AN APPLICATION OF GREY SYSTEM THEORY INTO
REAL ESTATE INVESTMENT DECISION-MAKING

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Master of Science Thesis
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Abstract

Real estate investment decision-making is an important part of project development, which directly affects the risk level of real estate project development and the return on investment size. Real estate investment has features of high risks and high uncertainty. The failure of real estate investment will not only affect the enterprises’ survival and development, and may even affect the entire national economy. Therefore, increasing in scientificity in decision-making of real estate enterprises investment is extremely essential and necessary.

Grey system theory (Julong.D 1990) is a new approach to study on little data, uncertainty and poor information. It takes "small samples" and "poor information"
uncertainty system as research objects, which has characteristics of "Partly known information, partly unknown information". And mainly through generating, developing and taking useful information from “known information” to realize effective control on gray system operation and the accurate evolution rule.

In this paper, I will introduce grey system theory into real estate investment area, build appropriate mathematical model to have detail analysis on the main decision factors in the field of real estate investment such as where and what kind of property to invest and what is the optimal amount to invest. Here the methods will be used include Grey clustering analysis, Grey relation analysis and GM(1,1) forecasting model.

The paper first talk about the meaning and significance behind real estate investment and then have a detail introduction about grey system theory, how grey has applied to other areas. And the comparison between grey system and conventional decision model will be explored to see the advantages existing in grey system theory. Afterwards accordingly build a gray model and apply a real case into this model to prove its feasibility.
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1 INTRODUCTION

1.1. Background of Research Study

Real estate means the industry engaged in real estate development, operation and management, which plays an extremely important role in the national economy. Real estate is closely related to people’s standard of living. Comfortable living and happy working are the two prime preconditions to guarantee the harmony of society. In another words, having a house is related to the basic security of people's lives; living in good, comfortable environment is related to people's quality of life, and therefore the government's housing policy and real estate development policies have much to do with the social development goals. Real estate industry is an important pillar of national economy. It plays an important role in the development of industries of iron and steel, building materials and household appliances which have significant contribution to the stability and development of financial industry and the whole society.

In the last few decades, real estate had experienced dramatic development. It has become one of strong economic power in many countries. Just in a short time, many buildings spring up even though we already have so many in the market. Because people and developer saw the consistently going up of property price and they believe such an important industry will have to get support from the country, in consequence, people struggle to invest in real estate hoping that they can share some benefits from it. Wildcat and non-scientific investment in real estate leads to imbalance of supply and demand in real estate market which can form a serious financial bubble. Once this bubble finally crashes, it will cause a huge impact and fluctuation in the whole economy. Like Dubai World, its “Dubai dream” or “luxury heaven” collapses over night without any warning. So we should have a more scientific and accurate method to make investment decision in order to avoid wildcat and sense less investment.
Real estate investment decision-making process is a complex process, in this process, policy-makers face a number of risk factors of uncertainty, and the information that the decision-makers can take advantage of is not sufficient. How to use limited information to make the most accurate decision that is the problem policy-makers is facing. In order to improve the scientificity of real estate investment and ensure the healthy and fast development of real estate market, a more scientific and practical decision-making method is needed for investors.

1.2 Main Purpose

Gray system theory have been applied to many other areas such as in Hydropower Project, Construction Project and Power prediction, but not yet too much study on real estate sector. In this paper we try to build a gray model which fits into real estate business and to see its feasibility and how it works in real estate sector.

1.3 Literature Review

Real estate investment development has the characteristics of huge investment, long production cycle and slow cash flow, but is also constrained to geographical, economic, policy and many other factors, which make it have great risks. Therefore, the decision-making will be the key to be successful. For the investors themselves, and even the country's economic development project, both are important. There already have been many papers or theories to study on real estate investment decision.

In the current real estate market, the most commonly used investment decision-making method is discount-cash-flow (Dixit, 1994). It includes Net Present Value and Internal Rate of Return. However, these two methods have certain
constrains that they both study on certainty elements involving in a project. And in real case, it is very difficult to define the net operating income and discount rate of each year. Developers have to do either optimistic or pessimistic choice in order to keep on or just give up (Myers, 2001). But real estate investment itself is an uncertainty process. All the investment determinants keep changing with the change of external environment and the change of government policy such as product price, cost, interest rate and other relevant market information. Hence NPV and IRR do not take changing in investment process into consideration, which cannot give an accurate investment decision.

Later on than this certainty investment method, single-objective decision-making model is introduced and widely used by investors. Kaiyun.Y (2006) puts forward the concept of marginal profit and applied into computer models to select the optimal project with the biggest profit margin. But real estate investment decision-making is actually a multi-objective decision-making problem. We have to consider all aspects such as the amount, place and type of properties instead of just making a decision on whether or not to make an investment. Therefore, a more advanced investment analysis method should be introduced in order for investors to make a more accurate and right investment decision.

