Transmit Beamforming for Single-user Large-Scale MISO Systems with Sub-connected Architecture and Power Constraints

Author 1, Author 2, and Author 3

***Abstract*—This letter considers optimal transmit beamforming for a sub-connected large-scale MISO system with RF chain and per-antenna power constraints. The system is configured such that each RF chain serves a group of antennas. For the hybrid scheme, necessary and sufficient conditions to design the optimal digital and analog precoders are provided. It is shown that, in the optimum, the optimal phase shift at each antenna has to match the channel coefficient and the phase of the digital precoder. In addition, an iterative algorithm is provided to find the optimal power allocation. We study the case where the power constraint on each RF chain is smaller than the sum of the corresponding per-antenna power constraints. Then, the optimal power is allocated based on two properties: Each RF chain uses full power and if the optimal power allocation of the unconstraint problem violates a per-antenna power constraint then it is optimal to allocate the maximal power for that antenna.**

***Index Terms*—Large-scale, massive MIMO, sub-connected architecture, hybrid beamforming, per-antenna power constraints.**

I. Introduction

In recent years, large-scale multiple-input multiple-output (massive MIMO) wireless communication has received much attention due to its envisioned application in 5G wireless systems. The motivation for massive MIMO is to use a very large number of antennas which enhance the spectral efficiency significantly [1] and higher frequencies (mmWave) which reduce the antennas' size and radiated energy [2], [3].

From a hardware perspective, we distinguish between two configurations of the large-scale antenna system, namely fully-connected and sub-connected [3]–[7]. In the fully-connected architecture [3]–[5], each antenna is connected to all RF chains through analog phase shifters and adders, i.e., each analog precoder output is a combination of all RF signals. One of the biggest drawbacks of this architecture is the requirement of a large number of RF adders and phase shifters which results in both high hardware cost and power consumption. Different from the fully-connected architecture, a sub-connected architecture has a reduced complexity where each RF chain is connected to a subset of transmit antennas only. Since this sub-connected architecture requires no adder and less phase shifters, it is less expensive to implement than the fully-connected one but results in less freedom for the signalling. Previous studies of transmit strategies for sub-connected architectures have been done in [6], [7]. However, these works assume a sum power constraint only. Since the RF chain has a physical limitation, it is reasonable to impose a power constraint on each RF chain. Furthermore, since each RF chain serves more than one antenna, power dividers are used to split the output powers to its connected antennas. Since it will be optimal to use the maximal power per RF chain and since the splitting ratio of the power divider has a physical limitation, it is reasonable to apply a power constraint on each antenna to limit the energy per-antenna.

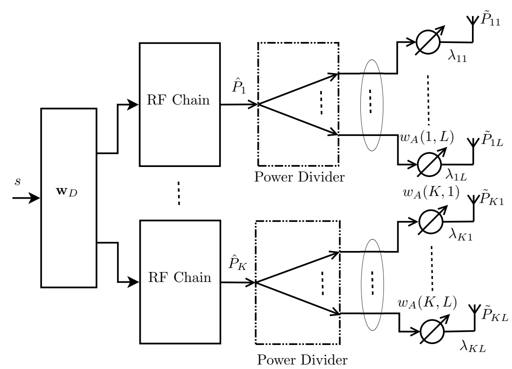

Fig. 1: Transmitter architecture for large-scale antenna system with sub-connected architecture and power constraints.

Previous works studied optimal transmit strategies for the MISO channel with per-antenna power constraints [8], [9] and joint sum and per-antenna power constraints [10]. However, the problem has so far not been studied for sub-connected architectures. In this letter, we focus on studying the optimal transmit strategy for a single-user large-scale MISO system with sub-connected architecture, per RF chain and per-antenna power constraints. Necessary and sufficient conditions to design the optimal digital and analog precoders for the hybrid beamforming are provided. The hybrid beamforming scheme is considered when the number of RF chains is strictly smaller than the number of antennas.

