

# Operator Theory Analysis and Mathematical Physics

## OTAMP 2022

Hybrid Conference

*Place:* Stockholm University, Kräftriket house 6, room 306

*Zoom:* <https://stockholmuniversity.zoom.us/j/69265888794>

### Organizers:

Fritz Gesztesy (Baylor)  
Jan Janas (Krakow)  
Pavel Kurasov (Stockholm)  
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Annemarie Luger (Stockholm)  
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### Contents

<b>1</b>	<b>Schedule Overview</b>	<b>2</b>
<b>2</b>	<b>Schedule day by day</b>	<b>3</b>
<b>3</b>	<b>Abstracts</b>	<b>7</b>
3.1	Abstracts – Afternoons . . . . .	7
3.2	Abstracts – Young researcher session . . . . .	22
<b>4</b>	<b>List of participants</b>	<b>27</b>
4.1	Speakers . . . . .	27
4.2	Other registered participants . . . . .	28

# 1 Schedule Overview

## OTAMP 2022 - Schedule

Monday 27/6    Tuesday 28/6    Wednesday 29/6    Thursday 30/6

### Young researcher session

08:10-08:25	Kechar
08:30-08:45	Khaldi
08:50-09:05	Matallah
09:10-09:25	Mezghiche
09:30-09:45	Saadi
09:50-10:05	Loucif

10:20-10:35	Saouli
10:40-10:55	Zid
11:00-11:15	Melkemi
11:20-11:35	Djeribia
11:40-11:55	Anissa

12:45-13:00	Opening			
13:00-13:25	Smilansky	Simonov	Behrndt	Breuer
13:30-13:55	Karlsson	Read	Holzmann	Harrell
14:00-14:25	Wood	Kennedy	Rohleder	Exner

14:30-15:00	Closing			
15:00-15:25	Pankrashkin	Denisov	Mantile	
15:30-15:55	Gupta	Liu	Dehimi	
16:00-16:25	Karlovych	Nichols	Eskisehirli	

16:30-17:00				
17:00-17:25	Fischer	Simon	Fillman	
17:30-17:55	Safronov		Trapasso	

## 2 Schedule day by day

### Monday 27/6

- 12:45-13:00 **Opening**
- 13:00-13:25 **Uzy Smilansky**  
*Can one hear a real symmetric matrix?*
- 13:30-13:55 **Anders Karlsson**  
*A new fixed-point theorem with applications to operator theory*
- 14:00-14:25 **Ian Wood**  
*Spectrum of the Maxwell Equations for a Flat Interface between Homogeneous Dispersive Media*
- Break
- 15:00-15:25 **Konstantin Pankrashkin**  
*Asymptotics of Robin eigenvalues on sharp infinite cones*
- 15:30-15:55 **Shubham Gupta**  
*Symmetrization inequalities on one-dimensional integer lattice*
- 16:00-16:25 **Oleksiy Karlovykh**  
*When is the norm of the Riesz projection on a Banach function space equal to one?*
- Break
- 17:00-17:25 **Florian Fischer**  
*Characterisations of Criticality for Quasi-Linear Schrödinger Operators on Graphs*
- 17:30-17:55 **Oleg Safronov**  
*Absolutely continuous spectrum of a typical Schrödinger operator with a slowly decaying potential*

## Tuesday 28/6 Morning: Young researcher session

- 08:10-08:25 **Chahra Kechar**  
*Uniqueness of solutions for nonlinear Hadamard fractional differential equations with nonlocal conditions using fixed point theorem*
- 08:30-08:45 **Aya Khaldi**  
*Stability result for a nonlinear Kirchhoff type reaction-diffusion equation with variable-exponent nonlinearities*
- 08:50-09:05 **Hana Matallah**  
*Existence result for fractional reaction-diffusion problem using in image restoration*
- 09:10-09:25 **Lynda Mezghiche**  
*Periodic positive solutions of an iterative delay differential equations*
- 09:30-09:45 **Chaima Saadi**  
*Existence result for nonlinear fractional problem involving the distributional Riesz derivative*
- 09:50-10:05 **Sami Loucif**  
*Asymptotic behavior of Piezoelectric Beams system with delay*
- Break
- 10:20-10:35 **Nabil Saouli**  
*Approximate Solution for a class of inverse problem*
- 10:40-10:55 **Sohir Zid**  
*The  $\lambda$ -Aluthge transform and EP operator*
- 11:00-11:15 **Oussama Melkemi**  
*Note on Variable Hardy Spaces with general weight*
- 11:20-11:35 **Oussama Djeribia**  
*Positive  $p$ -majorizing operators and some results*
- 11:40-11:55 **Elgues Anissa**  
*On  $n$ -Ep operators on Hilbert space*

## Tuesday 28/6 afternoon

- 13:00-13:25 **Sergey Simonov**  
*Estimates of Green matrix entries of unbounded block Jacobi matrices*
- 13:30-13:55 **Larry Read**  
*Factorisation and Lieb-Thirring type inequalities for perturbed Schrödinger operators*
- 14:00-14:25 **James Kennedy**  
*What geometric structures in a metric graph limit the speed of diffusion?*
- Break
- 15:00-15:25 **Sergey Denisov**  
*Szegő condition, scattering, and vibration of Krein strings*
- 15:30-15:55 **Wencai Liu**  
*Fermi isospectrality for discrete periodic Schrödinger operators*
- 16:00-16:25 **Roger Nichols**  
*Strict Domain Monotonicity of the Principal Eigenvalue and a Characterization of Lower Semiboundedness for the Friedrichs Extension of Four-Coefficient Sturm-Liouville Operators*
- Break
- 17:00-17:55 **Barry Simon**  
*A tale of three coauthors: comparison of Ising models*