In the literature, grey system model has been widely used and debated in many other areas. Hongjian.G(2005) has applied this method in investment scale control of urban construction process. Guoping.W (2006) again takes advantage of this method in construction. He made a noise prediction during the construction process to effectively control the impact of noise on the local people’s life. But very few studies focus on real estate investment. The prediction of property price (Dajiang.L, 2004) is the only literature we usually see related to Grey Forecasting model. So here I am going to make deeper application of grey system theory in real estate investment in order to provide a better and an alternative investment method for investors.
2 CONVENTIONAL METHOD IN REAL ESTATE INVESTMENT

2.1 Current situation and characteristics of real estate development investment

(1) Real estate development project has the characteristic of investment irreversibility, because real estate has a bad liquidity and once a property has been built, it is not removable. In another word, once investment on real estate is formulated, it is hard to retrieve your money and everything will become sunk cost. For the construction that already been built, it is difficult to cash your constructions because of its rather high costs (Zeng, 2004).

(2) There are a lot of risks and uncertain factors existing in real estate investment, which includes financial risk, market risk, interest rate risk, natural surrounding and market competition etc. Hence, it is almost impossible to estimate each year’s cash flow and discount rate during the whole project period. The real cash flow and discount rate will change as the whole financial investment environment and market environment changes.

(3) Real estate development project has the characteristic of flexible management and operation, which namely is soft management. Developers can adjust their project strategy, and even adjust their property structure and property scale according to their competitors’ behavior in order to occupy a dominant position.

(4) Real estate development investment can be delayed. Developer can have flexible decision on if start, when to start and what is the project scale etc. these relevant decisions by virtue of market situation in order to avoid the loss caused by giving up project or a rush starting.

(5) The real value of real estate development project lies not only on its net cash flow, also we have to consider about its strategic value and coordination with other projects deriving from it.
2.2 Conventional investment decision

The conventional real estate development investment decision analyzes on the basis of discounted cash flow (DCF). It mainly works by evaluating projects through the discounted value of cash flow generated during the development process. And decide if we go into investment according to the size of net present value. It includes net present value (NPV) and internal rate of return (IRR) (Dixit & Dick, 2002) and this method has the following assumptions:

(1) Investment is reversible. Developers can retrieve their investment through cash input of the project. Or they can get money back when the market situation is not good for investment.

(2) The income generated from investment is predictable or can be sure. Investors are able to have accurate prediction of each year’s cash flow during the period of project and determine its rate of return and risk-adjusted value. In this case we can calculate all the cash flow generated by this project, investment risk and its investment rate of return.

(3) Investors can only adopt rigid investment strategy, not take the influence of delaying investment strategy on expected return into consideration. That means investors mainly work according to the principle of net present value method. When the NPV is bigger than zero, we invest, and instead if NPV is smaller than zero, we refuse to invest. In terms of if we will be in the same situation by delaying investment, NPV will not take it into consideration.

(4) During the whole project life cycle, we have some certain assumptions on expected changes in terms of the investment’s internal and external environment. If it exceeds the expected changes we have assumed when we were in the process of decision, NPV will not take the changes out of expected ones into consideration. There is no soft management in this method.

(5) The project valued by net present value method is independent. It does not have correlation effects between valued project and one another project and does not
have influence between valued project and corporate strategy.

2.2.1 Net Present Value (NPV)

Net present value works by discounting each year’s cash flow generated during the whole project period to the beginning of investment calculation by a given discount rate, and sum up all the discounted values. Its mathematic formula is as follows:

\[
\text{NPV} = \sum_{t=0}^{n} (\text{CI} - \text{CO})_t \left(1 + i_e\right)^{-t}
\]

Where, CI represents the cash flowing into the project during real estate development project life cycle; CO represents the cash flowing out; (CI-CO)_t represents the cash flow in year t; n represents real estate project cycle; i_e represents the benchmark yield or the expected discount rate in the field of real estate. There are two kinds of situation about the net present value from development project calculation: NPV $> 0$ or NPV $< 0$. When NPV $> 0$, it means real estate project can get all the investment back on the basis of benchmark yield or acquire benefits after getting all the investment back. So the project is feasible, otherwise it is not feasible.

2.2.2 Internal rate of return (IRR)

IRR means to get the discount rate when net present value equals to zero during the real estate project cycle, which reflects the biggest expected rate of return in an investment project. And its formula is:

\[
\sum_{t=0}^{n} (\text{CI} - \text{CO})_t \left(1 + \text{IRR}\right)^{-t} = 0
\]

Once we get the IRR, we can compare the IRR with the benchmark yield or expected discount rate i_e in real estate sector. If IRR $> i_e$, we believe that development project is profitable, otherwise it is not. Generally speaking, the bigger the IRR is, the much bigger ability of acquiring benefits is.
2.3 The limitation of conventional real estate investment decision