II. System Model

We consider a sub-conneted architecture of a single-user large-scale MISO system as depicted in Fig 1. The transmitter is equipped with K RF chains and M antennas such that each RF chain is connected to a group of L antennas, i.e., $M = KL$. RF chains are indexed by $k \in \mathcal{K} = \{1, \ldots, K\}$ and antennas connecting to each RF chain are indexed by $l \in \mathcal{L} = \{1, \ldots, L\}$. The transmit data $\mathbf{s} \in \mathbb{C}^{N\times 1}$, where N is the number of data streams and $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_N$, is precoded by applying a baseband processing (digital precoder) $\mathbf{W}_D \in \mathbb{C}^{K\times N}$ followed by adjustable power dividers and analog phase shifters. In the hardware setup in Fig 1, adjustable power dividers as in [11] are used to distribute the power from the RF chains to the corresponding transmit antennas. To control the power allocation for each antenna, a block diagonal matrix $\mathbf{\Lambda} \in \mathbb{R}_+^{M\times K}$ is introduced. It is defined as $\mathbf{\Lambda} = \mathrm{BlockDiag}\{\mathbf{\Lambda}_1, \ldots, \mathbf{\Lambda}_K\}$ with $\mathbf{\Lambda}_k = [\lambda_{k1}, \ldots, \lambda_{kL}]^T \in \mathbb{R}_+^{L\times 1}$ and $\lambda_{kl} \geq 0, \forall k, l$. Since λ_{kl}^2 denotes the power fraction transmitted from the l-th antenna, $\sum_{l=1}^{L}\lambda_{kl}^2=1$. Additionally, a diagonal matrix describing analog phase shifters (analog precoder) $\mathbf{W}_A\in\mathbb{C}^{M\times M}$ is used to adjust the phase for each individual antenna. The analog precoder can be written as $\mathbf{W}_A=\operatorname{diag}\{\mathbf{w}_A(1),\ldots,\mathbf{w}_A(K)\}$ with complex phase shift diagonal matrix $\mathbf{w}_A(k)=\operatorname{diag}\{w_A(k,1),\ldots,w_A(k,L)\}\in\mathbb{C}^{L\times L}$ and $|w_A(k,l)|^2=1$, $\forall k,l$.

The channel coefficient vector denoted as $\mathbf{h}=[\mathbf{h}_1^T,\ldots,\mathbf{h}_K^T]^T\in\mathbb{C}^{M\times 1}$ with $\mathbf{h}_k=[h_{k1},\ldots,h_{kL}]^T$, $k\in\mathcal{K}$, is known at both transmitter and receiver. Without loss of generality, we assume that $|h_{kl}|>0$, $\forall k\in\mathcal{K}$,$\forall l\in\mathcal{L}$. Then, the received signal can be written as

$$y=\mathbf{h}^H\mathbf{W}_A\mathbf{\Lambda}\mathbf{W}_D\mathbf{s}+z, \tag{1}$$

where $z\sim\mathcal{CN}(0,1)$ is additive white Gaussian noise.

We consider individual power constraints at each transmit antenna $\tilde{P}_{kl}$ and power constraints at each RF chain $\hat{P}_k$, $\forall k\in\mathcal{K}$, $\forall l\in\mathcal{L}$. If $\hat{P}_k\geq\sum_{l=1}^{L}\tilde{P}_{kl},\forall k\in\mathcal{K}$, then we face the *per-antenna power constraints only* problem. If $\hat{P}_k\leq\sum_{l=1}^{L}\tilde{P}_{kl}$ for a certain $k\in\mathcal{K}$, i.e., the transmit power on the k-th RF chain is more restricted than the total transmit power on antennas connecting to the k-th RF chain, we face the optimization problem where both sum and per-antenna power constraints can be active. In this work, we focus on the latter case only. Solutions to the other problems follow straight forwardly from this solution. We are interested in finding the optimal precoding matrices $\mathbf{W}_A$, $\mathbf{W}_D$ and the optimal power allocation matrix $\mathbf{\Lambda}$ that achieve the capacity of the point-to-point MISO channel (1). This is the standard problem of finding the optimal covariance matrix of the zero mean Gaussian distributed input, but here with a certain covariance matrix structure $\mathbf{W}_A\mathbf{\Lambda}\mathbf{W}_D\mathbf{W}_D^H\mathbf{\Lambda}\mathbf{W}_A^H$ reflecting the hardware design. Thus, the optimization problem is given as follows