## Wednesday 29/6

- 13:00-13:25 **Jussi Behrndt**  
*The generalized Birman-Schwinger principle*
- 13:30-13:55 **Markus Holzmann**  
*Spectral transitions for two dimensional Dirac operators with singular potentials*
- 14:00-14:25 **Jonathan Rohleder**  
*A new approach to the hot spots conjecture*
- Break
- 15:00-15:25 **Andrea Mantile**  
*On the origin of the Minnaert resonances*
- 15:30-15:55 **Souheyb Dehimi**  
*Triangle inequalities for closed linear operators*
- 16:00-16:25 **Beyaz Basak Eskisehirli**  
*Spectral properties of quasi-parabolic composition operators on the Hardy space of the polydisc*
- Break
- 17:00-17:25 **Jake Fillman**  
*The Spectrum of the Doubling Map Model is an Interval*
- 17:30-17:55 **S. Ivan Trapasso**  
*Wave packet analysis of Feynman path integrals*

## Thursday 30/6

- 13:00-13:25 **Jonathan Breuer**  
*Fluctuations of orthogonal polynomial ensembles and recurrence coefficients*
- 13:30-13:55 **Evans Harrell**  
*The diagonal of the heat kernel on metric graphs*
- 14:00-14:25 **Pavel Exner**  
*Quantum graphs:  $\mathcal{T}$ -asymmetry and  $\mathcal{PT}$ -symmetry*
- 14:30 **Closing**

## 3 Abstracts

### 3.1 Abstracts – Afternoons

#### **The generalized Birman-Schwinger principle**

**Jussi Behrndt**

Graz University of Technology

In this talk we discuss a generalized Birman-Schwinger principle in the non-self-adjoint context. In particular, we provide a detailed discussion of geometric and algebraic multiplicities of eigenvalues of the basic operator of interest (e.g., a Schrödinger operator) and the associated Birman-Schwinger operator, and additionally offer a careful study of the associated Jordan chains of generalized eigenvectors of both operators.

This talk is based on a joint paper with Tom ter Elst (Auckland, New Zealand) and Fritz Gesztesy (Baylor, US).

#### **Fluctuations of orthogonal polynomial ensembles and recurrence coefficients**

**Jonathan Breuer**

The Hebrew University of Jerusalem

Orthogonal polynomial ensembles are a special class of point processes that arise naturally in various contexts, such as random matrix theory and combinatorics. The reason for the name is an intimate connection with orthogonal polynomials. In the talk we review some results connecting the asymptotic behavior of orthogonal polynomial ensembles on the real line with properties of the recurrence coefficients of the associated orthogonal polynomials, and sketch the extension of these results to the unit circle case. This is joint work with Daniel Ofner.

## Triangle inequalities for closed linear operators

Souheyb Dehimi

Department of Mathematics, Faculty of Mathematics and Informatics,  
University Mohamed El Bachir El Ibrahimi, Bordj Bou Arréridj, El-Anasser  
34030, Algeria.

The purpose of our works is to investigate when the relations of the types  $|ST| = |S||T|$ ,  $|S \pm T| \leq |S| + |T|$  and  $||S| - |T|| \leq |S \pm T|$  hold in an unbounded operator setting.

We have presented a generalization of the famous Reid inequality related to linear operators. Also, we gave a necessary conditions to make sure that the relation  $|ST| = |S||T|$  holds for closed operators  $S$  and  $T$ .

We also proved the triangle inequalities for closed linear operators. As consequences, we obtain a characterization of invertibility for the class of unbounded normal operators.

## Szegő condition, scattering, and vibration of Krein strings

Sergey Denisov

University of Wisconsin - Madison

The Poisson-finite measure  $\mu$  on the real line belongs to the Szegő class  $\mathbf{Sz}(\mathbb{R})$  if its logarithmic integral converges, i.e.,

$$\int_{\mathbb{R}} \frac{\log \mu'}{1+x^2} dx > -\infty.$$

Measures that satisfy this condition find applications in complex analysis, theory of stationary Gaussian stochastic processes, etc. I will give the characterization of such measures both in terms of the Hamiltonian in canonical system which  $\mu$  generates and in terms of dynamics of the unitary evolution group. The applications to 1d Dirac systems and the theory of vibrating strings will be discussed.

This is a joint work with Roman Bessonov. The work is supported by NSF Grant DMS-2054465 and Van Vleck Professorship research award.

## Spectral properties of quasi-parabolic composition operators on the Hardy space of the polydisc

**Beyaz Basak Eskisehirli**

Istanbul University

In this talk, we give a Fredholm criteria for the operators in the  $C^*$ -algebra generated by certain Toeplitz operators and Fourier multipliers. With help of the obtained results we also completely characterize the essential spectra of quasi-parabolic composition operators on the Hardy spaces of the polydisc. This is joint work with U. Gul.

