Kap 1 (1.1, 1.2, 1.4, 1.5, 1.7)

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Enhetsvektor $\quad \bar{e}_a = \frac{\bar{a}}{|\bar{a}|} = \frac{\bar{a}}{a}$

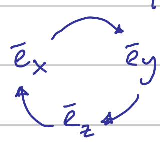
Skalärprodukt $\quad \bar{a}\cdot\bar{b} = ab\cos\alpha \qquad 0 \leq \alpha \leq \pi$

$$\bar{a}\cdot\bar{b} = a_x b_x + a_y b_y + a_z b_z$$

Beloppet av en vektor $\quad a = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Vektorprodukt $\quad |\bar{a}\times\bar{b}| = ab\sin\alpha \qquad 0 \leq \alpha \leq \pi$

$$\bar{a}\times\bar{b} = \begin{vmatrix} \bar{e}_x & \bar{e}_y & \bar{e}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$



när vi kryssar medurs följer vi bara cirkeln men när vi kryssar moturs blir det ombytt tecken. t.ex. $\bar{e}_x \times \bar{e}_y = \bar{e}_z$

$$\bar{e}_z \times \bar{e}_y = -\bar{e}_x$$

Dubbla vektorprodukt $\quad \bar{a}\times\bar{b}\times\bar{c} = \bar{b}(\bar{a}\cdot\bar{c}) - \bar{c}(\bar{a}\cdot\bar{b})$

1.1 - Givet $\bar{a}=(2,-1,-2)$; $\bar{b}=(0,-3,4)$; $\bar{c}=(1,-1,2)$

c) Söker $\bar{a}_{//}$ och $\bar{a}_{\perp}$ gentemot $\bar{b}$

* hitta $\bar{e}_b$

$$\bar{e}_b = \frac{(0,-3,4)}{\sqrt{9+16}} = \frac{1}{5}(0,-3,4)$$

* projicera $\bar{a}$ på $\bar{e}_b \Rightarrow a_{//}$

$$\bar{a}_{//} = (\bar{a}\cdot\bar{e}_b)\bar{e}_b = \left(\begin{vmatrix}2\\-1\\-2\end{vmatrix}\cdot\frac{1}{5}\begin{vmatrix}0\\-3\\4\end{vmatrix}\right)\bar{e}_b = \frac{-5}{5}\cdot\bar{e}_b = -\bar{e}_b$$

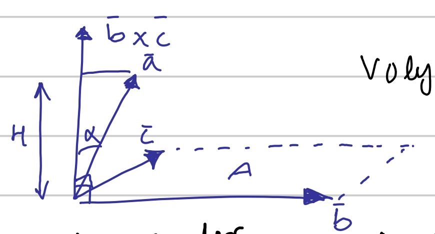
$$= \underline{\underline{\frac{1}{5}(0,3,-4)}}$$

* hitta $a_{\perp}$

$$\bar{a} = \bar{a}_{//} + \bar{a}_{\perp} \Leftrightarrow \bar{a}_{\perp} = \bar{a} - \bar{a}_{//} = \begin{vmatrix}2\\-1\\-2\end{vmatrix} - \begin{vmatrix}0\\3/5\\-4/5\end{vmatrix} = \begin{vmatrix}2\\-8/5\\-6/5\end{vmatrix} =$$

$$\underline{\underline{\frac{2}{5}(1,-4,-3)}}$$

f) Söker volymen hos parallellepiped



$$\text{Volymen} = |\overbrace{\bar{a}\cdot\bar{b}\times\bar{c}}^{*}| = a\,\underbrace{|\bar{b}\times\bar{c}|}_{A}\cos\alpha =$$

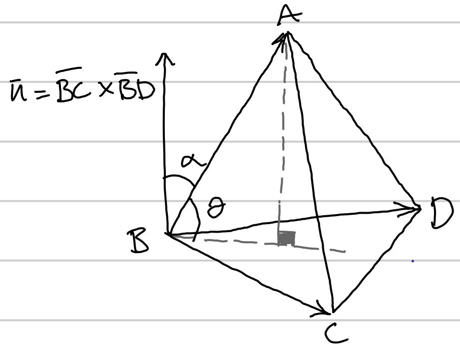
$$= A\,\underbrace{a\cos\alpha}_{H} = \text{Volym}$$

