

2. 1. 1.

Sölet: $\sigma(x)$

Reverse engineering:

vill ha

$$\sigma(x) = \frac{N(x)}{A(x)}$$

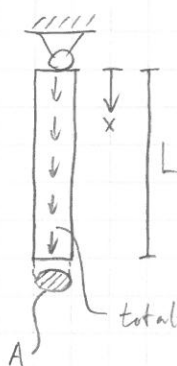
②

Snitta + Jmv

①

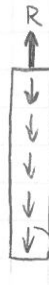
Frläggning + Jmv

definition



$$\text{totalt} = Q (= mg)$$

① Frlägg + Jmv



$$\begin{aligned} \text{Jmv } x\text{-led:} \\ Q - R = 0 \Leftrightarrow \\ R = Q \end{aligned}$$

② Snitta + Jmv

$$R = Q$$

$$Q_I = Q \frac{x}{L}$$

$$N(x)$$

$$Q_{II} = Q - Q \frac{x}{L} = Q \left(1 - \frac{x}{L}\right)$$

Jmv del II x-led:

$$Q \left(1 - \frac{x}{L}\right) - N(x) = 0 \Leftrightarrow$$

$$\Leftrightarrow N(x) = Q \left(1 - \frac{x}{L}\right)$$

(prova att göra jämvikter i del I, du ska få samma svar. Tänk på att du har R!)

$$\sigma(x) = \frac{N(x)}{A} = \frac{Q}{A} \left(1 - \frac{x}{L}\right)$$

(dim-analys ok)

Alt 2 - Diff-ekvation

Sölet: $\sigma(x) = \frac{N(x)}{A}$

F.S. 6.3 s. 60 $\Rightarrow \frac{dN(x)}{dx} + K_x(x) A(x) = 0$

(s. 58 i gamla bkr F.S.)

$\underbrace{\left(\text{kraft per volym} = \frac{Q}{AL} \right)}_{Q/L}$

$\left(\text{Jämför } \frac{mg}{AL} = \frac{mg}{V} = \rho g \Rightarrow K_x = \rho g \right)$

$$\Rightarrow \frac{dN(x)}{dx} = -\frac{Q}{L} \Rightarrow N(x) = -\frac{Q}{L} \int dx = -\frac{Q}{L} x + C$$

Randvillkor: $N(L) = 0 \Rightarrow 0 = -\frac{Q}{L} L + C \Rightarrow C = Q \Rightarrow N(x) = -\frac{Q}{L} x + Q \Rightarrow$

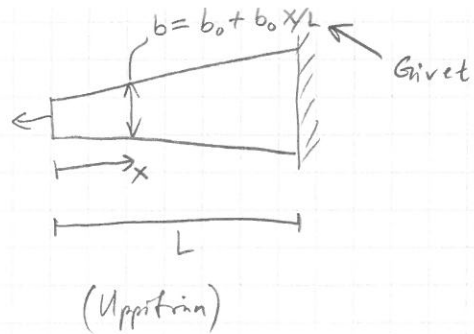
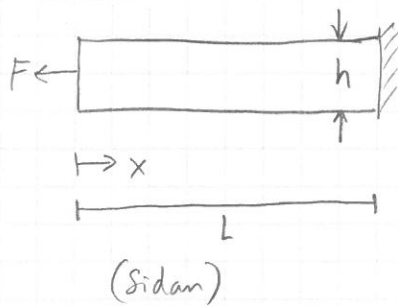
$$\Rightarrow N(x) = Q \left(1 - \frac{x}{L}\right) \Rightarrow \sigma(x) = \frac{Q}{A} \left(1 - \frac{x}{L}\right)$$

Alt. $N(x) = N(0) + \int_0^x -\frac{Q}{L} dx = Q - \frac{Q}{L} (x-0) = Q \left(1 - \frac{x}{L}\right)$

Q

minste vara samma!

2.1.4.

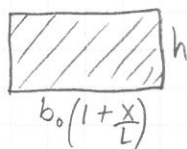


Sökt: $\sigma(x)$

Huvr?:

$$\sigma(x) = \frac{N(x)}{A(x)}$$

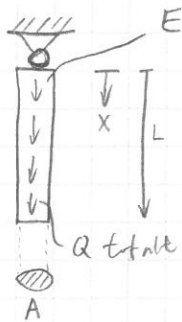
(Snitt)



$$\Rightarrow \sigma(x) = \frac{F}{b_0 h (1 + x/L)} = \frac{FL}{b_0 h (x + L)}$$

dim analys ok

2.1.15.