## Quantum graphs: $\mathcal{T}$ -asymmetry and $\mathcal{PT}$ -symmetry

**Pavel Exner**

Doppler Institute for Mathematical Physics  
and Applied Mathematics, Prague

The focus of this talk are quantum graphs with the vertex coupling violating the time-reversal invariance. As a case study we analyze the simplest situation in which the asymmetry is maximal at a fixed energy. This has an interesting consequence, namely that high-energy scattering depends crucially on the vertex parity; we will demonstrate implications of this fact for spectral and transport properties in several classes of graphs, both finite and infinite periodic ones. We will also discuss other time-asymmetric graphs and identify a class of such couplings which exhibits a nontrivial  $\mathcal{PT}$ -symmetry despite being self-adjoint. The results come from a common work with Marzieh Baradaran, Jiří Lipovský, and Miloš Tater.

# **The Spectrum of the Doubling Map Model is an Interval**

**Jake Fillman**

Texas State University

We consider discrete Schrödinger operators on the half-line with potentials generated by the doubling map and continuous sampling functions. We show that the essential spectrum of these operators is always connected. This result is obtained by computing a suitable subgroup of the range of the Schwartzman homomorphism associated with homotopy classes of continuous maps on the suspension of the standard solenoid and then showing that this subgroup characterizes the topological structure of the spectrum.

# **Characterisations of Criticality for Quasi-Linear Schrödinger Operators on Graphs**

**Florian Fischer**

University of Potsdam

A natural classification of random walks is the one into recurrent and transient ones. This is equivalent to the non-/validity of the Hardy inequality for the energy functional associated with the Laplace operator on the graph. The latter is an abstract inequality between functionals and can be generalised further. The corresponding theory is known as criticality theory. In this talk, we introduce quasi-linear Schrödinger operators on graphs and show many equivalent statements of a Hardy inequality to hold true. If the time permits, we also discuss the methods used in the proofs and the optimality of the corresponding Hardy weight. The talk is based on work in progress.

## **Symmetrization inequalities on one-dimensional integer lattice**

**Shubham Gupta**

Imperial College London

In this talk, I will talk about some new and interesting results on the discrete symmetrization inequalities on integers. In particular, I will prove Polya-Szegö inequality on integers and apply them to prove some power weight Hardy-type inequalities on the half line, that is, non-negative integers.

## **The diagonal of the heat kernel on metric graphs**

**Evans Harrell**

Georgia Institute of Technology

Denoting the heat kernel on a metric graph  $H(t, p, q)$ , we refer to  $h(t, q) := H(t, q, q)$  as its *diagonal*. The diagonal satisfies certain differential equations and trace formulae and has other useful properties. This talk will describe those and use them to give estimates of the full heat kernel, the eigenvalue spectrum, and the localization of eigenfunctions.

This talk is based on a publication with David Borthwick and Kenny Jones and on work in progress with David Borthwick, Anna Maltsev, and Haozhe Yu.

# Spectral transitions for two dimensional Dirac operators with singular potentials

Markus Holzmann

Graz University of Technology

In this talk the self-adjointness and spectral properties of a family of Dirac operators with singular  $\delta$ -shell potentials supported on smooth curves in  $\mathbb{R}^2$  are discussed. For a three-parameter group of coefficients of the singular interaction the self-adjoint realizations are described. It turns out that there is a critical combination of coupling constants for which there is a loss of Sobolev regularity in the domain of definition and a spectral transition occurs. More precisely, if the interaction support is a closed and compact curve, then there is an additional point in the essential spectrum in the critical case. If the interaction support is a straight line, then an interval of continuous spectrum collapses in the critical case to an eigenvalue of infinite multiplicity.

## When is the norm of the Riesz projection on a Banach function space equal to one?

Oleksiy Karlovykh

Universidade NOVA de Lisboa, Portugal

The lower estimate by Gohberg and Krupnik (1968) and the upper estimate by Hollenbeck and Verbitsky (2000) for the norm of the Riesz projection  $P$  on the Lebesgue space  $L^p$  lead to  $\|P\|_{L^p \rightarrow L^p} = 1/\sin(\pi/p)$  for every  $p \in (1, \infty)$ . Hence  $L^2$  is the only space among all Lebesgue spaces  $L^p$  for which the norm of the Riesz projection  $P$  is equal to one. Banach function spaces  $X$  are far reaching generalizations of Lebesgue spaces  $L^p$ . We prove that the norm of  $P$  is equal to one on the space  $X$  if and only if  $X$  coincides with  $L^2$  and there exists a constant  $C \in (0, \infty)$  such that  $\|f\|_X = C\|f\|_{L^2}$  for all functions  $f \in X$ . We also show that if the space  $X$  is separable or rearrangement-invariant, then the norm of  $P$  on  $X$  is equal to one if and only if the norm of the backward shift operator  $S$  on the abstract Hardy space  $H[X]$  built upon  $X$  is equal to one. This is a joint work with Eugene Shargorodsky (King's College London, United Kingdom).

# **A new fixed-point theorem with applications to operator theory**

**Anders Karlsson**

University of Geneva, Uppsala University

For complete metric spaces with a bicombing (which includes all Banach spaces and spaces of nonpositive curvature in a weak sense) every isometry needs to fix a point in a certain metric compactification of the space. This new metric fixed-point theorem accommodates conventionally fixed-point free isometric examples of Kakutani, Edelstein, Alspach, and Prus. It implies a mean ergodic theorem in any Banach space (which classically is not true in general) that in turn implies the usual Carleman-von Neumann ergodic theorem. Finally, we apply the fixed-point theorem together with some geometric arguments to the space of positive operators and show that every bounded, invertible operator of a Hilbert space has an invariant non-constant metric functional.