vi använder oss av * och inte V=AH, då vi behöver räkna ut a

$$V = \text{abs}(\bar{a}\cdot\bar{b}\times\bar{c}) = \text{abs}\left(\begin{vmatrix}2\\-1\\-2\end{vmatrix}\cdot\begin{vmatrix}e_x & e_y & e_z\\0 & -3 & 4\\1 & -1 & 2\end{vmatrix}\right) = \text{abs}\left(\begin{vmatrix}2\\-1\\-2\end{vmatrix}\cdot\begin{vmatrix}-2\\4\\3\end{vmatrix}\right) =$$

$$= \text{abs}(-14) = 14\ \text{v.e}$$

1.2



Givet: $A(1,2,4)$; $B(1,0,2)$; $C(2,1,3)$; $D(4,1,1)$

Söker: vinkel θ

Lösning

För att kunna hitta vinkel θ, vill vi först hitta normalvektorn $\bar{n}$ från BCD för att senare beräkna vinkel α med hjälp av skalär produkt.

OBS! Eftersom vi är bara intresserade av vinklar behöver vi inte normalisera vektorerna.

vektor $\bar{n}$

$$\bar{n} \perp \bar{BC}, \bar{n} \perp \bar{BD} \longrightarrow \bar{n} = \bar{BC} \times \bar{BD}$$

$$\bar{n} = (1,1,1) \times (3,1,-1) = \begin{vmatrix} e & e & e \\ 1 & 1 & 1 \\ 3 & 1 & -1 \end{vmatrix} = (-2, 4, -2)$$

vinkel α - skalär produkt

$$\bar{n} \cdot \bar{BA} = n \cdot BA \cdot \cos\alpha \longrightarrow \cos\alpha = \left(\frac{\bar{n} \cdot \bar{BA}}{n \cdot BA}\right)$$

$$(-2,4,-2)(0,2,2) = 8-4 = \sqrt{2^2+4^2+2^2}\sqrt{2^2+2^2}\cos\alpha$$

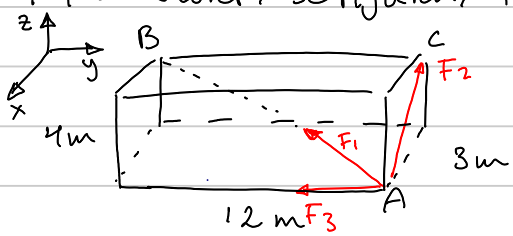
$$\cos\alpha = \left(\frac{4}{\sqrt{24}\sqrt{8}}\right) = \frac{4}{4\sqrt{12}} = \frac{1}{2\sqrt{3}}$$

vinkel θ

$$\alpha + \theta = \pi/2 \longrightarrow \theta = \pi/2 - \alpha; \cos\alpha = \sin(\pi/2 - \alpha) = \sin(\theta) = \frac{1}{2\sqrt{3}}$$

Svar $\theta = \arcsin\left(\frac{1}{2\sqrt{3}}\right) \approx 16,8^\circ$

1.4 – Givet: se figuren, $F_1 = 260\,N$; $F_2 = 75\,N$; $F_3 = 60\,N$



Söker: * Summan av $\bar{F} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$

* Bestäm α, β, γ

Lösning

* Hitta enhetsvektorer $\bar{e}_{AC}$ & $\bar{e}_{AB}$

$$\bar{e}_{AB} = \frac{\bar{r}_{AB}}{r_{AB}} = \left\{ \bar{r}_{AB} = \underbrace{\begin{vmatrix} 0 \\ 0 \\ 4 \end{vmatrix}}_{B} - \underbrace{\begin{vmatrix} 3 \\ 12 \\ 0 \end{vmatrix}}_{A} = \begin{vmatrix} -3 \\ -12 \\ 4 \end{vmatrix} \right\} = \frac{(-3, -12, 4)}{\sqrt{9+144+16}} =$$

$$= \frac{1}{13}(-3, -12, 4)$$

$$\bar{e}_{AC} = \frac{\bar{r}_{AC}}{r_{AC}} = \left\{ \bar{r}_{AC} = \underbrace{\begin{vmatrix} 0 \\ 12 \\ 4 \end{vmatrix}}_{C} - \underbrace{\begin{vmatrix} 3 \\ 12 \\ 0 \end{vmatrix}}_{A} = \begin{vmatrix} -3 \\ 0 \\ 4 \end{vmatrix} \right\} = \frac{-3, 0, 4}{\sqrt{9+16}} =$$

$$= \frac{1}{5}(-3, 0, 4)$$

* Bestäm $\bar{F}_1$, $\bar{F}_2$ & $\bar{F}_3$

$$\bar{F}_1 = F_1\,\bar{e}_{AB} = \frac{260\,N}{13}(-3, -12, 4) = 20\,N(-3, -12, 4)$$

