i \leftarrow i + 1$.
  until $|\mu_i(\mathrm{tr}(\boldsymbol{Q}_{JSPC,i}(s)) - P_{tot})| < \epsilon$, $\mathrm{diag}(\boldsymbol{Q}_{JSPC,i}(s)) \leq \hat{\boldsymbol{P}}$;
  Return $\mathbf{Q}_{opt}(s) = \mathbf{Q}_{JSPC}(s)$
end if

Proposition 4 ([11]). *Let Π be a set of matrices with diagonal elements +1 or -1. There exists a $\pi \in \Pi$ and K such that $\boldsymbol{R}_i^{-1}\pi$ is decreasing in i for some π and for all iterations $k \geq K$, and*

(1) *if* $P_{tot} > \sum_{k=1}^{N_t} \hat{P}_k$ *then* $(\mathrm{diag}(\boldsymbol{Q}_k^{(2)}(\hat{\boldsymbol{p}}, s)) - \hat{\boldsymbol{P}})\pi \succeq 0$,
(2) *if* $\min_k(\hat{P}_k) \leq P_{tot} \leq \sum_{k=1}^{N_t} \hat{P}_k$ *then* $(\mathrm{tr}(\boldsymbol{Q}_k^{(2)}(\hat{\boldsymbol{p}}, s) - P_{tot})\pi \succeq 0$ *and* $(\mathrm{diag}(\boldsymbol{Q}_k^{(2)}(\hat{\boldsymbol{p}}, s)) - \hat{\boldsymbol{P}})\pi \preceq 0$.

From Proposition 4, we obtain that since $\mathbf{R}_i^{-1}\pi$ is decreasing in i, Algorithm 1 always converges to the optimum.

The trade-off between transmission rate and secrecy rate as denoted in (10) is then characterized. The curved section of the boundary of the rate region is parametrized by s, $0 \leq s \leq s_{\max}$. A specific rate pair on the boundary then can be found by performing a line search.

VI. Numerical Examples

In this section, illustrative numerical examples for the optimization problems with sum power constraint only and per-antenna power constraint only with two antennas at the transmitter, and one antenna at legitimate receiver and eavesdropper each are shown. We first provide a MISO wiretap channel with two transmit antennas. The complex channel coefficients corresponds to legitimate receiver and eavesdropper are given as $\mathbf{h}_r = [0.3737 + 0.8912i, 0.9795 + 1.2926i]^T$ and $\mathbf{h}_e = [0.4387 + 0.7655i, 0.3816 + 0.7952i]^T$. The powers on

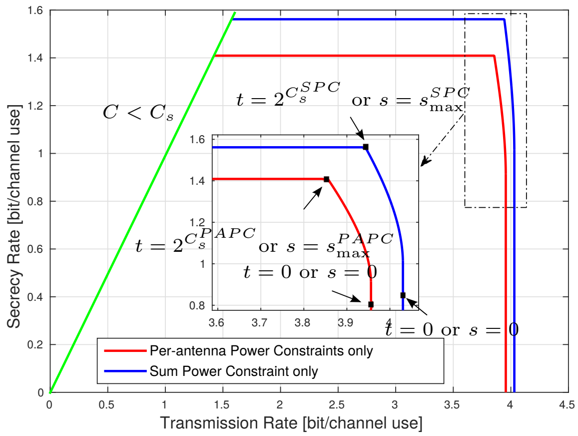

Fig. 3: The optimal regions between the transmission rates and the secrecy rate with sum power constraint only $P_{tot} = 15$ and per-antenna power constraints only $\hat{P}_1 = 5$ and $\hat{P}_2 = 10$

maximum transmit power on antennas are set as $\hat{P}_1 = 5$ and $\hat{P}_2 = 10$. The sum power constraint $P_{tot} = 15$.

Fig. 3 depicts optimal regions between the transmission rate and the secrecy rate of the wiretap channel with two different sets of power constraints: sum power constraint only and per-antenna power constraints only. The figure shows that the regions are fully characterized by the curved sections which can be obtained from the optimal solutions in Sections IV. A, IV. B, and V. A. It also shows the optimal trade-off between the transmission rate and the secrecy rate. For instance, we can see that the strategies that maximize the secrecy rates, $t = 2^{C_s^{SPC}}$ with $C_s^{SPC} = 1.5783$ for the case with sum power constraint only and $t = 2^{C_s^{PAPC}}$ with $C_s^{PAPC} = 1.4182$ for the case with per-antenna power constraints only. These respectively correspond to $s_{\max}^{SPC} = 0.3349$ and $s_{\max}^{PAPC} = 0.3742$.