# **What geometric structures in a metric graph limit the speed of diffusion?**

**James Kennedy**

University of Lisbon

We study the question of whether the presence of whether the presence of certain geometric structures in a metric graph, such as cycles or paths of a given length, is enough by itself to bound from above the smallest nonzero eigenvalue of the Laplacian equipped with standard and/or Dirichlet vertex conditions on that graph.

Intuitively and heuristically, we are asking whether having a long cycle, or edge, or path, or other kind of embedded subgraph, is enough by itself to put a brake on the speed of convergence of a diffusion process to equilibrium, in the tradition of earlier works which showed (for example) that diameter alone is not enough to control this eigenvalue (the spectral gap) in the case of standard conditions only.

Here we will present a number of positive and negative results based on so-called surgery methods and/or careful test function arguments. For example, a lower bound on the girth, i.e. the shortest cycle length (appropriately

modified if Dirichlet vertices are present), is enough to control the smallest eigenvalue if at least one Dirichlet vertex is present, but is not enough to control the spectral gap if all vertices are equipped with standard conditions.

This is based on joint work with Gregory Berkolaiko, Pavel Kurasov and Delio Mugnolo.

## Fermi isospectrality for discrete periodic Schrödinger operators

Wencai Liu

Texas A&M University

Let  $\Delta + V$  be the discrete Schrödinger operator, where  $\Delta$  is the discrete Laplacian on  $\mathbb{Z}^d$  and the potential  $V : \mathbb{Z}^d \rightarrow \mathbb{R}$  is  $\Gamma$ -periodic. We prove three rigidity theorems for discrete periodic Schrödinger operators in any dimension  $d \geq 3$ :

1. if  $V$  and  $Y$  are Fermi isospectral (that is, at some energy level, Fermi varieties of the  $\Gamma$ -periodic potential  $V$  and the  $\Gamma$ -periodic potential  $Y$  are the same), and  $Y$  is a separable function, then  $V$  is separable as well;
2. if potentials  $V$  and  $Y$  are Fermi isospectral and both  $V = \bigoplus_{j=1}^r V_j$  and  $Y = \bigoplus_{j=1}^r Y_j$  are separable functions, then, up to a constant, lower dimensional decompositions  $V_j$  and  $Y_j$  are Floquet isospectral,  $j = 1, 2, \dots, r$ ;
3. if a potential  $V$  and the zero potential are Fermi isospectral, then  $V$  is zero.

In particular, all conclusions in (1), (2) and (3) hold if we replace the assumption “Fermi isospectrality” with a stronger assumption “Floquet isospectrality”.

## On the origin of the Minnaert resonances

Andrea Mantile

Laboratoire de Mathématiques de Reims

It is well known that the presence, in a homogeneous acoustic medium, of a small inhomogeneity (of size  $\varepsilon$ ), enjoying a high contrast of both its mass density and bulk modulus, amplifies the generated total fields. This amplification is more pronounced when the incident frequency is close to the Minnaert frequency  $\omega_M$ . Here we explain the origin of such a phenomenon: at first we show that the scattering of an incident wave of frequency  $\omega$  is described by a self-adjoint  $\omega$ -dependent Schrödinger operator with a singular  $\delta$ -like potential supported at the inhomogeneity interface. Then we show that, in the low energy regime (corresponding in our setting to  $\varepsilon \ll 1$ ) such an operator has a non-trivial limit (i.e., it asymptotically differs from the Laplacian) if and only if  $\omega = \omega_M$ . The limit operator describing the non-trivial scattering process is explicitly determined and belongs to the class of point perturbations of the Laplacian. When the frequency of the incident wave approaches  $\omega_M$ , the scattering process undergoes a transition between an asymptotically trivial behaviour and a non-trivial one. (In collaboration with: A. Posilicano and M. Sini)

## Strict Domain Monotonicity of the Principal Eigenvalue and a Characterization of Lower Semiboundedness for the Friedrichs Extension of Four-Coefficient Sturm-Liouville Operators

Roger Nichols

University of Tennessee at Chattanooga

Abstract text.

# Asymptotics of Robin eigenvalues on sharp infinite cones

Konstantin Pankrashkin

Carl von Ossietzky University of Oldenburg

Let  $\Omega \subset \mathbb{R}^d$  be an open set, with sufficiently regular (for example, Lipschitz) boundary, either bounded or with suitable behavior at infinity, and  $\alpha > 0$ . Denote by  $T_\alpha^\Omega$  the Laplacian,  $T_\alpha^\Omega : u \mapsto -\Delta u$ , acting on suitable functions  $u$  defined in  $\Omega$  and satisfying the Robin boundary condition  $\partial_\nu u = \alpha u$  on  $\partial\Omega$ , where  $\partial_\nu$  is the outer normal derivative. Many authors studied the asymptotic behavior of the spectrum of  $T_\alpha^\Omega$  as  $\alpha$  tends to  $+\infty$ , in particular, it is shown under rather general assumptions that  $\inf \sigma(T_\alpha^\Omega) = \alpha^2 \inf_{x \in \partial\Omega} \inf \sigma(T_1^{U_x}) + o(\alpha^2)$ , where  $U_x$  denotes the so-called tangent cone to  $\Omega$  at  $x$ . Therefore, the Robin eigenvalues on infinite cones play a central role in the study of much more general domains.