In theory the conventional real estate investment method presented by NPV and IRR is feasible. However it has many disadvantages in real application. First, real estate development is irreversible and it is conflicts with the reversible assumption of net present value. Second, it is hard to have accurate prediction of cash flow and the changes of internal and external environment in the project because of high risks and market uncertainty in real estate development (Zhang, 2003). Real estate development investment has huge risks, net present value measures risks through discount rate. Hence investors usually choose comparatively big discount rate and accordingly reduce the economic value of investment project, which mostly cause investors miss a good invest opportunity (Zeng, 2004). The current investment theory believes that uncertainty is an opportunity, not threat. Because uncertainty may not only cause loss, also it may bring higher benefits, thereby this opportunity is valuable. Third, the investment decision valued by net present value method is singleness. It does not take the flexibility of operation into consideration and it ignores the characteristics of delaying and soft management in real estate development. Actually decision-maker can adjust operation strategy with the changes of markets in order to improve cash flow in a project. But conventional investment method is not able to reflect this advantage. The disadvantages that exist in conventional decision method mean that cannot scientifically reflect the real operation situation of investment capital in a real estate development project (Liu 2002). Hence it is difficult to meet the real requirements of real estate investment assessment.
3 INTRODUCTION OF GREY SYSTEM THEORY

Grey system theory is a new subject of system science created by Chinese Professor Julong L (1988) in the early of eighties. And he gave the definition and research method of grey system theory in the article of “The controlling problem of grey system theory”. He pointed out that there are four kinds of uncertain information which are fuzzy information, random information, grey information and uncertain information. Grey information means that people can only acquire part of information or the approximate range of the amount of information rather than getting the whole information and precise information because of the complexity of things, the interruption of various noises and the limited ability of receiving system. And this kind of partly known, partly unknown information comprise of grey information. Grey system is one between white system and black system. White system is a system whose internal characteristic is completely known and system information is perfectly sufficient. However, black system means its internal information is totally unknown to the outside world. We only can have observation and studies through its connection with outside world. Grey system’s internal information is partly known and partly unknown. The factors inside grey system have the relations of uncertainty. Investors in real estate field have to deal with many things under uncertainty and face a lot of risk factors. So decision making process in real estate investment project itself is a gray and lacking information process. And grey system theory just in place can meet the characteristics of real estate investment. It is feasible to apply grey system theory in real estate investment decision.

Grey system theory takes uncertainty system of partly known, partly unknown “small sample” and “poor information” as research target. It mainly works by developing and drawing valuable information through that partly known information and realize effective control and accurate description on gray system operation. At present, this increasingly mature theory has won the recognition and attention from both
international and domestic academic world and it is widely applied to regional
technology; analysis, prediction and decision of economic system (Deng, 1988).

3.1 Grey clustering method

Grey clustering model is a method that cluster some observed indicators or some
observations to several definable categories by virtue of whitening weight function.
One cluster can be seen as an aggregation of the same type of observing objects. Grey
clustering decision is one part of grey system theory. It works in a way that has an
overall assessment of observing objects according to many different decision
indicators in order to ascertain if the observing objects are satisfied with given
trade-off criteria (Deng, 1990). In this paper, I am going to use this grey clustering
model to handle advantage and disadvantage ranking problem in real estate
multi-objects decision-making.

3.1.1 Whitening weight function

Assume we have “n” clustering observing objects, “m” clustering indicators and “s”
different grey classes. Categorize indicator j into grey class k (k∈{1,2,…s}) according
to the observations x_{ij} (i= 1, 2, …n; j=1,2, …m) composed by observing object i (i=1, 2… n) concerning about indicator j (j= 1,2, …m), we call it grey clustering. And
then categorize the values composed by “n” observing objects concerning about
indicator j into “s” grey classes, we call it indicator j subtype. We define whitening
weight function of indicator j and k subtype as f_{k}^{j} (·) (Deng, 1990).
Assume whitening weight function f_{k}^{j} (·) of indicator j and k subtype is shown by the
following graph, we say x_{k}^{j}(1), x_{k}^{j}(2), x_{k}^{j}(3), x_{k}^{j}(4) are the turning points of f_{k}^{j} (·). And
the typical whitening weight function is shown as:

f_{k}^{j} | x_{k}^{j}(1), x_{k}^{j}(2), x_{k}^{j}(3), x_{k}^{j}(4)
If whitening weight function $f^k_{j}(\cdot)$ does not have number 1 and 2 turning points, it will be shown as follows; Here we call $f^k_{j}(\cdot)$ as lower measure white weight function.

$$f^k_{j}(x) = \begin{cases} 
0, & x \notin [x^k_j(1), x^k_j(4)] \\
\frac{x - x^k_j(1)}{x^k_j(2) - x^k_j(1)}, & x \in [x^k_j(1), x^k_j(2)] \\
1, & x \in [x^k_j(2), x^k_j(3)] \\
\frac{x^k_j(4) - x}{x^k_j(4) - x^k_j(3)}, & x \in [x^k_j(3), x^k_j(4)] 
\end{cases}$$

If whitening weight function $f^k_{j}(\cdot)$ does not have number 1 and 2 turning points, it will be shown as follows; Here we call $f^k_{j}(\cdot)$ as lower measure white weight function.