VII. Conclusions

In this paper, we studied the trade-off between the transmission rate and the secrecy rate of the Gaussian MISO wiretap channels considering different power constraint settings. The optimization problem is non-convex, thus difficult. However, using equivalent convex reformulations allow the characterization of the boundary of the rate region on which then the optimal rate pair can be found by a simple line search. In particular, for the optimization problem with sum power constraint only, the optimal transmit strategy is characterized by a simple closed-form solution. For the optimization problem with per-antenna power constraints only, a relaxed optimization problem can be used to compute a strategy that is optimal if the transmit covariance matrix is rank one, which always hold for $N_t \leq 2$. Since the optimality for $N_t > 2$ cannot be established, an efficient optimization algorithm based on primal-dual approach and semi-definite programming framework has been provided. The extension to the joint sum and per-antenna power constraints is similar as in [19]. Lastly, we are convinced that studies on optimal transmit strategies including more advanced power constraint settings are highly relevant for future wireless networks, in particular for massive MIMO setups.

Appendix

A. Proof of Proposition 1

To prove the proposition, we need to show the necessity and sufficiency . For the necessary part, we need to show that for $R_s(\mathbf{Q}) > 0$, $\mathbf{h}_r\mathbf{h}_r^H - \mathbf{h}_e\mathbf{h}_e^H$ has at least one positive eigenvalue. The secrecy rate can be written as

$$\begin{aligned} R_s(\mathbf{Q}) &= \log(1 + \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r) - \log(1 + \mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e) \\ &= \log\left(1 + \frac{\operatorname{tr}\{(\mathbf{h}_r\mathbf{h}_r^H - \mathbf{h}_e\mathbf{h}_e^H)\mathbf{Q}\}}{1 + \mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e}\right) > 0. \end{aligned} \tag{49}$$

Since $1 + \mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e > 0$, it follows that $\operatorname{tr}(\mathbf{A}\mathbf{Q}) > 0$ with $\mathbf{A} = \mathbf{h}_r\mathbf{h}_r^H - \mathbf{h}_e\mathbf{h}_e^H$. Since $\operatorname{tr}(\mathbf{A}\mathbf{Q}) = \sum_{i=1}^{N_t} \lambda_i(\mathbf{Q})\mathbf{v}_i^H\mathbf{A}\mathbf{v}_i > 0$, there must exist an $\hat{i} \in \{1, \ldots, N_t\}$ such that $\mathbf{v}_{\hat{i}}^H\mathbf{A}\mathbf{v}_{\hat{i}} > 0$. Thus, we have

$$\lambda_{\max}(\mathbf{A}) = \max_{\|\mathbf{v}_i\|=1} \mathbf{v}_i^H\mathbf{A}\mathbf{v}_i \geq \mathbf{v}_{\hat{i}}^H\mathbf{A}\mathbf{v}_{\hat{i}} > 0. \tag{50}$$

For the sufficient part, we need to show that if $\mathbf{A} = \mathbf{h}_r\mathbf{h}_r^H - \mathbf{h}_e\mathbf{h}_e^H$ has a positive eigenvalue, then there exists $\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})$ such that $R_s(\mathbf{Q}) > 0$.

Since $\mathbf{A}$ has a positive eigenvalue, there exist a vector $\mathbf{v} : \|\mathbf{v}\| = 1$ such that $\mathbf{v}^H\mathbf{A}\mathbf{v} > 0$. This implies that we can construct $\mathbf{Q} = \xi\mathbf{v}\mathbf{v}^H$, $\xi > 0$, such that $\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})$ and $\operatorname{tr}(\mathbf{A}\mathbf{Q}) > 0$. Then we have

$$\begin{aligned} R_s(\mathbf{Q}) &= \log(1 + \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r) - \log(1 + \mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e) \\ &= \log\left(1 + \frac{\operatorname{tr}(\mathbf{A}\mathbf{Q})}{1 + \mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e}\right) > 0. \end{aligned}$$

■

B. Proof of Theorem 1

First, we show that for every $w \in [0, 1]$ there exists a $t \in [0, t_{\max}]$ with $t_{\max} = 2^{C_s(\hat{\mathbf{p}})}$ such that $\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t)$ is an optimal transmit strategy, i.e., $R_{\sum}(\hat{\mathbf{p}}, w) = R_{\sum}(\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t), w)$.

For a given $w \in [0, 1]$ we assume that there exist no $t \in [0, t_{\max}]$ such that $\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t)$ is optimal. This implies that there exist a $\mathbf{Q}^\star$ so that $R_{\sum}(\hat{\mathbf{p}}, w) = R_{\sum}(\mathbf{Q}^\star, w)$ and

$$R_{\sum}(\mathbf{Q}^\star, w) > R_{\sum}(\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t), w) \ \forall t \in [0, t_{\max}]. \tag{51}$$

Following Lemma 1 we know that for an optimal $\mathbf{Q}^\star$ the corresponding values $D_r^\star$ and $D_e^\star$ are computed as

$$D_r^\star = (1 + \mathbf{h}_r^H\mathbf{Q}^\star\mathbf{h}_r)^{-1}, \tag{52}$$
$$D_e^\star = (1 + \mathbf{h}_e^H\mathbf{Q}^\star\mathbf{h}_e)^{-1}. \tag{53}$$