Let  $\omega \subset \mathbb{R}^n$  be a bounded domain with Lipschitz boundary. For  $\varepsilon > 0$  and  $n \in \mathbb{N}$  consider the infinite cone

$$\Omega_\varepsilon := \{(x_1, x') \in (0, \infty) \times \mathbb{R}^n : x' \in \varepsilon x_1 \omega\} \subset \mathbb{R}^{n+1}$$

and the associated Robin Laplacian  $T_\alpha^{\Omega_\varepsilon}$ . The dependence of the eigenvalues on the geometric parameter  $\varepsilon$  was previously addressed for  $n = 1$  only (in that case, the only admissible  $\omega$  are finite intervals and the resulting  $\Omega_\varepsilon$  are infinite sectors). In the present talk we discuss arbitrary dimensions  $n \geq 2$  and arbitrarily shaped “cross-sections”  $\omega$  and look at the spectral asymptotics as  $\varepsilon$  becomes small, i.e. as the cone becomes “sharp” and collapses to a half-line. It turns out that the main term of the asymptotics of individual eigenvalues is determined by the single geometric quantity

$$N_\omega := \frac{\text{Vol}_{n-1} \partial\omega}{\text{Vol}_n \omega}.$$

More precisely, for any fixed  $j \in \mathbb{N}$  and  $\alpha > 0$  the  $j$ th eigenvalue  $E_j(T_\alpha^{\Omega_\varepsilon})$  of  $T_\alpha^{\Omega_\varepsilon}$  exists for all sufficiently small  $\varepsilon > 0$  and satisfies

$$E_j(T_\alpha^{\Omega_\varepsilon}) = -\frac{N_\omega^2 \alpha^2}{(2j + n - 2)^2 \varepsilon^2} + O\left(\frac{1}{\varepsilon}\right) \text{ as } \varepsilon \rightarrow 0^+.$$

Joint work with Marco Vogel (Oldenburg).

# **Factorisation and Lieb-Thirring type inequalities for perturbed Schrödinger operators**

**Larry Read**

Imperial College London

The goal of the talk is to discuss the factorisation scheme for Schrödinger operators and its use for obtaining Lieb-Thirring type bounds. In one dimension, under certain conditions on the potential, we find an inequality comparing the eigenvalues of the perturbed and non-perturbed operator.

## **A new approach to the hot spots conjecture**

**Jonathan Rohleder**

Stockholm University

It is a conjecture going back to J. Rauch (1974) that the hottest and coldest spots in an insulated homogeneous medium such as an insulated plate of metal should converge to the boundary, for “most” initial heat distributions, as time tends to infinity. This so-called hot spots conjecture can be phrased alternatively as follows: the eigenfunction(s) corresponding to the first non-zero eigenvalue of the Neumann Laplacian on a Euclidean domain should take its maximum and minimum on the boundary only. This has been proven to be false for certain domains with holes, but it was shown to hold for several classes of simply connected or convex planar domains. One of the most recent advances is the proof for all triangles given by Judge and Mondal (Annals of Math. 2020). The conjecture remains open in general for simply connected or at least convex domains. In this talk we provide a new approach to the conjecture. It is based on a non-standard variational principle for the eigenvalues of the Neumann and Dirichlet Laplacians.

# Absolutely continuous spectrum of a typical Schrödinger operator with a slowly decaying potential

Oleg Safronov

University of North Carolina at Charlotte

We consider a family of operators  $-\Delta + \alpha V$  depending on a real parameter  $\alpha$ . We find conditions on the rate of the decay of  $V$  at the infinity guaranteeing that the absolutely continuous spectrum of this operator fills the positive half-line for almost every value of the parameter  $\alpha$ .

1. One set of such conditions requires  $V$  to be of the form

$$V(x) = \sum_{n \in \mathbb{Z}^d} v_n \omega_n \chi(x - n),$$

where  $\chi$  is the characteristic function of the unit cube  $[0, 1]^d$ , the factors  $\omega_n$  are bounded identically distributed independent random variables with zero expectations, and  $v_n$  satisfy

$$\sum_{n \neq 0} \frac{v_n^2}{|n|^{d-1}} < \infty, \quad d \geq 3.$$

2. Another theorem requires that

$$\int_{\mathbb{R}^d} \frac{|\nabla W|^2}{|x|^{d-1}} dx < \infty,$$

where  $W$  is defined in polar coordinates by

$$W(r, \theta) = \int_0^r V(\rho, \theta) d\rho.$$

## A tale of three coauthors: comparison of Ising models

Barry Simon

California Institute of Technology, USA

On Friday, Jan 14, I had a draft of a single author paper intended for the Lieb Festschrift. Six days later, the paper had three coauthors. This talk will explain the interesting story, expose some underlying machinery and sketch the proof of a lovely inequality on certain finite sums.

## Estimates of Green matrix entries of unbounded block Jacobi matrices

Sergey Simonov

St. Petersburg Department of Steklov Institute of Mathematics of the Russian Academy of Sciences; St. Petersburg State University; Alferov Academic University of the Russian Academy of Sciences

We consider a class of block Jacobi matrices

$$J = \begin{pmatrix} B_1 & A_1 & 0 & 0 & \cdots \\ A_1^* & B_2 & A_2 & 0 & \cdots \\ 0 & A_2^* & B_3 & A_3 & \cdots \\ 0 & 0 & A_3^* & B_4 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad (1)$$

for which sequences of norms of the entries  $\{\|A_n\|\}_{n=1}^\infty$  and  $\{\|B_n\|\}_{n=1}^\infty$  can be unbounded and such that the operator is self-adjoint on its maximal domain. We prove the estimate of norms of Green matrix (resolvent) entries

$$\|\{(J - \lambda I)^{-1}\}_{nk}\| \leq C(\lambda, \gamma) e^{-\gamma \sum_{l=\min\{n,k\}}^{\max\{n,k\}-1} \|A_l\|^{-1}}, \quad (2)$$

which depends on the rate of growth of norms of off-diagonal entries and on distance from the spectral parameter to the essential spectrum, if the latter is non-empty,  $\gamma \in (0, \text{dist}(\lambda, \sigma_{\text{ess}}(J))/2)$ . We will also discuss sharpness of the estimate and relations to former results. The talk is based on joint work with S. Naboko.

