Abstract

Renormalization is a powerful technique showing up in different contexts of mathematics and physics. In the context of circle diffeomorphisms, the renormalization operator is a dynamical system, which acts like a microscope and allows us to study the dynamics of a circle diffeomorphism on a small scale. The convergence of renormalization leads to a proof of the so-called rigidity theorem, which classifies the dynamics of circle diffeomorphisms geometrically: the conjugacy between $\mathcal{C}^3$ circle diffeomorphism with Diophantine rotation number and the corresponding rotation is $\mathcal{C}^1$.

In this thesis, we define the renormalization of circle diffeomorphisms and study its dynamics. In particular, we prove that the renormalization of orientation preserving $\mathcal{C}^3$ circle diffeomorphisms with irrational rotation number of bounded type converges to rotations at exponential speed. We also introduce the necessary relevant concepts such as rotation number, distortion and non-linearity and discuss some of their properties.

This thesis is a summary and supplement to the book *One-Dimensional Dynamics: From Poincaré to Renormalization.*

Keywords: circle diffeomorphism; rotation number; renormalizaton; rigidity.