Then for $w \in [0, 1]$ we have:

$$\begin{aligned} R_{\sum}(\mathbf{Q}^\star, w) &= D_r^\star(1 + \mathbf{h}_r^H\mathbf{Q}^\star\mathbf{h}_r) - \log(D_r^\star) - 1 \\ &\quad + w(-D_e^\star(1 + \mathbf{h}_e^H\mathbf{Q}^\star\mathbf{h}_e) + \log(D_e^\star) + 1) \\ &\leq \max_{\mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}})} D_r^\star(\mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r - w\frac{D_e^\star}{D_r^\star}\mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r) + D_r^\star \\ &\quad - \log(D_r^\star) - 1 - wD_e^\star - w\log(D_r^\star) - w. \end{aligned} \tag{54}$$

Following (15) and (16) we know that the optimal solution for the latter of (54) is computed as

$$\begin{aligned}\mathbf{Q}^\star &= \arg\max_{\mathbf{Q}\in\mathcal{S}(\hat{\mathbf{p}})} \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r - w\frac{D_e^\star}{D_r^\star}\mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r \\ &= \mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, w\frac{D_e^\star}{D_r^\star}) = \mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t), \end{aligned} \tag{55}$$

where $0 \le t = w\frac{D_e^\star}{D_r^\star} \le t_{\max}$. This implies that

$$R_{\sum}(\mathbf{Q}^\star, w) \le R_{\sum}(\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t), w), \tag{56}$$

for $t = w\frac{D_e^\star}{D_r^\star} \in [0, t_{\max}]$. However, this contradicts with (51). Thus, it follows that for every $w \in [0,1]$ there exists a $t \in [0, t_{\max}]$ such that $\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t)$ is an optimal transmit strategy.

Next, we show that for every $t \in [0, t_{\max}]$ there exists a $w \in [0,1]$ such that $\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t)$ is an optimal transmit strategy, i.e., $R_{\sum}(\hat{\mathbf{p}}, w) = R_{\sum}(\mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t), w)$. Suppose that $\mathbf{Q}^\star$ is an optimal solution of (16), then from Lemma 1 we know that for a given $\mathbf{Q}^\star$ the corresponding values $t \in [0, t_{\max}]$ is given by $t = w\frac{D_e^\star}{D_r^\star}$ with

$$D_r^\star = (1 + \mathbf{h}_r^H\mathbf{Q}^\star\mathbf{h}_r)^{-1}, \tag{57}$$

$$D_e^\star = (1 + \mathbf{h}_e^H\mathbf{Q}^\star\mathbf{h}_e)^{-1}, \tag{58}$$

and $\mathbf{Q}^\star$ must satisfy the KKT condition of (18) which is given as follows.

$$\begin{aligned} &\frac{\partial}{\partial\mathbf{Q}}\phi^{(1)}(\mathbf{Q}, t) \quad = \mathbf{D} + \mu\mathbf{I} - \mathbf{M} && (59)\\ &\mathrm{tr}(\mathbf{Q}) \le P_{tot} \qquad \mathbf{e}_k^T\mathbf{Q}\mathbf{e}_k \le \hat{P}_k, \forall k \in \mathcal{I} \qquad \mathbf{Q} \succeq 0 \\ &\mu \ge 0 \qquad \mathbf{D} \succeq 0 \qquad \mathbf{M} \succeq 0 \\ &\mu(\mathrm{tr}(\mathbf{Q}) - P_{tot}) = 0 \quad \mathrm{tr}(\mathbf{D}(\mathbf{Q} - \hat{\mathbf{P}})) = 0 \qquad \mathbf{M}\mathbf{Q} = 0. \end{aligned}$$

On the other hand, we have

$$\begin{aligned} &\frac{\partial}{\partial\mathbf{Q}}\phi^{(1)}(\mathbf{Q}, t)\Big|_{\mathbf{Q}=\mathbf{Q}^\star,\ t=t} \\ &= D_r^\star\mathbf{h}_r\mathbf{h}_r^H - wD_e^\star\mathbf{h}_e\mathbf{h}_e^H \\ &= \mathbf{h}_r(1 + \mathbf{h}_r^H\mathbf{Q}^\star\mathbf{h}_r)^{-1}\mathbf{h}_r^H - w\mathbf{h}_e(1 + \mathbf{h}_e^H\mathbf{Q}^\star\mathbf{h}_e)^{-1}\mathbf{h}_e^H \\ &= \frac{\partial}{\partial\mathbf{Q}}R_{\sum}(\mathbf{Q}, w)\Big|_{\mathbf{Q}=\mathbf{Q}^\star,\ w=t\frac{D_r^\star}{D_e^\star}}. \end{aligned} \tag{60}$$

Therefore, we can conclude from (59) and (60) that the optimal transmit strategy $\mathbf{Q}^\star$ is also an optimal solution of (11) with $0 \le w = t\frac{D_r^\star}{D_e^\star} \le 1$. ■

