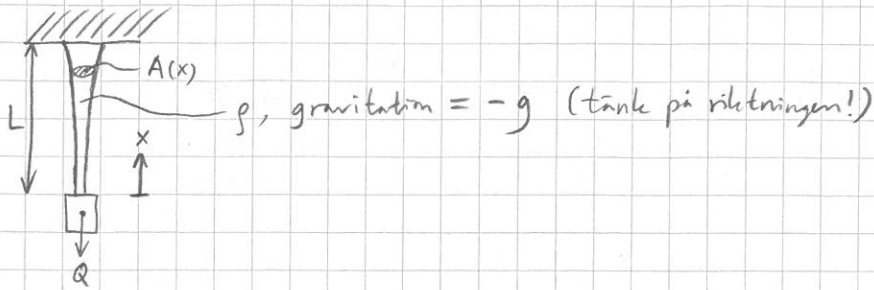


2.1.9.



Sökt: $A(x)$ så att $\sigma(x) = \text{konstant}$

Hur?: Diff. elev. F.S. s. 60, elev. 6.3 (s. 58 i gamla blå F.S.)

$$\Rightarrow N'(x) + K_x(x) A(x) = 0 \quad K_x = \text{kraft/volym} = \frac{-mg}{V} = -\rho g$$

Given $(-\rho g)$

$$N(x) = \underbrace{\sigma}_{\text{konst.}} A(x) \Rightarrow N'(x) = \sigma A'(x)$$

$$\Rightarrow \sigma A'(x) + (-\rho g) A(x) = 0 \Leftrightarrow A'(x) = \frac{\rho g}{\sigma} A(x) \Rightarrow$$

$$\Rightarrow A(x) = C \cdot \exp\left(\frac{\rho g}{\sigma} x\right), \quad C \text{ okänd konstant}$$

$$\text{Randvillkor: } N(0) = Q = \sigma A(0) \Rightarrow A(0) = \frac{Q}{\sigma} \Rightarrow$$

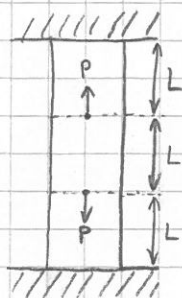
$$\Rightarrow A(0) = \frac{Q}{\sigma} = C \cdot \underbrace{\exp\left(\frac{\rho g}{\sigma} \cdot 0\right)}_1 = C \Rightarrow$$

$$\Rightarrow A(x) = \frac{Q}{\sigma} \exp\left(\frac{\rho g}{\sigma} x\right)$$

$$\text{Dim. analys: } \frac{N}{N/m^2} = m^2 \Rightarrow \frac{Q}{\sigma} \text{ ok}$$

$$\frac{(kg/m^3) \cdot (m/s^2) \cdot (m)}{(N/m^2)} = \frac{\overset{N}{kg} \cdot \frac{m}{s^2}}{m^3} \cdot \frac{m^2}{N} \cdot m = \frac{Nm^3}{Nm^3} = 1 \Rightarrow \exp(\dots) \text{ ok}$$

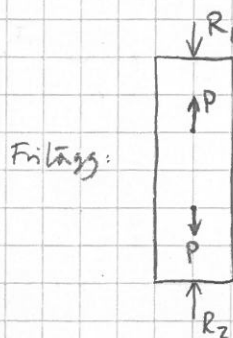
2.1.30.



Konstant area, A
E-modul, E

Sökt: Reaktionskrafter R_1, R_2

Har? Reaktionskrafter \leftarrow Frlägg + Jmv.

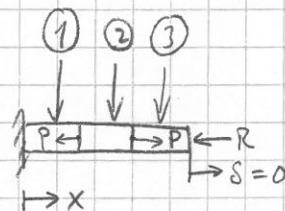


$$\Rightarrow \text{Jmv} \downarrow: R_1 - P + P - R_2 = 0 \Leftrightarrow R_1 = R_2 = R$$

(Alt. från symmetri)

Vad vet man mer?

Stela väggar $\Rightarrow \delta_{\text{tot}} = 0$ totalt sett, jämför



$$\delta = \int_0^{3L} \epsilon dx = \int_0^{3L} \frac{\sigma}{E} dx \leftarrow \text{Aha! Behöver } \frac{N}{A}!$$

\Rightarrow Smitta!

