MIXED SHIMURA VARIETIES AND OTHER ADVANCED TOPICS ABOUT FAMILIES OF MIXED HODGE STRUCTURES

WUSHI GOLDRING

Contents

1. Course name

"Mixed Shimura varieties and other advanced topics about families of Mixed Hodge structures".

2. Responsible teacher/examiner for the course

Wushi Goldring

3. Course Plan

3.1. **Background.** Ever since its introduction, Deligne's theory of mixed Hodge structures – and more generally his conjectural framework of mixed motives – has been fundamental in the development of algebraic geometry. In particular, starting from the pure theory, one important way that mixed objects arise is via degeneration. The bases of natural families of schemes, such as moduli spaces, are often not proper over their own base. It is technically crucial to construct and understand "compactifications" of such moduli spaces. If the original moduli space parameterizes pure Hodge structures or pure motives of some kind, then the hope is that points on the boundary of compactifications will parameterize mixed Hodge structures or mixed motives.

The above general viewpoints has been carried out particularly well in the theory of Shimura varieties, which is closely related to variations of Hodge structure of weight one. Compactifications of (pure) Shimura varieties were understood in terms of families of mixed Hodge structures by Pink in his thesis [8]. In particular, Pink introduced the notions of 'mixed Shimura datum' and 'mixed Shimura variety'. On the one hand, Pink explained why these notions were the natural generalization of Deligne's original notions of 'Shimura datum' and 'Shimura variety' as given in [3]. On the other, Pink showed that the boundary stratification of compactifications of Shimura varieties are naturally understood in terms of mixed Shimura varieties. In particular, the mixed Hodge theory viewpoint allowed Pink to show that mixed Shimura varieties and compactifications of pure Shimura varieties admit 'canonical models' (defined by a universal property, unique up to isomorphism) over their reflex field (a number field).

3.2. **Course Aims.** Our first aim is to understand Pink's notion of mixed Shimura variety. Second, we may consider one of the following generalizations of Pink's work: (1) Compactifications of integral models of Hodge-type Shimura varieties using mixed Shimura varieties, following Madapusi-Pera [7] (2) Klingler's notion [6] of 'mixed Hodge datum' which generalizes 'mixed Shimura varieties' to families of mixed Hodge structure of higher weight. In other words, mixed Hodge data provide the natural mixed framework for mixed Griffiths-Schmid manifolds [4] and their partial compactifications [5].

Date: September 14, 2020.

3.3. **Prerequisites.** It will be assumed that students are comfortable with Deligne's theory of pure Shimura varieties as in [3], and with the basics of mixed Hodge structures as in [1, 2]. Otherwise a student can request approval from the teacher/examiner.

3.4. **Timetable.** Upon approval, the course will meet for two hours every week throughout the term Fall 2020. Given the pandemic, the course will meet on zoom. If the course is successful, we will consider continuing the course during Spring 2021.

3.5. **Examination.**

3.5.1. *Examination Form.* PhD students will be examined based on presentations made to the course. Active participation by PhD students will be required. There will be a discussion during every meeting and it will be expected that students contribute to the discussion during most meetings.

3.5.2. *Grading scale.* Pass/Fail

3.5.3. *Credits.* 7.5HP

References

[1] P. Deligne. Théorie de Hodge. II. *Publ. Math. IHES*, 40:5–57, 1971.

[2] P. Deligne. Théorie de Hodge. III. *Publ. Math. IHES*, 44:5–77, 1974.

[3] P. Deligne. Variétés de Shimura: Interprétation modulaire, et techniques de construction de modèles canoniques. In A. Borel and W. Casselman, editors, *Automorphic forms, representations and L-functions, Part 2*, volume 33 of *Proc. Symp. Pure Math.*, pages 247–289. Amer. Math. Soc., Providence, RI, 1979. Proc. Sympos. Pure Math., Oregon State Univ., Corvallis, OR., 1977.

[4] P. Griffiths and W. Schmid. Locally homogeneous complex manifolds. *Acta. Math.*, 123:253–302, 1969.

[5] K. Kato and S. Usui. *Classifying spaces of degenerating polarized Hodge structures*, volume 169 of *Annals of Math. Studies*. Princeton Univ. Press, 2009.

[6] B. Klinger. Hodge loci and atypical intersections: conjectures. Preprint, available at `https://www2.mathematik.hu-berlin.de/~klingleb/papers.html`.

[7] K. Madapusi. Toroidal compactifications of integral models of Shimura varieties of Hodge type. Preprint, arXiv:1211.1731.

[8] Richard Pink. *Arithmetical compactification of mixed Shimura varieties*, volume 209 of *Bonner Mathematische Schriften [Bonn Mathematical Publications]*. Universität Bonn, Mathematisches Institut, Bonn, 1990. Dissertation, Rheinische Friedrich-Wilhelms-Universität Bonn, Bonn, 1989.

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