Simultaneous Measurement Imputation and Outcome Prediction for Achilles Tendon Rupture Rehabilitation

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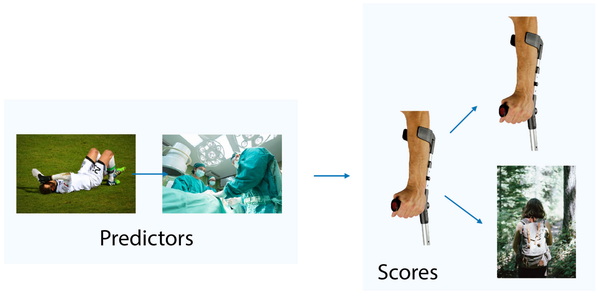
Achilles Tendon Rupture Rehabilitation

- ATR is a **complete** tear through the Achilles tendon.
  - Hard to collect data (costly, painful measurements)
  - Lengthy healing process with abundant complications
  - Lack of understanding of the healing process and hard to predict healing outcome
  - Typical soft tissue injury: method developed for ATR can be applied to all other soft tissue injuries

- A patient journey



- End-to-end probabilistic framework:
  Missing data **imputation** and rehabilitation outcome **prediction**

Cohort

	Length	Weight	.....	DVT_2	...	ATRS_3_pain	...	ATRS_12_stiff
1	172	79.8	.....	×	...	6	...	8
2	×	76.5	.....	0	...	4	...	×
3	×	×	...	1	...	8	...	10

Predictors → Data imputation | Scores → Prediction

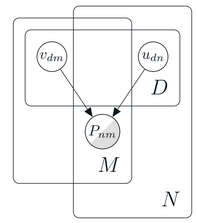
- Real dataset aggregated from previous studies [1, 2]
- $N = 442$ patients, $M = 297$ predictors (measurements) and $S = 63$ scores
- Missing entries: 69.5% in predictors, 64.2% in scores
- Several data types: integers, categories, real numbers, strings

Model

- **Data Imputation**

$$\underset{(N\times D)}{\mathbf{U}} \quad \underset{(D\times M)}{\mathbf{V}^T} = \underset{(N\times M)}{\mathbf{P}}$$

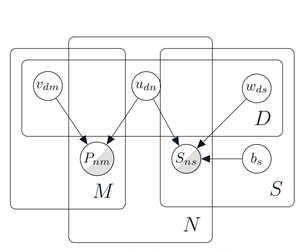
(a) Schematic



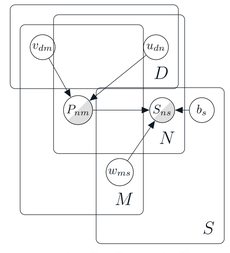
(b) Graphical model

$$p(\mathbf{P}|\mathbf{U},\mathbf{V},\sigma^2_{\mathbf{P}}) = \prod_{n=1}^{N}\prod_{m=1}^{M}\left[\mathcal{N}(P_{nm}|\mathbf{u}_n\mathbf{v}_m^T,\sigma^2_{\mathbf{P}})\right]^{\mathbf{I}(n,m)}$$

- **Simultaneous Data Imputation and Outcome Prediction**
  - Graphical models



(a) BLR with $\mathbf{U}$



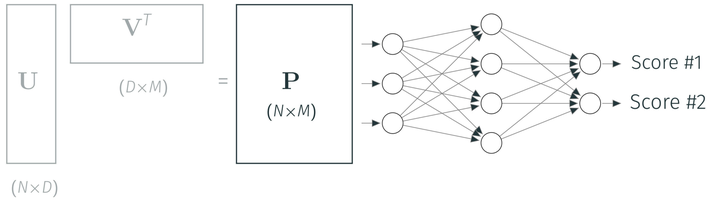
(b) BLR with $\mathbf{P}$

$$\mathrm{p(S|}\ \mathbf{W},\mathbf{b},\mathbf{X}) = \prod_{n=1}^{N}\prod_{s=1}^{S}\left[\mathcal{N}(S_{ns}\mid \mathbf{x}_n\mathbf{w}_s + b_s, \sigma^2_{\mathbf{S}})\right]^{\mathbf{I}'_{ns}}, \quad \mathbf{W}\sim\mathcal{N}(\mathbf{W}\mid\mathbf{0},\sigma^2_w\mathbf{1}), \quad \mathbf{b}\sim\mathcal{N}(\mathbf{b}\mid\mathbf{0},\sigma^2_{\mathbf{b}})$$

$$\mathrm{p(S|}\ \theta,\mathbf{X}) = \prod_{n=1}^{N}\prod_{s=1}^{S}\left[\mathcal{N}(S_{ns}\mid \mathbf{NN}(\mathbf{x}_n;\theta),\sigma^2_{\mathbf{S}})\right]^{\mathbf{I}'_{ns}}, \quad \mathbf{H}_l = \tanh\left(\mathbf{H}_{l-1}\mathbf{W}_l + \mathbf{b}_l\right) \quad \text{for } l = 1,...,L$$

$$\mathrm{w}_{lij}\sim\mathcal{N}(w_{lij}\mid 0,\sigma^2_{w_{lij}}), \quad \sigma^2_{w_{lij}} = \frac{2}{n_{lj,\mathrm{in}}+n_{lj,\mathrm{out}}}$$