$$f^k_{j}[-\infty, \infty, x^k_j(3), x^k_j(4)]$$
If whitening weight function \( f_{jk}^k (\cdot) \) has number 2 and 3 turning points overlapped, which is shown as follows. And we call it middle measure white weight function.

\[
f_{jk}^k(x) = \begin{cases} 
0, & x > x_{jk}^k(4) \\
1, & x \in [0, x_{jk}^k(3)] \\
\frac{x_{jk}^k(4) - x}{x_{jk}^k(4) - x_{jk}^k(3)}, & x \in [x_{jk}^k(3), x_{jk}^k(4)]
\end{cases}
\]

If whitening weight function \( f_{jk}^k (\cdot) \) does not have number 3 and 4 turning points, which is shown as follows. So we call it as upper measure white weight function.

\[
f_{jk}^k(x) = \begin{cases} 
0, & x > x_{jk}^k(1) \\
\frac{x - x_{jk}^k(1)}{x_{jk}^k(2) - x_{jk}^k(1)}, & x \in [x_{jk}^k(1), x_{jk}^k(2)] \\
\frac{x_{jk}^k(4) - x}{x_{jk}^k(4) - x_{jk}^k(2)}, & x \in [x_{jk}^k(2), x_{jk}^k(4)] \\
0, & x > x_{jk}^k(4)
\end{cases}
\]
3.1.2 Grey fixed weight clustering method

In the aspect of determining clustering coefficient, there are three different ways of setting coefficient in grey system theory. They are grey variable weight clustering method, grey equivalent weight clustering method and grey fixed weight clustering method respectively. And among these methods, the most commonly used method is grey fixed weight clustering method because it applies to the situation when the meaning of clustering indicators are different; the dimension of indicators are different and have great disparity in number among each indicators (Liu, 1999). If in this case we use grey variable weight clustering method instead, it may weaken the involvement of some indicators in grey clustering model.

Here we assume $x_{ij}$ ($i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$) is the observations of observing object $i$ ($i = 1, 2, \ldots, n$) concerning about indicator $j$ ($j = 1, 2, \ldots, m$), $f^k_j(\cdot)$ ($j = 1, 2, \ldots, m$; $k = 1, 2, \ldots, s$) is the whitening weight function of indicator $j$ and $k$ subtype. If the weight $\eta^k_j$
(j=1,2,…m; k=1,2,…s) of indicator j and k subtype has nothing to do with k, namely
any k1, k2∈(1,2,…s), there is always \( \eta_j^{k_1} = \eta_j^{k_2} \), then we can ignore the superscript k
of\( \eta_j \), note as\( \eta_j(j=1, 2,…m) \), and we call

\[
\sigma_i^k = \sum_{j=1}^{m} f_j^k(x_{ij}) \cdot \eta_j
\]

as grey fixed weight clustering coefficient of observing object i concerning about grey
class k.

So grey fixed weight clustering method can be processed by the following steps:

First, Give whiteness weight function of j decision indicator, k subtype

\( f_j^k(\cdot) \) (j=1,2,…m; k=1,2,…s).

Second, Decide clustering weight on different decision indicators \( \eta_j(j=1, 2,…m) \).

Third, From the whiteness weight function \( f_j^k(\cdot) \) (j=1,2,…m; k=1,2,…s) in step one,
clustering weight\( \eta_j(j=1, 2,…m) \) in step two and observations \( x_{ij} \) (i= 1, 2, …n;
j=1,2, …m) composed by observing object i (i= 1, 2… n) concerning about
indicator j (j= 1,2, …m), we can calculate coefficient of grey fixed weight
clustering model

\[
\sigma_i^k = \sum_{j=1}^{m} f_j^k(x_{ij}) \cdot \eta_j
\]

Forth, If \( \max_{1 \leq k \leq s}\{\sigma_i^k\} = \sigma_i^{k^*} \), we can be sure that observing object i belong to grey
class k*.

3.2 Grey relation decision model

Choosing property type is also an important part in investment process, which
influence the success of a project. We have to have an overall consideration on many
different indicators such as rate of return, standard deviation and cost to invest etc to
make a right decision.
(1) Select assessment indicators and build assessment indicators matrix

Now we assume there are m real estate investment projects to be selected and each project has n assessment indicators which together compose of assessment indicators set. So we can make m*n assessment indicators matrix.

\[ X = (x_{ij})_{m \times n} \]

Where: \( x_{ij} \) is the observation value of project i under assessment indicator j.

(2) Establish standard pattern

Translate assessment indicators set into both maximum and minimum these two major kinds of indicators. \( X_0 \) is the optimal results vector of many mutually exclusive investment projects in grey relation analysis, namely is standard pattern:

\[ X_0 = (x_{01}, x_{02}, \ldots, x_{0m}) \]

Where \( X_0 \) equals to maximize value of assessment indicators whose maximize values are optimal or minimize value of assessment indicators whose minimize values are optimal.