$$\max_{\mathbf{W}_A,\mathbf{W}_D,\mathbf{\Lambda}}\quad \log(1+\mathbf{h}^H\mathbf{W}_A\mathbf{\Lambda}\mathbf{W}_D\mathbf{W}_D^H\mathbf{\Lambda}\mathbf{W}_A^H\mathbf{h}) \tag{2}$$

$$\text{s.t.}\quad \forall k,l:\mathbf{e}_{kl}^T\mathbf{W}_A\mathbf{\Lambda}\mathbf{W}_D\mathbf{W}_D^H\mathbf{\Lambda}\mathbf{W}_A^H\mathbf{e}_{kl}\leq\tilde{P}_{kl}, \tag{2a}$$

$$\forall k:\sum_{l=1}^{L}\mathbf{e}_{kl}^T\mathbf{W}_A\mathbf{\Lambda}\mathbf{W}_D\mathbf{W}_D^H\mathbf{\Lambda}\mathbf{W}_A^H\mathbf{e}_{kl}\leq\hat{P}_k, \tag{2b}$$

$$\forall k,l:|w_A(k,l)|^2=1, \tag{2c}$$

where (2a), (2b), and (2c) are the per-antenna, RF chain, and phase shifter constraints. $\mathbf{e}_{kl}\in\mathbb{R}^{KL\times 1}$ is a Cartesian unit vector with a one at $((k-1)L+l)$-th position and zeros elsewhere. Since $\log(1+\mathbf{h}^H\mathbf{W}_A\mathbf{\Lambda}\mathbf{W}_D\mathbf{W}_D^H\mathbf{\Lambda}\mathbf{W}_A^H\mathbf{h})$ is an increasing function in $\mathbf{h}^H\mathbf{W}_A\mathbf{\Lambda}\mathbf{W}_D\mathbf{W}_D^H\mathbf{\Lambda}\mathbf{W}_A^H\mathbf{h}$, we can equivalently focus on the optimization problem to find an optimal $\mathbf{W}_A\mathbf{\Lambda}\mathbf{W}_D$ for the objective function $|\mathbf{h}^H\mathbf{W}_A\mathbf{\Lambda}\mathbf{W}_D|^2$ instead of (2).

III. Transmit beamforming design

If $L=1$, then we have fully digital precoding where every antenna has its own RF chain. In this case $\check{P}_i=\min\{\hat{P}_i,\tilde{P}_i\}$, $\forall i\in\{1,\ldots,K=M\}$ gives the per-antenna power constraint. Then, the optimization problem reduces to

$$\max_{\mathbf{\Lambda},\mathbf{W}_D}\ \mathbf{h}^H\mathbf{\Lambda}\mathbf{W}_D\mathbf{W}_D^H\mathbf{\Lambda}\mathbf{h},\ \text{s.t.}\ \mathbf{e}_i^T\mathbf{Q}\mathbf{e}_i\leq\check{P}_i,\ \forall i\in\{1,\ldots,M\}. \tag{3}$$

Following [8], the optimal solution of the optimization problem (3) is rank one for $L=1$, i.e., $\mathbf{W}_D=\mathbf{w}_D$ with elements $w_i=\sqrt{\check{P}_i}\frac{h_i^*}{|h_i|},\forall i\in\{1,\ldots,M\}$ and $\mathbf{\Lambda}=\mathbf{I}_M$.

In the following, we study the hybrid beamforming for the case where the number of RF chains is strictly smaller than the number of antennas, i.e., $K<M$ and $L>1$. The digital precoder $\mathbf{W}_D$ is designed under the assumption that an analog precoder $\mathbf{W}_A$ and a power allocation matrix $\mathbf{\Lambda}$ are given. For a given analog precoding $\mathbf{W}_A$ and a power allocation matrix $\mathbf{\Lambda}$, an equivalent channel $\mathbf{g}$ can be formulated as $\mathbf{g}=\mathbf{\Lambda}\mathbf{W}_A^H\mathbf{h}$. Then the convex optimization problem to find the digital precoder can be written as

$$\max_{\mathbf{W}_D}\ \mathbf{g}^H\mathbf{W}_D\mathbf{W}_D^H\mathbf{g}\qquad \text{s.t.}\ (2a),(2b). \tag{4}$$