Sölet: δ

Hur?: $\delta = \int_0^L \epsilon dx$, Obs! $\delta = \epsilon L$ gäller om och endast om ϵ är konstant

① känd sedan 2.1.1.
② Hookes $\epsilon \leftrightarrow \sigma$

$$\left. \begin{array}{l} \text{① Från 2.1.1.} \Rightarrow \sigma(x) = \frac{Q}{A} \left(1 - \frac{x}{L}\right) \\ \text{② Hookes lag} \Rightarrow \epsilon(x) = \frac{\sigma(x)}{E} \end{array} \right\} \Rightarrow \epsilon(x) = \frac{Q}{EA} \left(1 - \frac{x}{L}\right) = \frac{Q}{EAL} (L - x)$$

$$\Rightarrow \delta = \int_0^L \epsilon dx = \frac{Q}{EAL} \int_0^L (L - x) dx = \frac{Q}{EAL} \left[Lx - \frac{x^2}{2} \right]_0^L = \frac{QL}{2EA}$$

dim-analys: $\frac{Nm}{N/m^2 \cdot m^2} = m$ ok!

Alt.

om man föredrar det här

$$\Rightarrow \delta = \int_0^L \epsilon dx = \frac{Q}{EA} \int_0^L \left(1 - \frac{x}{L}\right) dx = \frac{Q}{EA} \left[x - \frac{x^2}{2L} \right]_0^L = \frac{QL}{2EA}$$

$\left(L - \frac{L^2}{2L} \right) - 0 = \frac{L}{2}$

Exempel titantrod, 1m:

$$\delta = \frac{mgL}{2EA} = \frac{\rho(LA)gL}{2EA} = \frac{\rho g L^2}{2E}$$

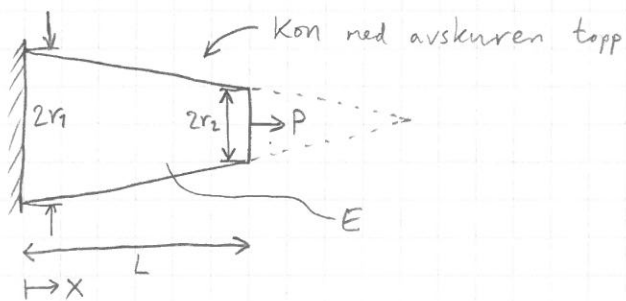
4500 kg/m³
9,82 m/s²
1m

$$105 \text{ GPa} = 105 \cdot 10^9 \frac{N}{m^2} = 105 \cdot 10^9 \frac{kg \cdot m}{m^2 \cdot s^2}$$

$$\Rightarrow \delta = \frac{4500 \cdot 9,82 \cdot 1^2}{2 \cdot 105 \cdot 10^9} \frac{\frac{kg}{m^3} \cdot \frac{m}{s^2} \cdot m^2}{\frac{kg \cdot m}{m^2 \cdot s^2}} \approx 2 \cdot 10^{-7} m = 0,2 \mu m \text{ (inte märkbart!)}$$

(hårstris: ca 70-90 μm)

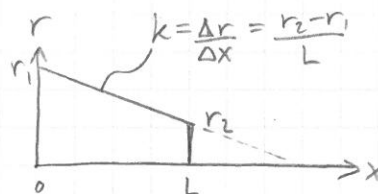
2.1.16.



Sölet: $\delta(P)$

Hur?: $\delta = \int_0^L \epsilon dx$

③ $\epsilon = \frac{\sigma}{E} \leftarrow \frac{P/A(x)}{E}$
 Hookes
 ② Given



① $A(r(x)) = \pi r^2$

$r(x) = r_1 + \frac{r_2 - r_1}{L} x$ (rät linjens ekv. $y = m + kx \Rightarrow r = r_1 + kx$)

② $\epsilon = \frac{\sigma}{E} = \frac{P}{EA(x)} = \frac{P}{\pi E} \cdot \frac{1}{r^2(x)}$

③ $\delta = \int_0^L \epsilon dx = \frac{P}{\pi E} \int_0^L \frac{1}{\left(\frac{r_2 - r_1}{L}x + r_1\right)^2} dx = \{ \text{Beta 7.4.6 s. 153} \} =$

$= \frac{P}{\pi E} \left[\frac{1}{a^2 x + ab} \right]_0^L = \dots = \underline{\underline{\frac{PL}{\pi E r_1 r_2}}}$