The MHD of Compact Structures in the Solar Corona

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Abstract

Simplified MHD Models for the equilibrium and motion of localised structures in the solar corona are considered. Attention is focussed on models for the support and dynamics of filaments and quiescent prominences. Mirror currents due to line tying in the photosphere can make an important contribution to the magnetic forces. Comparisons are made with work on the melon seed effect for a magnetised plasma blob (plasmoid). An important question is whether there is a net current through the structure (e.g. simple current filaments) or alternatively the currents are isolated (e.g. plasmoids). A filament, even with no net axial current, will be subject to a force, analogous to that for a plasmoid, in a nonuniform ambient external field.

Keywords: prominences, filaments, plasmoids, currents

1 Introduction

The structure of the solar corona is both spatially complex and highly dynamic. A comprehensive theoretical treatment requires the use of three-dimensional magnetohydrodynamic codes including the thermodynamics of the plasma over a wide range of densities and temperatures. Such codes are only relatively recently becoming available. Appropriate simplified treatments of the MHD of the corona therefore play a vital rôle in understanding specific phenomena.

The MHD description of coronal structures can be simplified if special arguments can be made to locate and describe the current system. A particular case is that of filaments and prominences which appear as clearly localised structures with enhanced density and relatively low temperature. The cool dense material is thermally isolated by magnetic fields, which also provide support against gravity. In contrast the low general density of the corona is such that magnetic fields should to a good first approximation be force-free. Moreover the low density implies a high Alfvén velocity: changes within compact dense structures are rapidly transmitted through the corona leading to changes lower down in the solar atmosphere i.e. in the photosphere. From this point of view the primary currents are those within the compact object, together with photospheric currents and possible current sheets formed in order to ensure magnetic flux conservation during the evolution and in the dynamical phases of the structures.

In the case of elongated filamentary structures, the first question is the existence of a net current. This together with an assumed background magnetic field and mirror currents due to line tying in the photosphere gives forces to balance gravity in the case of equilibrium e.g. quiescent prominences, or to drive mass motion (as in disparitions brusques). If the filament currents are believed to be induced by localised twisting of the magnetic flux, a coaxial current system should appear so that there is no change in the external field. There is then no net current and the magnetic forces will depend on the internal distribution of the currents.

Plasmoids, plasma blobs, formed by the break up of prominences and filaments, and in connection with solar flares, are subject to the melon seed effect. They are accelerated by the force on the equivalent magnetic dipole of the internal currents in a nonuniform external magnetic field. An analogous force for a line dipole should even apply for filaments, and is of particular interest if they carry no net axial current.

2 Prominences

Quiescent prominences are believed to be closely associated with current sheets. In the model of Kippenhahn and Schlüter [Kippenhahn and Schlüter 1957] the dense prominence material is supported against gravity in the horizontal field at the top of magnetic arches in the corona. Due to the low scale height the cool dense material forms a thin sheet with a current giving a vertical force to balance gravitation. In an active region, magnetic arches can be forced open by plasma ejection. The resulting current sheet is then a favourable location for prominence formation [Kuperus and Tandberg-Hansen 1967]. A closely related scenario is the formation of quiescent prominences in current sheets, far from active regions, where oppositely directed magnetic flux regions come in contact [Kuperus and Raadu 1974]. Here it should be noted that the direction of the current in these cases is opposite to that in the Kippenhahn–Schlüter model. Tearing modes modified by thermal instabilities can lead to the formation of current filaments within the sheets [Kuperus and Tandberg-Hansen 1967]. The equilibrium and dynamics of these can then be treated by identifying the magnetic forces associated with the large scale structure. The formation and movement of a current filament modifies the large scale magnetic field.

At the photosphere, where the density is much higher than in the corona, the Alfvén velocity is low, so that changes in the magnetic field can only slowly penetrate. The footpoints of coronal field lines are therefore effectively fixed in the photosphere: the line tying condition, that the vertical field component is unchanged, applies. Applying this condition when a current filament is introduced in the corona implies that surface currents are induced at the photosphere. The line tying condition is satisfied if these are set to be equivalent to an equal and opposite mirror image of the current filament. The resulting magnetic field leads to an upward Lorentz force on the filament giving support against gravity. For a total current J this supporting mirror force F_m is [Kuperus and Raadu 1974],

$$F_m = \frac{\mu_0 J^2}{4\pi h} = \frac{1}{h}\left[\frac{B_\phi^2}{\mu_0}\right]\pi r_f^2 \tag{1}$$

where B_ϕ is the azimuthal surface field of the filament with radius r_f at a height h above the photosphere.

In further developments of the filament model [van Tend and Kuperus 1978, van Tend 1979, Kuperus and van Tend 1981] the stability of filaments, the onset of coronal transients and the relation of erupting filaments to the triggering of solar flares has been investigated. The consequences of the time delay between a moving filament and the induced photospheric currents have been taken into consideration in a further refinement of filament models for prominence dynamics [Schutgens 1997a, Schutgens 1997b, van den Oord et al. 1998].

For a moving filament the high electrical conductivity of the corona imposes conditions of flux conservation. In particular the flux between the filament and the photosphere should, as a first approximation, be conserved. This leads to the formation of current sheets as in filament eruption models of solar flares [Kaastra 1985, Martens and Kuin 1989]. The presence of current sheets changes the magnetic force on the filament, and their dynamical evolution can be related to the development phases of two-ribbon flares (ibid).