Add vector \( X_0 \) to the assessment indicators matrix above, and we get new \((m+1)\times n\) assessment indicators matrix:

\[ X' = (x_{ij})_{(m+1) \times n} \]

(3) Gray target transformation

Because each indicator has different physical meaning; the dimension of indicators are different and have great disparity in number among each indicators. So we should remove dimension from original data and transfer original data to comparable data column. We introduce one way of removing dimension in the following:

Maximize indicator: \( t_{ij} = (x_{ij} - \text{min} x_{ij}) / (\text{max} x_{ij} - \text{min} x_{ij}) \),
Minimize indicator: 
\[ t_{ij} = \frac{(\text{max}_{ij} - x_{ij})}{(\text{max}_{ij} - \text{min}_{ij})} \]

where \( 1 \leq i \leq m+1 \)

After transferring, we change previous assessment indicators matrix \( X = (x_{ij})_{m \times n} \) to the new one \( T_{ij} = (t_{ij})_{(m+1) \times n} \). And at the same time, standard pattern is converted to Standard bull's-eye: \( X_0 = (1 \ 1 \ 1 \ldots \ 1 \ 1 \ 1) \).

(4) Calculate the difference between two poles. And get new matrix.

Deal with \( T_{ij} = (t_{ij})_{(m+1) \times n} \) according to the formula \( \Delta_{ij} = |1 - t_{ij}| \), and we get a new \((m+1) \times n\) matrix:

\[ R_{ij} = (\Delta_{ij}) (m+1) \times n, \]

All the elements in the matrix of \( R_{ij} \), we call maximum value as the biggest difference of the two poles \( \Delta_{\text{max}} \), call the minimum value as the smallest difference of the two poles \( \Delta_{\text{min}} \), therefore we have difference information space as

\[ \Delta_{CR} = (\Delta, \rho, \Delta_{\text{max}}, \Delta_{\text{min}}) \]

Where \( \Delta \) is all of the difference information, namely \( \Delta_{ij} \); \( \rho \) is resolution coefficient, \( \rho \in (0,1) \), which usually assume as 0.5

(5) Calculate Bull's-eye coefficient and bull's-eye-degree to optimize decision-making

We note \( \varphi_{oi}(j) \) as the Bull's-eye coefficient of \( X_{ij} \) and \( X_{oj} \), so we have:

\[ \varphi_{oi}(j) = \frac{(\Delta_{\text{min}} + \rho \Delta_{\text{max}})}{(\Delta_{ij} + \rho \Delta_{\text{max}})} \]

Take \( \varphi_{oi}(j) \) as element making Bull's-eye coefficient matrix \( \varphi_{oi}(j)_{m \times n} \), and calculate bull's-eye-degree,

\[ y_{0i} = \frac{1}{n} \sum_{j=1}^{n} \varphi_{oi}(j) \]

Where \( n \) is the number of assessment indicators set.

Rank each investment project according to size of bull's-eye-degree. The bigger the bull's-eye-degree is, the better the project is. And here we can realize the
optimization grey relations of many mutually exclusive investment projects.

3.3 Grey forecasting model

In recent years, grey forecasting theory has receiving increasingly attention from scientific researchers. It has been applied to industry, agriculture and scientific studies etc. many fields and achieve satisfied results in prediction. During the process of building grey forecasting model, in essence, it mainly works by accumulating original data to a new data column which has strong regularity and builds a new model; then generate reduction model from this new data column by inverse treatment and finally acquire forecasting model from the above reduction model. Grey forecasting is a quantitative prediction by use of Grey Model, GM (1, n). And GM (1, 1) is the most commonly used grey forecasting model. It is a model composed by first-order differential equation where only contains one variable (Deng, 2002). And it is an effective forecasting model in real estate sales. After the establishment of a good model, it is necessary to test the accuracy of the model. The mainly accuracy test of grey system theory are relative error method, Correlation method and Posteriori difference method. Here we mainly make use of Posteriori difference method to test the accuracy of model.

The procedure of building grey forecasting model
(1) We only need one data column \( x^{(0)} \) to establish GM(1, 1) model.

Assume original data column as:

\[
x^{(0)} = \{ x^{(0)}(i) \}, \quad (i = 1, 2, \ldots, n)
\]

Use (1-AGO) which stands for cumulative increase to do a first-order accumulated increase generating a new column as

\[
x^{(1)}(i) = \sum_{s=1}^{t} x^{(0)}(s), X^{(1)} = \{ x^{(1)}(i) \}, \quad (i = 1, 2, \ldots, n)
\]
After accumulated generating, it weakens the influence of bad data in the original data column and makes the new column have the rule of exponential growth. And the solution of first-order differential equations can be considered to satisfy the first-order linear differential equation.

\[ \frac{dX^{(1)}}{dt} + aX^{(1)} = u \]

We take parameter column A as:

\[ A = [a \ u] \]

Where a works as model development parameter which reflects the development trend of \( X^{(1)} \) and original data column \( x^{(0)} \); u works as coordination parameter which reflects changing relations among data.