C. Proof of Theorem 3

From Theorem 1, we obtained that every optimal transmit strategy obtained from (18) is the same as the optimal transmit strategy obtained from (16) and is the optimal transmit strategy of (11). This implies that, for a given w, at the optimum we have $s = wD_e^\star$, $t = w\frac{D_e^\star}{D_r\star}$ and

$$\mathbf{Q}_{opt}^{(2)}(\hat{\mathbf{p}}, s) = \mathbf{Q}_{opt}^{(1)}(\hat{\mathbf{p}}, t) = \mathbf{Q}_{opt}(\hat{\mathbf{p}}, w). \tag{61}$$

Thus, it is sufficient to find the rank of the optimal transmit strategy by considering the following optimization problem

$$\max_{\mathbf{Q}}\ \phi^{(2)}(\mathbf{Q}, s),\ \mathrm{s.t.}\, \mathbf{Q} \in \mathcal{S}(\hat{\mathbf{p}}). \tag{62}$$

The Lagrangian for problem (62) is given by

$$\begin{aligned}\mathcal{L} = \log(1 + \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r) - s\mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e - \mathrm{tr}(\mathbf{D}(\mathbf{Q} - \hat{\mathbf{P}})) \\ - \mu(\mathrm{tr}(\mathbf{Q}) - P_{tot}) + \mathrm{tr}(\mathbf{M}\mathbf{Q}), \end{aligned} \tag{63}$$

where $\mathbf{D} = \mathrm{diag}\{\nu_k\}$ is a diagonal matrix of Lagrangian multiplier for the per-antenna power constraints, μ is the Lagrangian multiplier for the sum power constraint, $\mathbf{M}$ is the Lagrangian multiplier for the positive semi-definite constraint, and $\hat{\mathbf{P}} = \mathrm{diag}\{\hat{P}_k\}$, $\forall k \in \mathcal{I} = \{1, \ldots, N_t\}$, is a diagonal matrix of the per-antenna power constraints.

Taking the first derivative of the Lagrangian above and set equal to zero, we have

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\mathbf{Q}} &= \mathbf{h}_r(1 + \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r)^{-1}\mathbf{h}_r^H - s\mathbf{h}_e\mathbf{h}_e^H - \mathbf{D} - \mu\mathbf{I} + \mathbf{M} \\ &= \mathbf{h}_r(1 + \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r)^{-1}\mathbf{h}_r^H - \mathbf{K} + \mathbf{M} \stackrel{!}{=} 0, \end{aligned} \tag{64}$$

where $\mathbf{K} = s\mathbf{h}_e\mathbf{h}_e^H + \mathbf{D} + \mu\mathbf{I}$.

By using the slackness condition $\mathbf{M}\mathbf{Q} = 0$, we obtain $\mathbf{h}_r\mathbf{h}_r^H\mathbf{Q} = (1 + \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r)\mathbf{K}\mathbf{Q}$ by multiplying (64) with $\mathbf{Q}$ from the right. On the other hand, from the KKT condition of the convex optimization problem (62), we know that in the optimum either the sum power constraint is active or all per-antenna power constraints are active. i.e., we have either $\mu > 0$ or $\mathbf{D} \succ 0$. This implies that, in the optimum, $\mathbf{K}$ has full rank and

$$\mathrm{rank}(\mathbf{Q}_{opt}(\hat{\mathbf{p}}, w)) = \mathrm{rank}(\mathbf{h}_r\mathbf{h}_r^H\mathbf{Q}) \le \mathrm{rank}(\mathbf{h}_r\mathbf{h}_r^H) = 1.$$

Since $\mathrm{rank}(\mathbf{Q}_{opt}(\hat{\mathbf{p}}, w)) = 0$ is not optimal, the optimal rank of $\mathbf{Q}_{opt}(\hat{\mathbf{p}}, w))$ is one. This proves Theorem 3. ■

D. Proof of Theorem 4

By using singular value decomposition, for a given t, we have $\mathbf{h}_r\mathbf{h}_r^H - t\mathbf{h}_e\mathbf{h}_e^H = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H$. Let $\tilde{\mathbf{Q}} = \mathbf{V}^H\mathbf{Q}\mathbf{V}$, we obtain $\tilde{\mathbf{Q}} \succeq 0$. Then

$$\begin{aligned}\phi^{(1)}(\mathbf{Q}, t) = \mathrm{tr}\{(\mathbf{h}_r\mathbf{h}_r^H - t\mathbf{h}_e\mathbf{h}_e^H)\mathbf{Q}\} = \mathrm{tr}\{\mathbf{\Lambda}\tilde{\mathbf{Q}}\} \\ = \mathrm{tr}\{\mathbf{\Lambda}\,\mathrm{diag}(\tilde{\mathbf{Q}})\} \le \lambda_{\max}P_{tot} \end{aligned} \tag{65}$$

with $\lambda_{\max}$ is the largest entry in $\mathbf{\Lambda}$ and $\mathrm{tr}(\tilde{\mathbf{Q}}) = \mathrm{tr}(\mathbf{Q}) = P_{tot}$.