3 Plasmoid Dynamics

A plasmoid is a self contained structure containing plasma, and possibly an internal current system giving a local magnetic field. Such structures have been associated with the break up of filaments and quiescent prominences, as well as more energetic phenomena such as solar flares. Since there is no net current through the plasmoid, forces exerted by the external field are a result of the local distortion. This is the basis of the melon seed effect. The external field is excluded from the plasmoid due to its high electrical conductivity. The nonuniform distribution of magnetic pressure over the surface of the plasmoid then leads to a net force directed away from the stronger magnetic field region. In a simple model the plasmoid currents provide a dipole field, which in the case of a spherical blob can exactly cancel the radial component of a constant external field at the surface. In a nonuniform external field the net force is then given by the force on the dipole. For a non-magnetised blob the external magnetic pressure is balanced by the internal gas pressure, so that the acceleration is essentially driven by thermal energy. An alternative possibility is that there is an internal force-free magnetic field, which can balance the ambient field at the surface. In this case it is the magnetic energy that drives the acceleration, and highly supersonic velocities are possible [Raadu et al 1987]. The external Alfvén velocity will set an upper limit since the formations of shocks with dissipation can be expected when this velocity is approached.

The force on the distributed current $\mathbf{j}$ within the plasmoid can be found from the energy U_B of the current system in the external field $\mathbf{B}$. If it is assumed that $\mathbf{B}$ is nearly uniform over the plasmoid the vector potential A is approximately $(\mathbf{B} \times \mathbf{r})/2$. The energy is then given by [Bleaney and Bleaney 1965],

$$U_B \;=\; -\int (\mathbf{A}.\mathbf{j})\, dV \;=\; -\frac{1}{2}\mathbf{B}.\int (\mathbf{r} \times \mathbf{j})\, dV \;=\; -\mathbf{B}.\boldsymbol{\mu} \tag{2}$$

whereby the equivalent magnetic dipole moment $\boldsymbol{\mu}$ of the current distribution is (ibid),

$$\boldsymbol{\mu} \;=\; \frac{1}{2}\int (\mathbf{r}\times\mathbf{j})\,dV \tag{3}$$

Considering the change in energy when the direction of the moment $\boldsymbol{\mu}$ is changed then gives the torque on the dipole. Strictly speaking the derivations of Eq. (2) and Eq. (3) assume that the magnetic field $\mathbf{B}$ is uniform. However it can be argued that the energy dependence on a magnetic field $\mathbf{B}$ that is changing in time should be the same as that resulting from the motion of the dipole through a nonuniform field. Equating the work done on the plasmoid as it moves through the field to the change in the magnetic energy U_B the force on the dipole can be found to be,

$$\mathbf{F}_B \;=\; -grad\,(-\mathbf{B}.\boldsymbol{\mu}) \;=\; \boldsymbol{\mu}.\boldsymbol{\nabla}\mathbf{B} \tag{4}$$

In the case of a spherical plasmoid with radius r_p in a nearly uniform magnetic field $\mathbf{B}$ the magnetic moment $\boldsymbol{\mu}$ must be antiparallel to $\mathbf{B}$ in order for the normal component at the surface to be zero. Requiring that the net radial field is zero determines $\boldsymbol{\mu}$, the magnetic dipole moment,

$$\mathbf{B} \;=\; -2\frac{\mu_0}{4\pi r_p^3}\boldsymbol{\mu} \tag{5}$$

The force on the plasmoid is then found to be,

$$\mathbf{F}_p \;=\; -f_3 V_p \boldsymbol{\nabla}\left[\frac{B^2}{2\mu_0}\right] \tag{6}$$

where V_p is the volume of the plasmoid (in this case a sphere of radius r_p), and f_3 ($= 3/2$ for a sphere) is a shape factor depending on the plasmoid geometry.

The equation of motion for a plasmoid subjected only to the melon seed effect can now be written as

$$\rho\frac{D\mathbf{v}_p}{Dt} \;=\; -f_3 \boldsymbol{\nabla}\left[\frac{B^2}{2\mu_0}\right] \tag{7}$$

where ρ is the average density of the material within the plasmoid and $\mathbf{v}_p$ its velocity. From this it is clear that acceleration can lead to velocities of the order of the Alfvén velocity $B/\sqrt{\mu_0\rho}$. Since the plasmoid density ρ, in most cases of interest, is higher than the ambient coronal density, the Alfvén velocity will be lower than in the corona. It can however be much greater than the internal thermal velocity. The energy source is the assumed internal magnetic field [Raadu et al 1987].

4 Discussion

In the case of a filament with no net current, such as is expected if the internal magnetic field is twisted by localised motions at the footpoints in the photosphere, the distribution of the internal currents becomes significant. In a nonuniform external field, the condition that there is no normal surface field component implies that the filament must have a net line dipole. There is a force on this line dipole that is similar to that for a plasmoid. Following analogous arguments, the force per unit length on the filament is found to be,

$$\mathbf{F}_f = -f_2 S_f \boldsymbol{\nabla}\left[\frac{B^2}{2\mu_0}\right] \tag{8}$$

where S_f is the cross sectional area (πr_f^2 for a circle) and the shape factor f_2 (= 2 for a circle) depends on the shape of the cross section. Comparing this with Eq. (1) it can be seen that this force may be comparable to the mirror force.

In the case of a plasmoid, mirror forces should also arise as a consequence of line tying in the photosphere. However, the mirror currents give an equivalent dipole field and this falls off rapidly with height. The mirror force in this case can be expected to be very weak, except when the plasmoid is close to the photosphere.

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