$$\bar{F}_2 = F_2\,\bar{e}_{AC} = \frac{75\,N}{5}(-3, 0, 4) = 15\,N(-3, 0, 4)$$

$$\bar{F}_3 = F_3\,(-)\bar{e}_y = 60\,N\,(0, -1, 0)$$

* Räkna ut $\bar{F}$

$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 = \begin{vmatrix} -60 - 45 \\ -240 - 60 \\ 80 + 60 \end{vmatrix} = \begin{vmatrix} -105 \\ -300 \\ 140 \end{vmatrix}$$

$$F = \sqrt{105^2 + 300^2 + 140^2} = \sqrt{120625} = 347{,}31\,N \approx \underline{\underline{350\,N}}$$

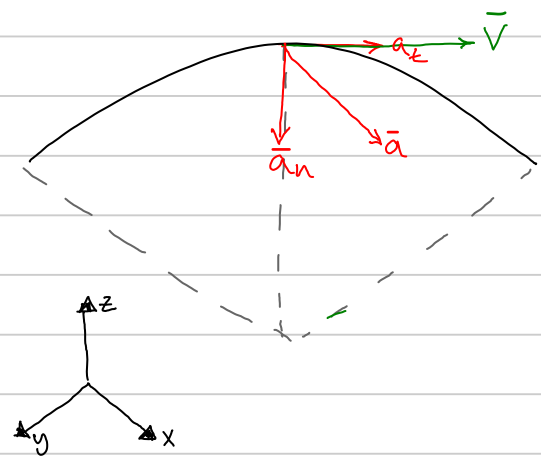
* Bestäm α, β, γ

$$\bar{e}_F = \frac{\bar{F}}{F} = \left(\frac{-105}{\sqrt{120625}}, \frac{-300}{\sqrt{120625}}, \frac{140}{\sqrt{120625}} \right) = (\cos\alpha, \cos\beta, \cos\gamma)$$

$$\alpha = \arccos -0{,}302 \approx \underline{\underline{107{,}60^0}}; \quad \beta = \arccos(-0{,}864) \approx \underline{\underline{149{,}74^0}}$$

$$\gamma = \arccos 0{,}403 \approx \underline{\underline{66{,}25^0}}$$

1.5



Givet: $\bar{v} = 10(4,12,3)\ m/s$

Söker $\bar{a} = (10,-2,1)\ m/s^2$

$\bar{a}_t$ och $\bar{a}_n$

Lösning

För att beräkna $\bar{a}_t$ projecerar vi $\bar{a}$ på $\bar{v}$.

$$\bar{a}_t = \frac{\bar{a}\cdot\bar{v}}{v}\cdot\bar{v} = \frac{(10,-2,-1)\cdot\cancel{10}(4,12,3)}{\cancel{10^2}(4^2+12^2+3^2)}\cdot\cancel{10}(4,12,3)\left(\frac{ms^{-1}\cdot ms^{-2}\cdot ms^{-1}}{m^2 s^{-2}}\right)$$

$$= \frac{13}{169}(4,12,3)\ ms^{-2} = \frac{1}{13}(4,12,3)\ m/s^2$$

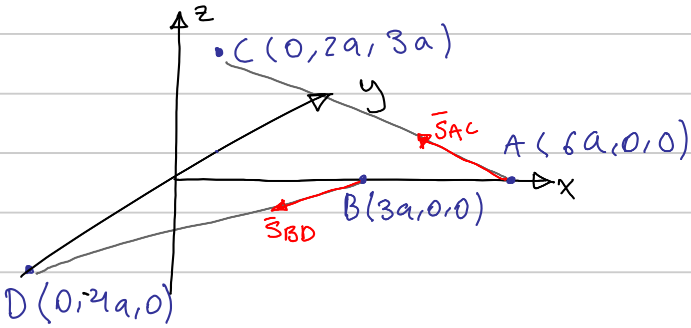
För att beräkna an använder vi oss av sambanden

$$\bar{a}_t + \bar{a}_n = \bar{a} \longrightarrow \bar{a}_n = \bar{a} - \bar{a}_t$$

$$\bar{a}_n = (10,-2,-1) - \frac{1}{13}(4,12,3) = \frac{126,-28,-16}{13}\ m/s^2$$