Following Proposition 2 in [10], we can conclude that the optimal digital precoder $\mathbf{W}_D$ also has rank one, i.e., beamforming is optimal. Therefore, it is sufficient to consider a digital precoder that can be denoted as $\mathbf{W}_D=\mathbf{w}_D\in\mathbb{C}^{K\times 1}$. Moreover, it means that it is optimal to have only one data stream, i.e., $N=1$, which we will assume in the following.

In hybrid precoding, the analog precoder controls the phase for each antenna. Since it is sufficient to consider a digital precoder of rank one, the phase of the digital precoder can be merged with the analog precoder or simply choosen to be equal to zero. Because of this we assume, without loss of generality, that $\mathbf{w}_D\in\mathbb{R}_+^{K\times 1}$ in the following.

Next, we will derive the optimal analog precoder, the characterization of the amplitude of the optimal digital precoder and optimal power allocation.

A. Analog precoder

By assuming that a digital precoder $\mathbf{w}_D\in\mathbb{R}_+^{K\times 1}$ and the power allocation $\mathbf{\Lambda}$ are given, we can obtain a necessary condition for the optimal analog precoder $\mathbf{W}_A^\star$ by solving the following optimization problem

$$\max_{\mathbf{W}_A}\ \mathbf{h}^H\mathbf{W}_A\mathbf{\Lambda}\mathbf{w}_D\mathbf{w}_D^H\mathbf{\Lambda}\mathbf{W}_A^H\mathbf{h}\qquad \text{s.t.}\ (2a),(2c). \tag{5}$$

Proposition 1. *Let $e^{i\angle h_{kl}}=\frac{h_{kl}}{|h_{kl}|}$ $\forall k,l$. Then the optimal analog precoder has elements given as*

$$w_A^\star(k,l)=e^{-i\angle h_{kl}},\ \forall k\in\mathcal{K},\forall l\in\mathcal{L}. \tag{6}$$

Proof. Given a power allocation $\mathbf{\Lambda}$ and a digital precoder $\mathbf{w}_D$, then we have

$$\begin{aligned}
&\max_{|w_A(k,l)|^2=1\forall k,l}|\mathbf{h}^H\mathbf{W}_A\mathbf{\Lambda}\mathbf{w}_D|^2\\
&=\max_{|w_A(k,l)|^2=1\forall k,l}\left|\sum_{k=1}^{K}\sum_{l=1}^{L}|h_{kl}|e^{i\angle h_{kl}}w_A(k,l)\lambda_{kl}w_D(k)\right|^2\\
&\leq\left(\sum_{k=1}^{K}\sum_{l=1}^{L}|h_{kl}|\lambda_{kl}\sqrt{P_k}\right)^2.
\end{aligned}\tag{7}$$

The upper bound (7) is achieved if $w_A(k,l)=e^{-i\angle h_{kl}}$, $\forall k\in\mathcal{K},\forall l\in\mathcal{L}$. This proves Theorem 1. □

Proposition 1 shows that it is optimal to match the phase at each antenna to the channel coefficient. Therefore, it is sufficient to design the optimal analog precoder by aligning phases of phase shifters to the channel coefficient such that the signal coherently adds up at the receiver.

In the next part, we derive the amplitude of the optimal digital precoder coefficient and the optimal power allocation matrix $\mathbf{\Lambda}$ under the assumption $\hat{P}_k \leq \sum_{l=1}^{L} \tilde{P}_{kl}$, $\forall k$.

B. Power allocation

The following proposition shows that it is optimal for the digital precoder to transmit with maximum power on all RF chains.