Equation (65) holds with equality if $\tilde{\mathbf{Q}}$ is diagonal and has a unique nonzero entry equal to P_{tot} corresponding to the largest entry of $\mathbf{\Lambda}$. This implies that $\mathbf{Q}$ and $\mathbf{h}_r\mathbf{h}_r^H - t\mathbf{h}_e\mathbf{h}_e^H$ share the same eigenvectors and $\mathbf{Q}$ has rank one. Therefore, we have $\mathbf{Q}_{SPC}^{(1)}(t) = P_{tot}\mathbf{v}\mathbf{v}^H$ where $\mathbf{v}$ is the eigenvector associated with the largest eigenvalue of $\mathbf{h}_r\mathbf{h}_r^H - t\mathbf{h}_e\mathbf{h}_e^H$ for a given t. This proves Theorem 4. ■

E. Proof of Proposition 2

Consider the optimization problem

$$\max_{\mathbf{Q}}\ \phi^{(1)}(\mathbf{Q}, t),\ \mathrm{s.t.}\ \mathbf{Q} \in \mathcal{S}_{PAPC}. \tag{66}$$

The Lagrangian for the problem (66) is given by

$$\mathcal{L} = \mathbf{h}_r^H\mathbf{Q}\mathbf{h}_r - t\mathbf{h}_e^H\mathbf{Q}\mathbf{h}_e - \mathrm{tr}(\mathbf{D}(\mathbf{Q} - \hat{\mathbf{P}})) + \mathrm{tr}(\mathbf{M}\mathbf{Q}), \tag{67}$$

where $\mathbf{D} = \mathrm{diag}\{\nu_k\}$ is a diagonal matrix of Lagrangian multiplier for the per-antenna power constraints, $\mathbf{M}$ is the

Lagrangian multiplier for the positive semi-definite constraint, and $\hat{\mathbf{P}} = \text{diag}\{\hat{P}_k\}$, $\forall k \in \mathcal{I} = \{1, \ldots, N_t\}$, is a diagonal matrix of the per-antenna power constraints. Based on the KKT conditions, we then obtain a set of optimality conditions as $\mathbf{h}_r\mathbf{h}_r^H - t\mathbf{h}_e\mathbf{h}_e^H = \mathbf{D} + \mathbf{M}, \mathbf{MQ} = 0, \mathbf{D} \succ 0$, and Hermitian $\mathbf{Q}, \mathbf{M} \succeq 0$.

Since $\mathbf{D} \succ 0$ has full-rank, at the optimum the power constraint must be met with equality, i.e., $q_{kk} = \hat{P}_k$, $\forall k \in \mathcal{I}$, otherwise we can always increase the power and get higher rate. This proves Proposition 2. ■

F. Proof of Theorem 5

Consider an optimization problem (23). The Lagrangian for problem (23) is given by

$$\mathcal{L} = \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - t\mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e - \sum_{k \neq l} \lambda_{kl}(|q_{kl}(t)|^2 - \hat{P}_k \hat{P}_l) - \sum_k \mu_k (q_{kk} - \hat{P}_k), \quad (68)$$

where λ_{kl} and μ_k are the Lagrange multipliers, and $k, l \in \mathcal{I}$. Taking the first derivative of (68) and set it equal to zero, we have

$$\frac{\partial \mathcal{L}}{\partial q_{kl}} = h_{rk}^* h_{rl} - t h_{ek}^* h_{el} - \lambda_{kl} q_{kl}(t) \stackrel{!}{=} 0, \quad (69)$$

or equivalently

$$q_{kl}(t) = \frac{h_{rk}^* h_{rl} - t h_{ek}^* h_{el}}{\lambda_{kl}}. \quad (70)$$

Similar to [17], the optimal value of q_{kl} in (70) is obtained when its constraint is satisfied with equality, i.e., $|q_{kl}(t)|^2 = \hat{P}_k \hat{P}_l$. By combining this condition with (70), we have the value of $q_{kl}(t)$ as in (24). ■

G. Proof of Proposition 3

Given function $\phi^{(1)}(\mathbf{Q}, t) = \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - t\mathbf{h}_e^H \mathbf{Q} \mathbf{h}_e : \mathbf{Q} \to \mathbb{R}_+$. From [28], it follows that for any positive semi-definite Hermitian matrices $\mathbf{Q}_1 \succeq \mathbf{Q}_2$, we have $\phi^{(1)}(\mathbf{Q}_1, t) \geq \phi^{(1)}(\mathbf{Q}_2, t)$. This implies that for (25) the optimal solution is achieved when the optimal transmit strategy allocates the maximal sum power P_{tot}, i.e., $\text{tr}(\mathbf{Q}_{JSPC}^{(1)}(t)) = P_{tot}$. ■

H. Proof of Theorem 6

The case with $\mathcal{P} = \emptyset$ is obvious, we focus to prove the remaining case. By applying [19, Lemma 2], we obtain that if any optimal power of sum power constraint only solution violated the per-antenna power constraints, it has to set equal to the maximal individual power, i.e., $|q_k|^2 = \mathbf{e}_k^T \mathbf{Q}_{JSPC}^{(1)}(t) \mathbf{e}_k = \hat{P}_k$, $\forall k \in \mathcal{P}$. Therefore, for the rest of this proof, we focus to show that the remaining optimal power can be computed by solving a reduce optimization problem.