Then we try to solve the parameter vector of the differential equation by least-squares method.

\[ A = [a \ u] = (B^T B)^{-1} B^T Y \]

\[ B = \begin{bmatrix}
-\frac{1}{2} [X^{(1)}(1) + X^{(1)}(2)] & 1 \\
-\frac{1}{2} [X^{(1)}(2) + X^{(1)}(3)] & 1 \\
\vdots & \vdots & \ddots & \ddots \\
-\frac{1}{2} [X^{(1)}(i - 1) + X^{(1)}(i)] & 1 \\
\end{bmatrix} \]

\[ Y = (x^{0}(2), x^{0}(3), \ldots, x^{0}(n)) \]

Take a and u we got above to first-order linear function, we get solution as:

\[ \hat{x}^1(t + 1) = \left[ x^{0}(1) - \frac{u}{a} \right] e^{-at} + \frac{u}{a}, \quad (t = 0, 1, \cdots, n - 1) \]

Do (r-IAGO) which stands for cumulative decrease to get grey forecasting model of
original data

\[\hat{x}^0(i) = \hat{x}^1(i) - \hat{x}^1(i - 1), (i = 2, 3, \ldots, n)\]

(2) Residual analysis

The residual generated from grey forecasting model is

\[q(k) = X(k) - \bar{x}(k) \quad (k = 1, 2, \ldots, n)\]

The average of residual is

\[\bar{q} = \frac{1}{n} \sum_{k=1}^{n} q(k) \quad (k=1, 2, \ldots, n)\]

The variance of the original data is

\[S_1^2 = \frac{1}{n} \sum_{k=1}^{n} (x^{(0)}(k) - \bar{x})^2, \quad \bar{x} = \frac{1}{n} \sum_{k=1}^{n} x^{(0)}(k) \quad (k = 1, 2, \ldots, n)\]

The variance of residuals is

\[S_1^1 = \frac{1}{n} \sum_{k=1}^{n} (q(k) - \bar{q})^2, \quad (k = 1, 2, \ldots, n)\]

The mean variance ratio is

\[C = \frac{S_2}{S_1}\]

The small error probability is

\[P = P\{|q(k) - \bar{q} < 0.6745S_1|\}, (k = 1, 2, \ldots, n)\]
Where the better the smaller of mean variance ratio is (Because small C reflects that $S_2$ is small and $S_1$ is big, which means the variance of residuals is small and the variance of the original data is big. It means that residuals are comparatively concentrated and the range of swing is small; original data is comparatively scattered and the range of swing is big. Hence a good simulation effects require the ratio of $S_2$ and $S_1$ is as small as possible). And the better the bigger of small error probability P is. C and P together decide the precision of forecasting model. Generally we divide model into four precision levels.

<table>
<thead>
<tr>
<th>Grade of precision</th>
<th>First level</th>
<th>Second level</th>
<th>Third level</th>
<th>Forth level</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>&lt;0.35</td>
<td>&lt;0.50</td>
<td>&lt;0.65</td>
<td>≥0.65</td>
</tr>
<tr>
<td>P</td>
<td>&gt;0.95</td>
<td>&gt;0.80</td>
<td>&gt;0.70</td>
<td>≤0.70</td>
</tr>
</tbody>
</table>

3.4 Advantages of taking grey system theory

Real estate investment decision is a complicated process. During this process, decision maker will face a good deal of uncertainty factors and the information they can take use of in the decision process is far from enough. Hence, how to use limited information to make the most accurate decision is the most important situation that investors are facing. Grey system theory has a certain specialty in this area. As an advanced decision method, it has the following specialty.

1. The information or data that needed in creating model is less than many other methods. Usually we can build model with 4 or more data (Deng,2002).
2. There is no need to know the distribution characteristics of original data in grey clustering model and grey relation analysis model. We can transfer random data
into regular sequence by cumulating original data.

3. Grey system theory is the combination of qualitative and quantitative. It is able to deal with multi-objectives decision problems.

4. The calculation method is simple and it can be achieved entirely by hand calculations.

3.5 How Grey Theory has been applied to other areas

In the last twenty years, grey system theory established its academic status as a new scientific technology by its powerful strength of life. Chinese Academy of Sciences Professor Chen Keqiang (1995) has pointed out that “At the beginning of new born of all social science, few of which can have significant breakthrough and rapid development within 10 years, however, grey system theory is one of them.” Grey system theory has received more and more attention widely from international and domestic academic world by virtue of its broad development prospect. Grey system theory has applied extensionally to industry, agriculture, education, medicine, hydro power and economics etc. many scientific fields, and it successfully deal with a lot of realistic problem in production, life and the field of scientific studies.