Proposition 2. *For the case where $\hat{P}_k \leq \sum_{l=1}^{L} \tilde{P}_{kl}$ for all k, the optimal solution of* (4) *allocates full power on all RF chains, i.e., $\sum_{l=1}^{L} P_{kl}^\star = \hat{P}_k$, $\forall k$.*

Proof. Let $\mathbf{q} = \mathbf{W}_A \mathbf{\Lambda} \mathbf{w}_D$, $\mathcal{Q} := \{\mathbf{q} : \mathbf{e}_{kl}^T \mathbf{q}\mathbf{q}^H \mathbf{e}_{kl} \leq \tilde{P}_{kl}, \sum_{l=1}^{L} \mathbf{e}_{kl}^T \mathbf{q}\mathbf{q}^H \mathbf{e}_{kl} \leq \hat{P}_k \forall k, l\}$. Suppose there exists an *optimal* $\mathbf{q}^\star$ such that there exists a $\bar{k} \in \mathcal{K}$ for which $\mathbf{e}_{\bar{k}l}^T \mathbf{q}^\star \mathbf{q}^{\star H} \mathbf{e}_{\bar{k}l} = P_{\bar{k}l}^\star$ and $\sum_{l=1}^{L} P_{\bar{k}l}^\star < \hat{P}_{\bar{k}}$, then the maximum value of (4) can be calculated as

$$
\begin{aligned}
f^\star &= \max_{\mathbf{q}\in\mathcal{Q}} |\mathbf{h}^H \mathbf{q}|^2 = \max_{\mathbf{q}\in\mathcal{Q}} \left| \sum_{\substack{k=1,\\ k\neq\bar{k}}}^{K} \sum_{l=1}^{L} h_{kl} q_{kl} + \sum_{l=1}^{L} h_{\bar{k}l} q_{\bar{k}l} \right|^2 \\
&= \left(\sum_{\substack{k=1,\\ k\neq\bar{k}}}^{K} \sum_{l=1}^{L} |h_{kl}| \sqrt{P_{kl}^\star} + \sum_{l=1}^{L} |h_{\bar{k}l}| \sqrt{P_{\bar{k}l}^\star} \right)^2 \\
&= \left(\sum_{\substack{k=1,\\ k\neq\bar{k}}}^{K} f_k^\star + f_{\bar{k}}^\star \right)^2 .
\end{aligned} \tag{8}
$$

Since $\sum_{l=1}^{L} P_{\bar{k}l}^\star = P_{\bar{k}}^\star < \hat{P}_{\bar{k}} \leq \sum_{l=1}^{L} \tilde{P}_{\bar{k}l}$, there exists a j and a $P_{\bar{k}j}$ with $P_{\bar{k}j}^\star < P_{\bar{k}j} \leq \tilde{P}_{\bar{k}j}$ and $\hat{P}_{\bar{k}} - P_{\bar{k}j} \geq \sum_{\substack{l=1\\ l\neq j}}^{L} P_{\bar{k}l}^\star$, so that $f_{\bar{k}}' = \sum_{\substack{l=1\\ l\neq j}}^{L} |h_{\bar{k}l}| \sqrt{P_{\bar{k}l}^\star} + |h_{\bar{k}j}| \sqrt{P_{\bar{k}j}} > f_{\bar{k}}^\star$. It follows that $f' = (\sum_{\substack{k=1,\\ k\neq\bar{k}}}^{K} f_k^\star + f_{\bar{k}}')^2 > (\sum_{\substack{k=1,\\ k\neq\bar{k}}}^{K} f_k^\star + f_{\bar{k}}^\star)^2 = f^\star$. This contradicts with the optimality of (8). This implies that the optimal solution of (4) must meet all RF chain power constraints with equality, i.e., $\sum_{l=1}^{L} P_{kl}^\star = P_k^\star = \hat{P}_k \; \forall k$. □

Proposition 2 implies that it is sufficient for the optimization to consider only transmit strategies which allocate full power on all RF chains, i.e., the RF chain power constraints are always active. Accordingly, $w_D(k) = \sqrt{\hat{P}_k}$ is optimal.