# Can one hear a real symmetric matrix?

Uzy Smilansky

The Weizmann institute, Rehovot, Israel

The question asked in the title is addressed from two points of view: First, we show that providing enough (term to be explained) spectral data, suffices to reconstruct uniquely *generic* (term to be explained) matrices. The method is well defined but requires somewhat cumbersome computations. Second, restricting the attention to banded matrices with band-width much smaller than the dimension, one can provide more spectral data than the number of unknown matrix elements. We make use of this *redundancy* to reconstruct *generic* banded matrices in a much more straight-forward fashion where the ?cumbersome computations? become unnecessary. Explicit criteria for a matrix to be in the non-generic set are provided.

## Wave packet analysis of Feynman path integrals

S. Ivan Trapasso

Università degli Studi di Genova (Italy)

The Feynman path integral formulation of quantum mechanics is universally recognized as a milestone of modern theoretical physics. Roughly speaking, the core principle of this picture provides that the integral kernel of the time-evolution operator shall be expressed as a “sum over all possible histories of the system”. In spite of the suggestive heuristic insight, the quest for a rigorous theory of Feynman path integrals is far from over, as evidenced by the wide variety of mathematical approaches developed over the last seventy years.

Among the several proposed frameworks, the closest one to Feynman’s original intuition is probably the time-slicing approximation due to E. Nelson. In short, if  $U(t)$  is the Schrödinger time evolution operator with Hamiltonian  $H = H_0 + V$  (free particle plus a suitable potential perturbation), then the Trotter product formula holds for all  $f \in L^2(\mathbb{R}^d)$ :

$$U(t)f = e^{-\frac{i}{\hbar}t(H_0+V)}f = \lim_{n \rightarrow \infty} E_n(t)f, \quad E_n(t) = \left( e^{-\frac{i}{\hbar} \frac{t}{n} H_0} e^{-\frac{i}{\hbar} \frac{t}{n} V} \right)^n.$$

Integral representations for the approximate propagators  $E_n(t)$  can be derived, so that the Trotter formula allows one to give a precise meaning to path integrals by means of a sequence of integral operators.

Notwithstanding the convergence results in suitable operator topologies, a closer inspection of Feynman's writings suggests that his original intuition underlay the much more difficult and widely open problem of the pointwise convergence of the integral kernels of the approximation operators  $E_n(t)$  to that of  $U(t)$ . We recently addressed this problem by means of function spaces and techniques arising in the context of harmonic analysis. The toolkit of Gabor wave packet analysis has been fruitfully applied to the study of path integrals only in recent times, leading to promising outcomes.

We exploit techniques of Gabor analysis of pseudodifferential operators to prove that the problem of pointwise convergence has a positive answer under the previous assumptions. Precisely, we prove stronger convergence results which imply uniform convergence on compact subsets for the integral kernels in the Trotter formula.

We will also discuss the issue of rates of convergence for such results, obtained with a modification of the Trotter approximate propagators.

## **Spectrum of the Maxwell Equations for a Flat Interface between Homogeneous Dispersive Media**

**Ian Wood**

University of Kent

We determine and classify the spectrum of a non-selfadjoint operator pencil generated by the time-harmonic Maxwell problem with a nonlinear dependence on the frequency. More specifically, we consider a one-dimensional reduction for the case of two homogeneous materials joined at a planar interface. The dependence on the spectral parameter, i.e. the frequency, is in the dielectric function and we make no assumptions on its form. In order to allow also for non-conservative media, the dielectric function is allowed to be complex, yielding a non-selfadjoint problem, for which the various standard types of essential spectra do not coincide.

This is joint work with Malcolm Brown (Cardiff), Tomas Dohnal (Halle) and Michael Plum (Karlsruhe).

## 3.2 Abstracts – Young researcher session

### On n-Ep operators on Hilbert space

**Elgues Anissa**

University of Batna 2, Algeria

Let  $T$  be a bounded linear operator with closed range on a complex Hilbert space  $H$ . In this work we introduce the class of n-Ep operators, denoted [n-Ep], satisfying  $T^n T^+ = T^+ T^n$  for  $n \in N$ . We give some properties of these operators in general, secondly we extend the Kaplansky theorem and the Fuglede-Putnam commutativity theorem for normal operators to n-Ep operators.

### Positive p-majorizing operators and some results

**Oussama Djeribia and Amar Belacel**

University of Laghouat - Algeria

In our presentation, which falls within the theory of p-summability the linear case, we study the concept of majorizing operators and disjoint p-summing operators and the duality relationships between them, which was introduced by Dongyang Chen, Amar Belacel, Javier Alejandro Chávez-Domínguez in their work Chen, D., Belacel, A., Chávez-Domínguez, J. A. (2021). Positive p-summing operators and disjoint p-summing operators. Positivity, 25(3) (2021), 1045 – 1077. We prove Pietsch's domination theorem using Ky Fan lemma in the linear case.