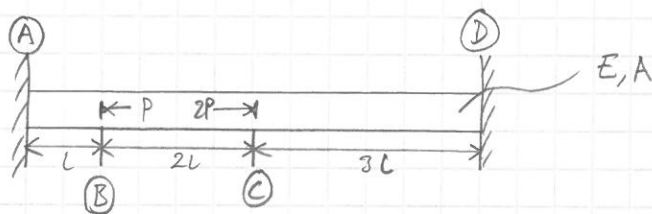
$$\begin{array}{cccc} R \rightarrow & \boxed{} & \rightarrow N_1 \leftarrow & \boxed{} & \rightarrow N_2 \leftarrow & \boxed{} & \rightarrow N_3 \leftarrow & \boxed{} & \leftarrow R \\ \text{Jmv: } R + N_1 = 0 & & -N_1 - P + N_2 = 0 & & -N_2 + P + N_3 = 0 & & -N_3 - R = 0 \\ \Rightarrow N_1 = -R & & \Rightarrow N_2 = N_1 + P & & \Rightarrow N_3 = N_2 - P & & \Rightarrow N_3 = -R \end{array}$$

$$\Rightarrow \begin{cases} N_1 = -R \\ N_2 = P - R \\ N_3 = -R \end{cases} \Rightarrow \begin{cases} \sigma_1 = -R/A \\ \sigma_2 = (P - R)/A \\ \sigma_3 = -R/A \end{cases} \leftarrow \text{Styckens konstant!} \Rightarrow \epsilon = \frac{\sigma}{E} \text{ är styckens konst.}$$

$$\delta_{\text{tot}} = \delta_1 + \delta_2 + \delta_3 = \epsilon_1 L + \epsilon_2 L + \epsilon_3 L = \frac{(-R)L}{EA} + \frac{(P-R)L}{EA} + \frac{(-R)L}{EA} = \frac{L}{EA} (P - 3R) = 0$$

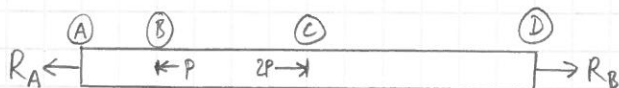
$$\Rightarrow P = 3R \Rightarrow R = P/3$$

2.1.31

Sölet: $\sigma_{AB}, \sigma_{BC}, \sigma_{CD}$

Hur? $\sigma_i = \frac{N_i}{A} \leftarrow \begin{matrix} \text{Snitt} \\ + J_{mv} \end{matrix} \leftarrow \begin{matrix} \text{Frlägg} \\ + J_{mv} \end{matrix}$
 $A \leftarrow \text{givet}$

Vid behov, deformationssamband (kompatibilitetsvillkor)

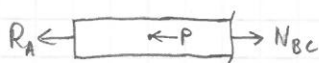
Lösning: ① Frlägg + J_{mv} .

$$J_{mv} \rightarrow: -R_A - P + 2P + R_B = 0 \Rightarrow R_A = R_B + P \Leftrightarrow R_B = R_A - P \quad (1)$$

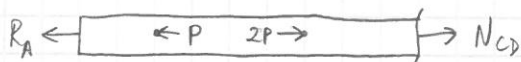
② Snittar: Inom A-B, inom B-C och inom C-D



$$J_{mv} \rightarrow: -R_A + N_{AB} = 0 \Rightarrow N_{AB} = R_A \quad (2)$$



$$J_{mv} \rightarrow: -R_A - P + N_{BC} = 0 \Rightarrow N_{BC} = R_A + P \quad (3)$$



$$J_{mv} \rightarrow: -R_A - P + 2P + N_{CD} = 0 \Rightarrow N_{CD} = R_A - P \quad (4)$$

Vi verkar sakna 1 ekv så att R_A , och därmed allt annat, kan bestämmas.5 okända: $R_A, R_B, N_{AB}, N_{BC}, N_{CD}$, 4 ekvationer \Rightarrow Deformationssamband!Stela väggar $\Rightarrow \delta = 0 = \delta_{AB} + \delta_{BC} + \delta_{CD} \Rightarrow$

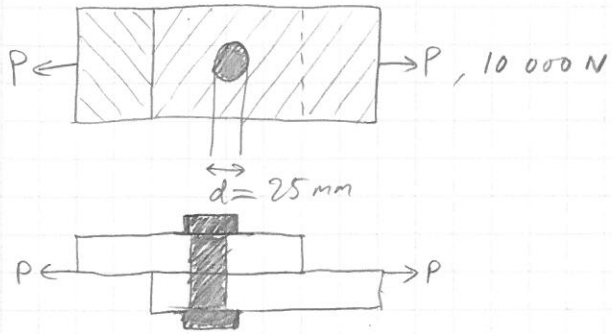
$$\left(\delta = \varepsilon L = \frac{\sigma L}{E} = \frac{NL}{EA} \right) \Rightarrow \frac{N_{AB} L}{EA} + \frac{N_{BC} (2L)}{EA} + \frac{N_{CD} (3L)}{EA}$$

$$\Rightarrow \frac{N_{AB} L}{EA} + 2 \frac{N_{BC} L}{EA} + 3 \frac{N_{CD} L}{EA} = 0 \Rightarrow N_{AB} + 2 N_{BC} + 3 N_{CD} = 0 \quad (5)$$

$$(2), (3), (4) \text{ i } (5) \Rightarrow (R_A) + 2 \cdot (R_A + P) + 3(R_A - P) = 6R_A - P = 0 \Rightarrow R_A = \frac{P}{6}$$

$$\Rightarrow \sigma_{AB} = \frac{N_{AB}}{A} = \frac{P}{6A}, \quad \sigma_{BC} = \frac{N_{BC}}{A} = \frac{7P}{6A}, \quad \sigma_{CD} = \frac{N_{CD}}{A} = -\frac{5P}{6A}$$