  - Schematics



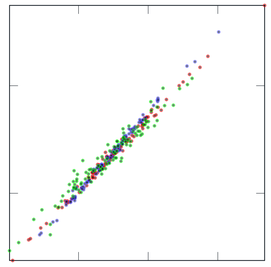
(a) BNN with $\mathbf{P}$

Experiments

MAE of Two-stage and End-to-end Methods

Component 2 Input	BLR $\mathbf{P}$	BLR $\mathbf{S}$	BLR $\mathbf{S}_{ATRS}$
$\hat{\mathbf{P}}$ 2-stage (mean)	0.228 ± 0.0014	0.230± 0.008	0.200 ± 0.010
$\hat{\mathbf{P}}$ 2-stage (OptSpace)	0.224± 0.0028	0.207 ± 0.009	0.193 ± 0.010
$\hat{\mathbf{P}}$ 2-stage (SoftImpute)	0.2049 ± 0.002	0.206 ± 0.008	0.192 ± 0.010
$\hat{\mathbf{P}}$ 2-stage (SVP)	0.316 ± 0.003	0.205 ± 0.012	0.200 ± 0.014
$\hat{\mathbf{P}}$ 2-stage (IALM)	0.237 ± 0.008	0.201 ± 0.011	0.201 ± 0.010
$\hat{\mathbf{P}}$ 2-stage (PMF)	0.164 ± 0.002	0.220 ± 0.006	0.201 ± 0.007
$\hat{\mathbf{U}}$ 2-stage (PMF)	0.164 ± 0.002	0.237 ± 0.006	0.208 ± 0.006
$\hat{\mathbf{P}}$ EE (proposed)	0.181 ± 0.001	0.202± 0.003	0.195± 0.005
$\hat{\mathbf{U}}$ EE (proposed)	0.178± 0.001	**0.164**± 0.004	**0.146**±0.005
Component 2 Input	BNN $\mathbf{P}$	BNN $\mathbf{S}$	BNN $\mathbf{S}_{ATRS}$
$\hat{\mathbf{P}}$ 2-stage (mean)	0.228 ± 0.0014	0.233 ± 0.005	0.202 ± 0.005
$\hat{\mathbf{P}}$ 2-stage (OptSpace)	0.224± 0.0028	0.203 ± 0.008	0.187± 0.009
$\hat{\mathbf{P}}$ 2-stage (SoftImpute)	0.2049 ± 0.002	0.201 ± 0.007	0.186 ± 0.008
$\hat{\mathbf{P}}$ 2-stage (SVP)	0.316 ± 0.003	0.194 ± 0.010	0.187 ± 0.010
$\hat{\mathbf{P}}$ 2-stage (IALM)	0.237 ± 0.008	0.187± 0.011	0.187 ± 0.009
$\hat{\mathbf{P}}$ 2-stage (PMF)	0.164 ± 0.002	0.207 ± 0.007	0.190 ± 0.007
$\hat{\mathbf{U}}$ 2-stage (PMF)	0.164 ± 0.002	0.208± 0.007	0.190 ± 0.007
$\hat{\mathbf{P}}$ EE (proposed)	0.158 ± 0.001	**0.152**±0.004	**0.143**±0.004
$\hat{\mathbf{U}}$ EE (proposed)	0.167±0.001	0.174±0.003	0.152± 0.003

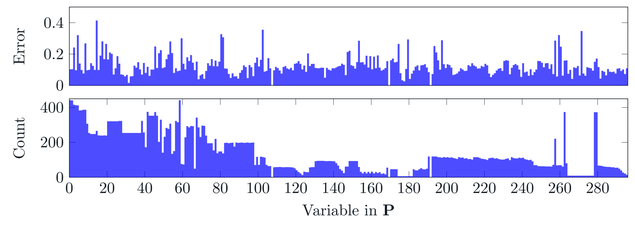
Simulation Experiments: Recovered and True Traits



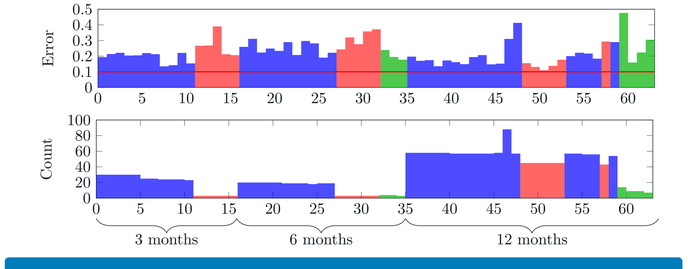
Outcome Prediction at Various Timestamps

	Discharge $\hat{\mathbf{P}}$	3 Month $\mathbf{P}_3$	6 Month $\mathbf{P}_6$
MAE $\mathbf{S}_3$	0.177 ± 0.006		
MAE $\mathbf{S}_{ATRS-3}$	0.173 ± 0.005		
MAE $\mathbf{S}_6$	0.172± 0.007	0.178± 0.006	
MAE $\mathbf{S}_{ATRS-6}$	0.167 ± 0.009	0.169±0.010	
MAE $\mathbf{S}_{12}$	0.138 ± 0.006	0.140 ± 0.006	**0.132**± 0.006
MAE $\mathbf{S}_{ATRS-12}$	0.111 ± 0.003	0.114 ± 0.004	**0.108**±0.003

Predictors P: Per-variable MAE and Number of Data



Per-variable MAE and Number of Data
Scores: ATRS, FAOS, and other scores



References

[1] E Domeij-Arverud et al. "Ageing, deep vein thrombosis and male gender predict poor outcome after acute Achilles tendon rupture". In: *Bone Joint J* (2016).

[2] Kars P Valkering et al. "Functional weight-bearing mobilization after Achilles tendon rupture enhances early healing response: a single-blinded randomized controlled trial". In: *Knee Surgery, Sports Traumatology, Arthroscopy* (2017).