# Uniqueness of solutions for nonlinear Hadamard fractional differential equations with nonlocal conditions using fixed point theorem

Chahra Kechar

Department of Mathematics and Informatics, University of Souk Ahras,  
Algeria

Fractional differential equations arise from a variety of applications including in various fields of science and engineering. In particular, problems concerning qualitative analysis of fractional differential equations have received the attention of many authors.

Fractional differential equations involving Riemann-Liouville and Caputo fractional derivatives have been studied extensively by several researchers. However, the literature on Hadamard differential equations is not yet as enriched.

The aim of this paper is to prove the existence and uniqueness of solutions for a nonlinear implicit Hadamard fractional differential equation with nonlocal conditions in a weighted Banach space. Our results are based on the Banach and Krasnoselskii fixed point theorems. An example is given to illustrate our obtained results.

## Stability result for a nonlinear Kirchhoff type reaction-diffusion equation with variable-exponent nonlinearities

Aya Khaldi

Department of mathematics, Laboratory of Applied Mathematics and History and Didactics of Mathematics (LAMAHS). University of 20 August 1955, Skikda, Algeria.

We consider a class of Kirchhoff type reaction-diffusion equations with variable exponents and source terms

$$u_t - M \left( \int_{\Omega} |\nabla u|^2 dx \right) \Delta u + |u|^{m(x)-2} u_t = |u|^{r(x)-2} u,$$

We prove with suitable assumptions on the variable exponents  $r(\cdot), m(\cdot)$  the stability result using potential with small positive initial energy, the stability being based on Komornik's inequality.

## **Asymptotic behavior of Piezoelectric Beams system with delay**

**Sami Loucif**

University of Larbi Tebessi, Tebessa, Algeria

Our work will be related to the one-dimensional system of piezoelectric beams with distributed delay on the mechanical equation. By using Semigroups theory, we prove that this system accepts only one solution. Next, we find the energy expression related to this system, and by using technique of Lyapunov functional we demonstrate that this system is exponentially stable.

## **Existence result for fractional reaction-diffusion problem using in image restoration**

**Matallah Hana, Messouad Maouni and Lakhal Hakim**

Laboratory of Applied Mathematics and History and Didactics of Mathematics "LAMAHS", Department of Mathematics, University 20 August 1955 Skikda, Algeria

This work propose new nonlinear parabolic reaction-diffusion model of fractional order, where the study is a generalization of the work proposed by Nourddine Alaa in 2014 in which we apply the fractional derivative in the sense of the Caputo. This is based on the restoration of digital image such that a digital result is given on a noisy image in which this model is found to be effective in eliminating noise.

## **Note on Variable Hardy Spaces with general weight**

**Oussama Melkemi**

University Batna 2

In this paper, we introduce and explore the weighted Hardy spaces with variable exponent on domains, where we establish the atomic decomposition for this kind of space. Furthermore we study the geometric-weighted Hardy  $n$  spaces with variable exponent on bounded Lipschitz domain  $\mathbb{D}$  of  $\mathbb{R}$ .

## **Periodic positive solutions of an iterative delay differential equations**

**Lynda Mezghiche**

University of 20 August 1955, Skikda, Algeria

In this work, we are interested in proving the existence and uniqueness of periodic positive solutions for a class of first-order iterative delay differential equations arising in population dynamics with harvesting term. By virtue of Schauder's fixed point theorem and some useful properties of Green's functions method we establish the existence of periodic positive solutions and under a Banach contraction principle we obtain the uniqueness of the solution. Our results complement some previous ones in the literature.

## **Existence result for nonlinear fractional problem involving the distributional Riesz derivative**

**Chaima Saadi**

LAMAHIS, University of 20 August 1955, Skikda, Algeria

In this work, we study the existence of weak solution for non-linear fractional problem involving the distributional Riesz derivative in a fractional Sobolev space. we establish existence results by the application of the Schauder fixed point theorem with some condition on the nonlinear terms.

## Approximate Solution for a class of inverse problem.

Nabil Saouli

University of Badji Mokhtar Annaba, Algeria

This paper deals with the problem of determining an unknown source and an unknown initial condition in a abstract final value parabolic problem. This problem is ill-posed in the sense that the solutions do not depend continuously on the data. To solve the considered problem a modified Tikhonov regularization method is proposed. Using this method regularized solutions are constructed and under boundary conditions assumptions, convergence estimates between the exact solutions and their regularized approximations are obtained. Moreover numerical results are presented to illustrate the accuracy and efficiency of the proposed method.

## The $\lambda$ -Aluthge transform and EP operator

Sohir Zid and Safa Menkad

University of Batna 2, Batna, Algeria

**Abstract:** Let  $T \in \mathcal{B}(\mathcal{H})$  be a bounded linear operator on a Hilbert space  $\mathcal{H}$ , and let  $T = U|T|$  be the polar decomposition of  $T$ . For any  $\lambda \in [0, 1]$ , the  $\lambda$ -Aluthge transform of  $T$  is defined by  $\Delta_\lambda(T) = |T|^\lambda U |T|^{1-\lambda}$ . In this paper, we investigate when an operator and its  $\lambda$ -Aluthge transform both are EP.