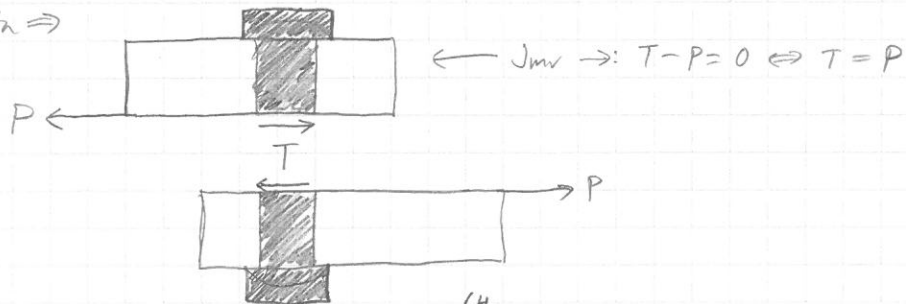
2.3.4.



Sökt: Medelskjuvspänning i bulten

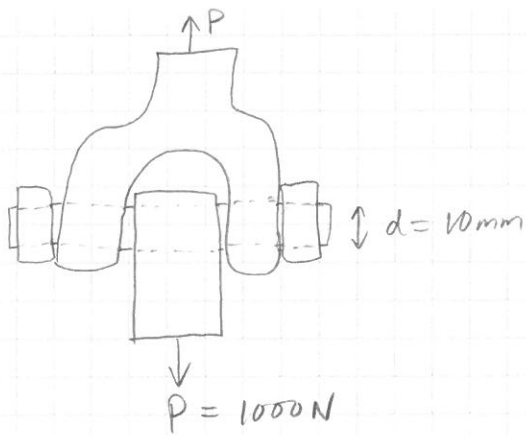
Hint: $\tau_m = \frac{T}{A}$ ← Skjuvar
 $A = \frac{\pi d^2}{4}$

Skjuvar \Rightarrow



$$\Rightarrow \tau_m = \frac{4P}{\pi d^2} = \frac{4 \cdot 10^4 \text{ N}}{\pi \cdot 25^2 \text{ mm}^2} = \frac{4 \cdot 4 \cdot 4 \cdot 10^4}{\pi \cdot 4 \cdot 4 \cdot 25 \cdot 25} \text{ MPa} = \frac{64}{\pi} \text{ MPa} \approx \underline{\underline{20 \text{ MPa}}}$$

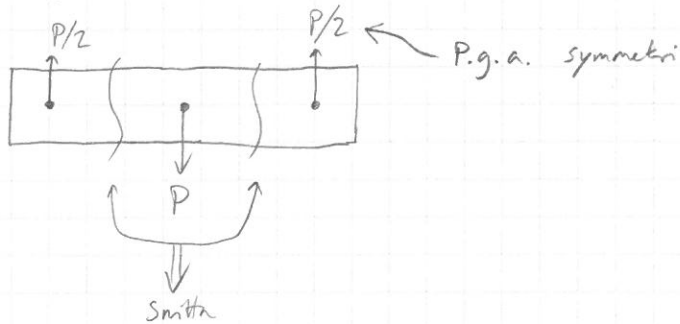
2.3.6.



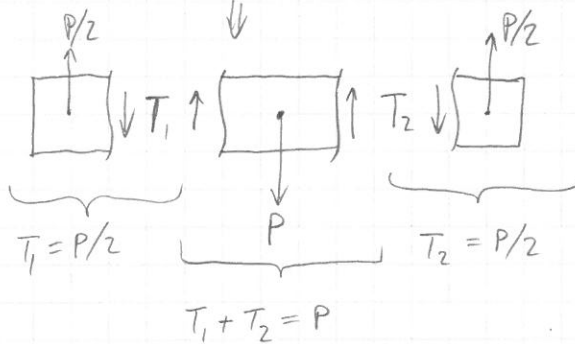
Sökt: Medelstygnspänning i bulten

Hur: $\tau_m = \frac{T}{A}$ ← Snitta ← Frilägg
 $\frac{\pi d^2}{4}$

① Frilägg: Förenkla geometrin till knutar



②



$$\Rightarrow T_1 = T_2 = T = P/2$$

$$\textcircled{3} \tau_m = \frac{T}{A} = \frac{4}{\pi d^2} \frac{P}{2} = \frac{2 \cdot 1000}{\pi \cdot 10^2} \frac{\text{N}}{\text{mm}^2} = \frac{20}{\pi} \text{ MPa} \approx \underline{\underline{6,4 \text{ MPa}}}$$