Next, the optimal power allocation $\mathbf{\Lambda}^\star \in \mathbb{R}_+^{M\times K}$ is designed under the assumption that the optimal digital and analog precoders are given, i.e., $w_D^\star(k) = \sqrt{\hat{P}_k} \; \forall k$ and $|w_A^\star(k,l)|^2 = 1$. Let $P_{kl} = |\lambda_{kl} w_A^\star(k,l) w_D^\star(k)|^2 = \hat{P}_k \lambda_{kl}^2 \; \forall k, l$. Then we have $\lambda_{kl} = \sqrt{\frac{P_{kl}}{\hat{P}_k}} \; \forall k, l$, or equivalently

$$
\mathbf{\Lambda}_k = \frac{1}{\sqrt{\hat{P}_k}} \left[\sqrt{P_{k1}}, \ldots, \sqrt{P_{kL}} \right]^T \; \forall k. \tag{9}
$$

From the proof of Proposition 2 we can easily see that the power allocation for one RF chain is independent from the allocation at all other RF chains. Thus, the problem reduces to the problem to find the optimal power allocation for one RF chain. For a given $k \in \mathcal{K}$, the optimal allocated powers $P_{kl}^\star$, $\forall l$ can be obtained by solving the following problem

$$
\begin{aligned}
\max_{P_{kl}, \forall l} \quad & \sum_{l=1}^{L} |h_{kl}| \sqrt{P_{kl}} \\
\text{s.t.} \quad & \forall l : P_{kl} \leq \tilde{P}_{kl}, \sum_{l=1}^{L} P_{kl} \leq \hat{P}_k.
\end{aligned} \tag{10}
$$

This problem is exactly the same as the optimization problem to find the optimal transmit strategy for MISO channels with joint sum and per-antenna power constraints [10]. Therefore, the solution of (10) can be approach by utilizing the the solutions of the sum power constraint only and per-antenna power constraints only problems as done in [10]. In accordance to that we need the optimal power allocation on a group of antennas connecting to one RF chain $k \in \mathcal{K}$ without per-antenna power constraints, which is given by a waterfilling solution [12]:

$$
P_{kl}^{WF} = \left(\frac{1}{\omega_k} - \frac{1}{|h_{kl}|^2} \right)^+, \forall l \tag{11}
$$

where ω_k satisfies $\sum_{l=1}^{L} \left(\frac{1}{\omega_k} - \frac{1}{|h_{kl}|^2} \right)^+ = \hat{P}_k$, for any $k \in \mathcal{K}$.

The optimal powers P_{kl}^{WF}, however, may violate the per-antenna power constraints $\tilde{P}_{kl}$ for some k, l. In this case, it is optimal to set those equal to the per-antenna power constraints $\tilde{P}_{kl}$. The remaining power allocations can then be obtained by solving a reduced optimization problem with a smaller total RF chain power, i.e., $\hat{P}_k - \sum_{l\in\{l\in\mathcal{L}: P_{kl}^{WF} \geq \tilde{P}_{kl}\}} \tilde{P}_{kl}$. The justification of this approach is in [10] and reformulated for the considered problem here in the following corollary.

Corollary 1 (Theorem 1 in [10])**.** *For a given $k \in \mathcal{K}$, let $\mathcal{P}_k := \{l \in \mathcal{L} : P_{kl}^{WF} \geq \tilde{P}_{kl}\}$ and $\mathcal{P} = \bigcup_{k=1}^{K} \mathcal{P}_k$. If $\mathcal{P} = \emptyset$ then $P_{kl}^\star = P_{kl}^{WF} \; \forall l$, else $P_{kl}^\star = \tilde{P}_{kl} \; \forall l \in \mathcal{P}_k$, and the remaining optimal powers can be computed by solving the reduced optimization problem*

$$
\begin{aligned}
\max_{P_{kl} \forall l \in \mathcal{P}_k^c} \quad & \sum_{l\in\mathcal{P}_k^c} |h_{kl}| \sqrt{P_{kl}} \\
\text{s.t.} \; \forall l \in \mathcal{P}_k^c : P_{kl} \leq \tilde{P}_{kl}, & \sum_{l\in\mathcal{P}_k^c} P_{kl} \leq \hat{P}_k - \sum_{l\in\mathcal{P}_k} \tilde{P}_{kl},
\end{aligned} \tag{12}
$$

where $\mathcal{P}_k^c = \mathcal{L} \setminus \mathcal{P}_k$.