## 4 List of participants

### 4.1 Speakers

Elgues	Anissa	Batna 2 Univ	Batna, Algeria
Jussi	Behrndt	Graz University of Mathematics	Graz, Austria
Jonathan	Breuer	The Hebrew University of Jerusalem	Jerusalem
Souheyb	Dehimi	University of Mohamed El Bachir El Ibrahimi	Bordj Bou Arréridj, Alg
Sergey	Denisov	University of Wisconsin - Madison	Madison, USA
Oussama	Djeribia	Jelfa Univ	Djelfa, Algeria
Beyaz Basak	Eskisehirli	Istanbul University	Istanbul, Turkey
Pavel	Exner	Czech Technical University	Prague, Czechia
Jake	Fillman	Texas State University	San Marcos, USA
Florian	Fischer	University of Potsdam	Potsdam, Germany
Shubham	Gupta	Imperial college London	London, UK
Evans	Harrell	Georgia Tech	Atlanta, GA, USA
Markus	Holzmann	Graz University of Technology	Graz, Austria
Oleksiy	Karlovyeh	University Lisbon	Lisbon, Portugal
Anders	Karlsson	Univ. Geneva, Univ. Uppsala	Geneva, Switzerland
Chahra	Kechar	Souk Ahras University	Souk Ahras, Algeria
James	Kennedy	Universidade de Lisboa	Lisbon, Portugal
Aya	Khalidi	University of 20 august 1955	Skikda, Algeria
Wencai	Liu	Texas A&M University	College Station, USA
Sami	Loucif	Larbi Tebessi University	Tebessa, Algeria
Andrea	Mantile	Université de Reims	Reims, France
Hana	Matallah	University of 20 August 1955,	Skikda, Algeria
Oussama	Melkemi	Batna 2 University	Batna, Algeria
Lynda	Mezghiche	University of 20 August 1955	Skikda, Algeria
Roger	Nichols	University of Tennessee at Chattanooga	Chattanooga, US
Konstantin	Pankrashkin	University of Oldenburg	Oldenburg, Germany
Larry	Read	Imperial College	London
Jonathan	Rohleder	Stockholm University	Stockholm, Sweden
Chaima	Saadi	University of 20 August 1955	Skikda, Algeria
Oleg	Safronov	University of North Carolina at Charlotte	USA
Nabil	Saouli	Badji Mokhtar Univ	Annaba, Algeria
Barry	Simon	California Institute of Technology	California, USA
Sergey	Simonov	St. Petersburg Department of V. A. Steklov Institute of Mathematics of the Russian Academy of Sciences	St. Petersburg, Russia
Uzy	Smilansky	The Weizmann institute	Rehovot, Israel
S. Ivan	Trapasso	University of Genoa	Genova, Italy
Ian	Wood	University of Kent	Canterbury, UK
Sohir	Zid	University of Batna 2	Batna, Algeria

## 4.2 Other registered participants

Aftab	Ali	Lahore University of Management Sciences	Lahore, Pakistan
YAGOUB	Ameur	Laghouat university ( Algeria)	Laghouat, Algeria
Meraou Mohammed	Amine	Djilali Liabes University	Sidi Bel Abbes, Algeria
Jockum	Aniansson	KTH	Stockholm, Sweden
Sergei	Avdonin	University of Alaska Fairbanks	Fairbanks, USA
Sabah	Baibeche	Université 20 Août 1955 Skikda	Skikda, Algeria
Meriem	Benaoued	Abdelhamid Iben Badis University	Mostaganem, Algeria
Kamel	Benyettou	Abdelhamid Ibn Badis of Mostaganem	Mostaganem, Algeria
Wissem	Boughamsa	20 August 1955 Skikda University	Skikda, Algérie
Abir	Bounaama	20 August 1955	Skikda, Algeria
Abir	Chaouche	Badji Mokhtar Annaba University	Annaba, Algeria
Omar	Choucha	University Kasdi Merbah Ouargla	Ouargla, Algeria
Wissem	Chougar	Laarbi Tebessi university	Tèbessa, Algeria
Zizai	Cui	University of Chicago	Chicago, United States
Fatima Siham	Djeradi	Laghouat Univ.	Laghouat, Algeria
Guedjiba Djalal	Eddine	University of Batna2	Batna, Algeria
Souilah	fairouz	University 20th August 1955	Skikda, Algeria
Fritz	Gesztesy	Baylor University	Waco, TX, USA
Amina	Hallal	Mohamed Cherif Messaadia University	Souk ahras, Algeria
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Marwa	Khemis	University of 20 August 1955	Skikda, Algeria
Rostyslav	Kozhan	Uppsala University	Uppsala, Sweden
Pavel	Kurasov	Stockholm University	Stockholm, Sweden
Ari	Laptev	Imperial College London	London, UK
Annemarie	Luger	Stockholm University	Stockholm, Sweden
HASSAN	MESSAOUDI	Mohamed-Cherif Messaadia Univ.	Souk Ahras, Algeria
Dinh Thi	NGUYEN	ENS Lyon	Lyon, France
Andrea	Posilicano	Università dell'Insubria	Como, Italy
Mario	Ruiz	National Autonomous University of Mexico	Mexico City, Mexico
Kushagra	Sachan	Indian Institute of Science (BHU), Varanasi	Varanasi, India
Christian	Seifert	Technische Universität Hamburg	Hamburg, Germany
Selim	Sukhtaiev	Auburn University	Auburn, Alabama, USA
Muhammad	Usman	Lahore University of Management Sciences	Lahore, Pakistan
Marcus	Vaknäs	Uppsala University	Uppsala, Sweden
Udit	Varma	IISER Bhopal	Bhopal
Fares	Yazid	Laghouat Univ.	Laghouat, Algeria