Since the rank of matrix $\mathbf{A} = \mathbf{h}_r\mathbf{h}_r^H - t\mathbf{h}_e\mathbf{h}_e^H$ is at most two, we can express $\mathbf{A}$ as $\mathbf{A} = \lambda_1 \mathbf{u}_1 \mathbf{u}_1^H + \lambda_2 \mathbf{u}_2 \mathbf{u}_2^H$. Then we have

$$\begin{aligned}
\arg\max_{\mathbf{Q} \in \mathcal{S}_{JSPC}} \phi^{(1)}(\mathbf{Q}, t) &= \arg\max_{\mathbf{q}:\mathbf{q}\mathbf{q}^H \in \mathcal{S}_{JSPC}} \mathbf{q}^H (\lambda_1 \mathbf{u}_1 \mathbf{u}_1^H + \lambda_2 \mathbf{u}_2 \mathbf{u}_2^H) \mathbf{q} \\
&= \arg\max_{\mathbf{q}:\mathbf{q}\mathbf{q}^H \in \mathcal{S}_{JSPC}} \sum_{l=1}^{2} \lambda_i |\mathbf{q}^H \mathbf{u}_l|^2 \\
&= \arg\max_{\mathbf{q}:\mathbf{q}\mathbf{q}^H \in \mathcal{S}_{JSPC}} \sum_{l=1}^{2} \lambda_l |\sum_{k=1}^{n} q_k^* u_{lk}|^2 \\
&\stackrel{(a)}{=} \arg\max_{\mathbf{q}' \in \mathbb{Q}'} \sum_{l=1}^{2} \lambda_l |\sum_{k \in \mathcal{P}} \sqrt{\hat{P}_k} u_{lk} + \mathbf{q}'^H \mathbf{u}_l'|^2 \\
&= \arg\max_{\mathbf{q}' \in \mathbb{Q}'} \sum_{l=1}^{2} \lambda_l |\mathbf{q}'^H \mathbf{u}_l'|^2
\end{aligned} \quad (71)$$

where (a) follows from $|q_k|^2 = \mathbf{e}_k^T \mathbf{Q}_{JSPC}^{(1)}(t) \mathbf{e}_k = \hat{P}_k$ $\forall k \in \mathcal{P}$, $\mathcal{P}^c = \mathcal{I} \setminus \mathcal{P}$, $\mathbb{Q}' := \{\mathbf{q}' : \sum_{k \in \mathcal{P}^c} |q_k|^2 \leq P_{tot} - \sum_{k \in \mathbb{P}} \hat{P}_k, |q_k|^2 \leq \hat{P}_k, k \in \mathcal{P}\}$ and $\mathbf{u}_l' = [u_{lk}]_{k \in \mathcal{P}^c}$. This proves Theorem 6. ■

I. Proof of Lemma 2

From (31), by multiplying $\mathbf{h}_r^H \check{\mathbf{K}}$ on the left, $\mathbf{Q}\mathbf{h}_r$ on the right and using slackness condition $\mathbf{MQ} = 0$, we have

$$\mathbf{h}_r^H \check{\mathbf{K}} \mathbf{h}_r (1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r)^{-1} \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r = \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r. \quad (72)$$

This is equivalent to

$$(\mathbf{h}_r^H \check{\mathbf{K}} \mathbf{h}_r - \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r - 1)(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r)^{-1} \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r = 0 \quad (73)$$

Since $(1 + \mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r)^{-1}$ is scalar and the problem is interesting only when $\mathbf{h}_r^H \mathbf{Q} \mathbf{h}_r > 0$, then we have

$$\mathbf{h}_r^H \check{\mathbf{K}} \mathbf{h}_r - \mathbf{h}_r^H \mathbf{Q} \mathbf{H}_r = 1. \quad (74)$$

This proves Lemma 2. ■

J. Proof of Theorem 8

Steps of proof of this theorem are similar as in [11]. From (32), multiplying both sides of the equation with $\check{\mathbf{h}}_r^H$ on the left and $\check{\mathbf{h}}_r$ on the right, we have

$$\check{\mathbf{h}}_r^H \mathbf{h}_r^H (\check{\mathbf{K}} - \mathbf{Q}) \mathbf{h}_r \check{\mathbf{h}}_r = \check{\mathbf{h}}_r^H \check{\mathbf{h}}_r. \quad (75)$$

or equivalent to

$$\mathbf{v}_1 \mathbf{v}_1^H (\check{\mathbf{K}} - \mathbf{Q}) \mathbf{v}_1 \mathbf{v}_1^H = \mathbf{v}_1 \sigma^{-2} \mathbf{v}_1^H. \quad (76)$$