Jun W (1996) has applied Grey Clustering Decision to select subscribing to scientific and technological periodicals. Selecting scientific and technological periodicals is one of comparatively complex task in library management, which is influenced by many factors. Some of those factors are certain but some others are uncertainty. So grey system theory is a useful tool to control the selecting of scientific and technological periodicals and pick up useful information. Grey clustering decision can have an overall consideration about variety of factors, but also it can categorize the selected periodicals into several categories in order to make sure which periodical is going to select first and which one is chosen as backup. In this paper, the author made use of grey fixed weight clustering model to select subscribing to scientific and
technological periodicals. Author made assumption that a certain library has 6 available periodicals to be subscribed and decide four decision-making indicators which are ① practicality, ② planning, ③ integrity, ④ saving to assess periodicals. Each of these indicators has a given weight, 0.48, 0.24, 0.13 and 0.15 respectively. And make several different clustering decisions according to “subscribe”, “probable subscribe” and “not subscribe”, finally give different grade (which has been categorized to three levels on grades, e.g. “subscribe”, “probable subscribe” and “not subscribe”) on these indicators based on different periodicals and follow model to build Whitening weight function to finally make decision that 3 of 6 periodicals belong to “subscribe”, 1 of 6 belongs to “probable subscribe” and the other two are “not subscribe”. Grey clustering model make overall assessment on different periodicals by use of four decision indicators and categorize 6 available periodicals into 3 grey classes. It facilitates an overall grasp of these periodicals and decide the scope of trade-offs.

Minzhen.H (2007) has also applied grey forecasting model in logistics demand forecast. Grey forecasting model has the advantage of needing less data and it is fit for short-medium term prediction. In this paper, the author built grey forecasting model by dealing with original historic data with a certain mathematic methods. It mainly works by transforming original data to a new data column with strong regularity, then generate reduction model from this new data column by inverse treatment and finally acquire forecasting model from the above reduction model. After prediction, the author made forecasting precision grade by virtue of Posteriori difference method. And we know that forecasting accuracy has reached level one by looking into Grey Forecasting Precision Grade, where C=0.09669 > 0.95. Finally we get forecasting result of logistic demand in 2007 is 79.569 thousand Tons.

Pei.Z(1992) applied grey clustering model into real estate investment risk analysis, where she quantized the decision indicators that can not be realized in sensitivity
analysis and it makes analysis more accuracy and intuitive. In the paper, the author took social risk, economic risk and politic risk as decision indicator to make assessment on three projects to see which has least risk. And the author get project 1 is low risk, project 2 belongs to high risk and project 3 is middle risk. However, the paper only has assessment on social, politic and economic risk these three basic elements, it may cause risk factors set not fully reflect the project's risk rating because of too few levels and general indicators, so the author suggested that we can refine decision indicators according to the real situation.

Chao.D(2002) applied grey forecasting model to predict when drought will happen. In this paper, the author works based on annual rainfall data of Shanxi Reservoir rainfall station in recent 40 years and apply grey forecasting model to have prediction on future drought. Actually it is a prediction on the outliers which can represent the year that drought will happen. Based on the real information in Shanxi Reservoir rainfall station, its average annual rainfall in recent years is around 1800mm, the maximum year rainfall is 2397mm in year 1990; the minimum year rainfall is 1169.8mm in year 1976. So the author took rainfall lower than 1400mm as outlier to have analysis and prediction to see which year may have drought. By taking use of grey forecasting model, results show that in the middle of 2017 and 2018 may have drought in Shanxi. Through model precision test also shows that it lies in the high accuracy level which indicates it has a reliable prediction.
4 CASE STUDY

4.1 Introduction of real estate development project

4.1.1 Background of development project

Grey system theory is a new subject in terms of real estate development decision-making. In this paper I just show how it works when we make real estate investment decision. Because of the limited data source and subjective factors in assessment system, inevitably I am going to make some assumptions in this application process.

Assume XXX real estate developing company is planning to develop and invest a real estate project in Stockholm. Now XXX Company has a preliminary plan to invest this project in four alternative districts which are Solna, Sollentuna, Sundbyberg and Taby. In addition, according to the type of property available in current market, they have four projects in terms of the type of property. They have four projects to invest which are

① Residence House  ② Shops  ③ Office Building  ④ Business Housing.

4.1.2 Decision Objectives:

1. Select appropriate district decision indicators to have a decision analysis on the four available districts chosen by the XXX Company, and finally choose the most fit for investing and developing district.

2. Based on the districts that have been chosen above, select the property type that fit into the development of the specific district that we have chosen above through researching and investigating the similar projects in the surrounding area.

3. After deciding the investing district and property type, we can have an analysis on the historic data in terms of the sales and price concerning the property type that we are going to invest and finally make prediction in the most economic investment scale and price.

According to the decision objectives, we can apply the grey system method
mentioned above to this decision process. Hence the application method is as follows:

(1) Apply grey clustering model to choose investment district.
(2) Apply grey relation model to choose investment property type.
(3) Apply grey forecasting model to have prediction on investment scale and property price.