If the waterfilling solution of the reduced optimization problem again violates a per-antenna power constraint, then Corollary 1 has to be applied again until the waterfilling solution of the reduced optimization problem does not violate any per-antenna power constraints. We have summarized the approach above to compute the optimal power allocation matrix $\mathbf{\Lambda}^\star$ in Algorithm 1 on the next page.

Algorithm 1: Optimal power allocation matrix

```
1  Compute optimal power allocation P_kl^{WF} using (11)
2  Denote 𝒫_k := {l ∈ ℒ : P_kl^{WF} ≥ P̃_kl} ∀k
3  Denote 𝒫 := ⋃_{k=1}^{K} 𝒫_k
4  if 𝒫 = ∅ then
5  |   P*_kl ← P_kl^{WF} ∀k, l
6  |   Go to 16
7  else
8  |   for k ∈ 𝒦 do
9  |   |   for l ∈ 𝒫_k do
10 |   |   |   P*_kl ← P̃_kl
11 |   |   end for
12 |   |   ℒ ← ℒ \ 𝒫_k, P̂_k ← P̂_k − Σ_{l∈𝒫_k} P̃_kl
13 |   end for
14 end if
15 Return to 1.
16 Form Λ*_k (as in (9)) and Λ* with optimal power P*_kl.
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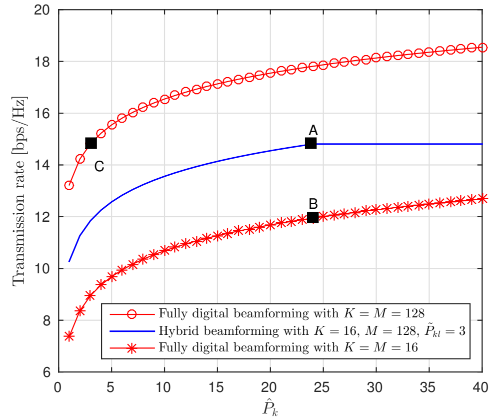

Fig. 2: Transmission rate of the large-scale antenna system with different RF chains and antennas configurations.

IV. Numerical results

In this section, we consider a large-scale MISO system with different transmit antenna configurations. We first evaluate the transmission rate for the case that the number of RF chains and the number of antennas are the same, i.e., fully digital beamforming is used. The system is considered with settings of 16 and 128 pairs of RF chains and transmit antennas respectively. In these settings, the per-antenna power constraints and the RF chain power constraints are the same. Next, we investigate a hybrid beamforming scheme that is configured with $M = 128$ transmit antennas and $K = 16$ RF chains. Each RF chain is designed to serve a group of $L = 8$ antennas. The per-antenna power constraint on each antenna is $\tilde{P}_{kl} = 3$. Curves in Fig. 2 are plotted by gradually increasing $\hat{P}_k$ from $\hat{P}_k = 1$ to $\hat{P}_k = 40$.

We can see from the figure that for the hybrid beamforming, if a RF chain power constraint is more restrictive than the sum of all individual powers of the group of antennas connected to that RF chain, i.e., $\hat{P}_k \leq \sum_{l=1}^{8} \tilde{P}_{kl} = 24$ (operating point A), then it is optimal to transmit with the maximal per RF chain power $\hat{P}_k$. After this value the RF power constraint is never active and it is optimal to transmit with the maximal individual power $\tilde{P}_{kl} = 3$ on all antennas.

Next, we compare operating point A of the hybrid beamforming scheme with operating points B and C of fully-digital beamforming schemes that both allocate the same total transmit power. We observe that: (i) By using the same number of RF chains while increasing the number of antennas, we can obtain a significantly higher transmission rate. (ii) With a smaller number of RF chains and the same number of antennas, we can achieve the same transmission rate as the one with fully digital beamforming.

V. Conclusions

In this letter, we provide necessary and sufficient conditions to design the hybrid beamforming for a single-user large-scale MISO system with a sub-connected architecture, RF chain and per-antenna power constraints. We further showed that phase matching is optimal and provided an algorithm to compute the optimal power allocation in closed-form using [10]. The numerical results illustrate that we obtain a significant higher capacity by increasing number of RF chains. However, this solution requires higher hardware cost and power consumption.

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