Next, by multiplying (76) with $\mathbf{v}_1^H$ on the left and $\mathbf{v}_1$ on the right, we get

$$\mathbf{v}_1^H (\check{\mathbf{K}} - \mathbf{Q}) \mathbf{v}_1 = \sigma^{-2}. \quad (77)$$

Since $\mathbf{V}_2^H \mathbf{v}_1 = 0$, from (77) we can deduce $\check{\mathbf{K}} - \mathbf{Q}$ as follows.

$$\begin{aligned}
\check{\mathbf{K}} - \mathbf{Q} &= [\mathbf{v}_1 \ \mathbf{V}_2] \begin{bmatrix} \sigma^{-2} & \mathbf{B} \\ \mathbf{B}^H & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{v}_1^H \\ \mathbf{V}_2^H \end{bmatrix} \\
&= \mathbf{v}_1 \sigma^{-2} \mathbf{v}_1^H + \mathbf{X},
\end{aligned} \quad (78)$$

where $\mathbf{X} = \mathbf{V}_2\mathbf{A}\mathbf{V}_2^H + \mathbf{v}_1\mathbf{B}\mathbf{V}_2^H + \mathbf{V}_2\mathbf{B}^H\mathbf{v}_1^H$. This implies that

$$\begin{aligned}\mathbf{Q} &= \check{\mathbf{K}} - \check{\mathbf{h}}_r^H\check{\mathbf{h}}_r - \mathbf{X} \\ &= (\mathbf{R} + s\mathbf{h}_e\mathbf{h}_e^H)^{-1} - \check{\mathbf{h}}_r^H\check{\mathbf{h}}_r - \mathbf{X} \\ &= \mathbf{R}^{-1} - \check{\mathbf{h}}_r^H\check{\mathbf{h}}_r - \mathbf{X} - \mathbf{Y},\end{aligned} \tag{79}$$

with $\mathbf{Y} = \mathbf{R}^{-1}s\mathbf{h}_e\mathbf{h}_e^H(\mathbf{I}_n + \mathbf{R}s\mathbf{h}_e\mathbf{h}_e^H)^{-1}\mathbf{R}^{-1}$ is obtain from inversion matrix lemma.

The remaining question is to find the values of $\mathbf{A}$ and $\mathbf{B}$ such that the rank condition in Theorem 3 is satisfied. By multiplying $\mathbf{V}_2^H$ on the left and $(\mathbf{R} + s\mathbf{h}_e\mathbf{h}_e^H)\mathbf{V}_2$ on the right of the (79), we have

$$\mathbf{A}=\big(\mathbf{I}_{n-1} - \mathbf{B}^H\mathbf{v}_1^H(\mathbf{R} + s\mathbf{h}_e\mathbf{h}_e^H)\mathbf{V}_2\big)\big(\mathbf{V}_2^H(\mathbf{R} + s\mathbf{h}_e\mathbf{h}_e^H)\mathbf{V}_2\big)^{-1}.$$

Similarly, by multiplying $\mathbf{V}_1^H$ on the left and $(\mathbf{R} + s\mathbf{h}_e\mathbf{h}_e^H)\mathbf{V}_2$ on the right of the (79), we have

$$\mathbf{B} = \left(\mathbf{v}_1^H\check{\mathbf{h}}_r^H\check{\mathbf{h}}_r(\mathbf{R} + s\mathbf{h}_e\mathbf{h}_e^H)\mathbf{V}_2\right)\left(\mathbf{V}_2^H(\mathbf{R} + s\mathbf{h}_e\mathbf{h}_e^H)\mathbf{V}_2\right)^{-1}.$$

This proves Theorem 8. ■

References

[1] A. Wyner, "The Wiretap Channel," *The Bell System Technical Journal*, vol. 54, pp. 1355–1387, 1975.

[2] I. Csiszár and J. Körner, "Broadcast Channel with Confidential Messages," *IEEE Trans. Inf. Theory*, vol. 24, no. 3, pp. 339–348, May 1978.

[3] S. Leung-Yan-Cheong and M. E. Hellman, "The Gaussian Wiretap Channel," *IEEE Trans. Inf. Theory*, vol. 24, no. 4, pp. 451–456, July 1978.

[4] A. Khisti and G. W. Wornell, "Secure Transmission With Multiple Antennas I: The MISOME Wiretap Channel," *IEEE Transactions on Information Theory*, vol. 56, no. 7, pp. 3088–3104, July 2010.

[5] A. Khisti and G. Wornell, "Secure Transmission with Multi-antenna II: The MIMOME Wiretap Channel," *Trans. Inf. Theory*, vol. 56, no. 11, pp. 5515–5532, 2010.