Svar: $\bar{a}_t = \frac{4,12,3}{13}\ m/s^2$; $\bar{a}_n = \frac{126,-38,16}{13}\ m/s^2$

1.7 – Givet: se figuren, $S_{AC} = 7$ kN, $S_{BD} = 5$ kN



Söker: α, vinkel mellan $\bar{S}_{BD}$ & $\bar{S}_{AC}$

$(\bar{S}_{AC})_{AO}$ & $(\bar{S}_{BD})_{AO}$

Lösningsmetod

- vinkel α ← $\bar{S}_{AC}$ & $\bar{S}_{BD}$ ← $\bar{e}_{AC}$ & $\bar{e}_{BD}$ ← $\bar{r}_{AC}$ & $\bar{r}_{BD}$
- $(\bar{S}_{AC})_{AO}$ & $(S_{BD})_{AO}$ ← $(\bar{S}_{AC})_{OA} = \bar{S}_{AC} \cdot \bar{e}_{OA}$ & $(S_{AB})_{OA} = S_{AB} \cdot \bar{e}_{OA}$

Lösning

* Hitta $\bar{e}_{AC}$ & $\bar{e}_{BD}$ & $\bar{e}_{AO}$

$$\bar{e}_{AC} = \frac{\bar{r}_{AC}}{r_{AC}} = \left\{ \bar{r}_{AC} = \begin{array}{c} C \\ \begin{vmatrix} 0 \\ 2a \\ 3a \end{vmatrix} \end{array} - \begin{array}{c} A \\ \begin{vmatrix} 6a \\ 0 \\ 0 \end{vmatrix} \end{array} = \begin{vmatrix} -6a \\ 2a \\ 3a \end{vmatrix} \right\} = \frac{a(-6,2,3)}{\sqrt{(6a)^2+(2a)^2+3a^2}} =$$

$$= \frac{1}{7}(6,2,3)$$

$$\bar{e}_{BD} = \frac{\bar{r}_{BD}}{r_{BD}} = \left\{ \bar{r}_{BD} = \begin{array}{c} D \\ \begin{vmatrix} 0 \\ -4a \\ 0 \end{vmatrix} \end{array} - \begin{array}{c} B \\ \begin{vmatrix} 3a \\ 0 \\ 0 \end{vmatrix} \end{array} = \begin{vmatrix} -3a \\ -4a \\ 0 \end{vmatrix} \right\} = \frac{a(-3,-4,0)}{a\sqrt{9+16}} =$$

$$= \frac{1}{5}(-3,-4,0)$$

$$\bar{e}_{AO} = -\bar{e}_x$$

* Bestäm $\bar{S}_{BD}$ och $\bar{S}_{AC}$

$$\bar{S}_{BD} = S_{BD} \cdot \bar{e}_{BD} = \frac{5\text{ kN}}{5}(-3,4,0) = 1\text{ kN}(-3,-4,0)$$

$$\bar{S}_{AC} = S_{AC} \cdot \bar{e}_{AC} = \frac{7\text{ kN}}{7}(-6,2,3) = 1\text{ kN}(-6,2,3)$$

* Räkna ut α

$$\bar{S}_{AC} \cdot \bar{S}_{BD} = S_{BD} \cdot S_{AC} \cdot \cos\alpha \Leftrightarrow \alpha = \arccos \frac{\bar{S}_{AC} \cdot \bar{S}_{BD}}{S_{AC} \cdot S_{BD}}$$

$$\alpha = \arccos\left(\frac{1 \cdot 1\ (kN)^2}{7 \cdot 5\ (kN)^2} \left| \begin{matrix} -3 \\ -4 \\ 0 \end{matrix} \right| \cdot \left| \begin{matrix} -6 \\ 2 \\ 3 \end{matrix} \right| \right) = \arccos\left(\frac{18-8}{35} \right)$$

$$= \arccos\left(\frac{10}{35} \right) \approx 73{,}39^{0} \approx \underline{\underline{70^{0}}}$$

* Bestäm $(\bar{S}_{AC})_{AO}$ & $(\bar{S}_{BD})_{AO}$

$$(\bar{S}_{AC})_{AO} = \bar{S}_{AC} \cdot \bar{e}_{AO} = kN(-6, 2, 3) \cdot (-1, 0, 0) = \underline{\underline{6\ kN}}$$

$$(\bar{S}_{BD})_{AO} = \bar{S}_{BD} \cdot \bar{e}_{AO} = kN(-3, -4, 0) \cdot (-1, 0, 0) = \underline{\underline{3\ kN}}$$