[6] F. Oggier and B. Hassibi, "The Secrecy Capacity of the MIMO Wiretap Channel," in *Proc. IEEE int. Symp. Inf. Theory*, 2008.

[7] J. Li and A. Petropulu, "Optimal Input Covariance for Achieving Secrecy Capacity in Gaussian MIMO Wiretap Channels," in *IEEE International Conference on Acoustics Speech and Signal Processing (ICASSP)*, March 2010.

[8] R. Liu and H. V. Poor, "Secrecy capacity region of a multiple-antenna gaussian broadcast channel with confidential messages," *IEEE Transactions on Information Theory*, vol. 55, no. 3, pp. 1235–1249, March 2009.

[9] Q. Shi, E. Song, and G. Chen, "Signaling Strategy Optimization for Gaussian MIMO Wiretap Channel," in *IEEE ICC*, 2012.

[10] Q. Li, M. Hong, H.-T. Wai, Y.-F. Liu, W.-K. Ma, and Z.-Q. Luo, "Transmit Solutions for MIMO Wiretap channels using Alternating Optimization," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 9, pp. 1714–1727, September 2013.

[11] M. Vu, "MIMO Capacity with Per-antenna Power Constraint," in *Global Communications Conference (GLOBECOM)*, 2011.

[12] Z. Pi, "Optimal MIMO Transmission with Per-antenna Power Constraint," in *Global Communications Conference (GLOBECOM)*, Dec 2012.

[13] M. Maamari, N. Devroye, and D. Tuninetti, "The Capacity of the Ergodic MISO Channel with Per-antenna Power Constraint and an Application to the Fading Cognitive Interference Channel," in *Proc. of International Symposium on Information Theory (ISIT)*, July 2014.

[14] D. Tuninetti, "On The Capacity of The AWGN MIMO Channel Under Per-Antenna Power Constraints," in *IEEE International Conference on Communications (ICC)*, June 2014.

[15] S. Shi, M. Schubert, and H. Boche, "Per-antenna Power Constrained Rate Optimization for Multiuser MIMO Systems," in *Proc. International ITG Workshop Smart Antennas.*, 2008.

[16] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraint," *IEEE Trans. on Signal Process.*, vol. 55, no. 6, pp. 2646–2660, June 2007.

[17] P. L. Cao, T. J. Oechtering, and M. Skoglund, "Optimal Transmission with Per-antenna Power Constraints for Multiantenna Bi-directional Broadcast Channel," in *The Ninth IEEE Sensor Array and Multichannel Signal Processing Workshop (SAM)*, July 2016.

[18] P. L. Cao, T. J. Oechtering, R. F. Schaefer, and M. Skoglund, "Optimal Transmission Rate for MISO Channels with Joint Sum and Per-antenna Power Constraints," in *IEEE International Conference on Communications (ICC)*, 2015.

[19] ——, "Optimal Transmit Strategy for MISO Channel with Joint Sum and Per-antenna Power Constraints," *IEEE Trans. on Signal Process.*, May 2016.

[20] S. Loyka, "The Capacity of Gaussian MISO Channels Under Total and Per-antenna Power Constraints." in *IEEE International Symposium on Information Theory (ISIT)*, Jul. 2016.

[21] P. L. Cao and T. J. Oechtering, "Optimal Transmit Strategy for MIMO Channels with Joint Sum and Per-antenna Power Constraints," in *International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, 2017.

[22] R. Liu, T. Liu, H. V. Poor, and S. Shamai, "New Results on Multiple-Input Multiple-Output Broadcast Channels With Confidential Messages," *IEEE Transactions on Information Theory*, vol. 59, no. 3, pp. 1346–1359, March 2013.

[23] E. Ekrem and S. Ulukus, "Capacity Region of Gaussian MIMO Broadcast Channels With Common and Confidential Messages," *IEEE Transactions on Information Theory*, vol. 58, no. 9, pp. 5669–5680, Sept 2012.

[24] ——, "Capacity-Equivocation Region of the Gaussian MIMO Wiretap Channel," *IEEE Transactions on Information Theory*, vol. 58, no. 9, pp. 5699–5710, Sept 2012.

[25] P. L. Cao and T. J. Oechtering, "Optimal Trade-off Between Transmission Rate and Secrecy Rate in Gaussian MISO Wiretap Channels," in *21th International ITG Workshop on Smart Antennas - WSA*, March 2017, pp. 1–8.

[26] J. Jose, N. Prasad, M. Khojastepour, and S. Rangarajan, "On Robust Weighted-sum Rate Maximixation in MIMO Interference Networks," in *IEEE International Conference on Communications (ICC)*, 2011.

[27] M. Vu, "MISO Capacity with Per-antenna Power Constraint," *IEEE Trans. on Communications*, vol. 59, no. 5, pp. 1268–1274, May 2011.

[28] E. A. Jorswieck and H. Boche, *Majorization and Matrix-monotone Functions in Wireless Communications*. Now Publishers Inc., 2007.

[29] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2009.