4.2 Decision on investment district

As mentioned above, there are four alternative investment districts, which are Solna, Sollentuna, Sundbyberg and Taby respectively. By looking through the literature concerning real estate investment, market analysis and discuss with professions in real estate field, we finalized on five decision indicators to assess investment districts. They are District economy, Location, Social environment, Traffic and Market demand. And the weight of each indicator is 0.24, 0.18, 0.16, 0.22, 0.20. It is a difficult and important task to give an appropriate weight to each indicator in building standard assessment system. As we found in many assessment systems, it usually depends largely on the assessment system makers’ subjective feeling and experience. Hence it does not have to be truth here. In this paper I refer to the weights from Real estate assessment system in Weidong L.’s paper (.

Apply grey clustering model to make investment district decision by use of decision indicators mentioned above. It works as follows:

(1) *Four observing objects*

- Solna ②Sollentuna ③Sundbyberg ④Taby

*Five decision indicators*

① District economy ② Location ③ Social environment ④ Traffic ⑤ Market demand

*Four Grey class*
(2) By analyzing the data from Datscha and discuss with professions I finalize on an 
assessment vector C by Quantitative scoring in terms of five decision indicators.

\[ C = \begin{bmatrix} 90 & 85 & 90 & 95 & 75 \\ 50 & 65 & 95 & 40 & 85 \\ 70 & 85 & 80 & 90 & 60 \\ 85 & 70 & 70 & 80 & 90 \end{bmatrix} \]

(3) White weight function of five decision indicators are respectively as follows (j=1, 2, ..., 5 Different indicator has the same white weight function):

① Upper measure white weight function

\[
f_1^j(x) = \begin{cases} 
0, & x < 65 \\
\frac{x - 65}{20}, & x \in [65, 85] \\
1, & x > 85 
\end{cases}
\]

② Middle measure white weight function

\[ f_2^j [50, 65, --, 80] \]
The lower measure white weight function is defined by:

\[ f^3_j(x) = \begin{cases} 
0, & x < 60 \\
1, & x \in [0,35] \\
\frac{60-x}{25}, & x \in [35,60] 
\end{cases} \]

\[ f^4_j(70,90,--,---) \]
(4) The weight of each indicator is as follows,

\[ \eta_1 = 0.24, \quad \eta_2 = 0.18, \quad \eta_3 = 0.16, \quad \eta_4 = 0.22, \quad \eta_5 = 0.20 \]

So we have decision coefficient of each indicator in four grey classes shown as follows,

\[ \sigma_1 = \sum_{j=1}^{5} f_j^4(x_1j) \cdot \eta_j \]

\[ = f_1^4(x_{11}) \cdot \eta_1 + f_2^4(x_{12}) \cdot \eta_2 + f_3^4(x_{13}) \cdot \eta_3 + f_4^4(x_{14}) \cdot \eta_4 + f_5^4(x_{15}) \cdot \eta_5 \]

\[ = f_1^4(90) \cdot 0.24 + f_2^4(85) \cdot 0.18 + f_3^4(90) \cdot 0.16 + f_4^4(95) \cdot 0.22 + f_5^4(75) \cdot 0.20 \]

\[ = 1 \times 0.24 + 1 \times 0.18 + 1 \times 0.16 + 1 \times 0.22 + 0.5 \times 0.20 \]

\[ = 0.90 \]
\[ \sigma^2_1 = \sum_{j=1}^{5} f_j^2(x_{1j}) \cdot \eta_j \]
\[ = f_1^2(x_{11}) \cdot \eta_1 + f_2^2(x_{12}) \cdot \eta_2 + f_3^2(x_{13}) \cdot \eta_3 + f_4^2(x_{14}) \cdot \eta_4 + f_5^2(x_{15}) \cdot \eta_5 \]
\[ f_1^2(90) \cdot 0.24 + f_2^2(85) \cdot 0.18 + f_3^2(90) \cdot 0.16 + f_4^2(95) \cdot 0.22 + f_5^2(75) \cdot 0.20 \]
\[ = 1 \times 0.24 + 1 \times 0.18 + 1 \times 0.16 + 1 \times 0.22 + 1 \times 3 \times 0.20 \]
\[ = 0.867 \]

\[ \sigma^3_1 = \sum_{j=1}^{5} f_j^3(x_{1j}) \cdot \eta_j \]
\[ = f_1^3(x_{11}) \cdot \eta_1 + f_2^3(x_{12}) \cdot \eta_2 + f_3^3(x_{13}) \cdot \eta_3 + f_4^3(x_{14}) \cdot \eta_4 + f_5^3(x_{15}) \cdot \eta_5 \]
\[ = f_1^3(90) \cdot 0.24 + f_2^3(85) \cdot 0.18 + f_3^3(90) \cdot 0.16 + f_4^3(95) \cdot 0.22 + f_5^3(75) \cdot 0.20 \]
\[ = 0 \times 0.24 + 0 \times 0.18 + 0 \times 0.16 + 0 \times 0.22 + 0 \times 0.20 \]
\[ = 0 \]

\[ \sigma^4_1 = \sum_{j=1}^{5} f_j^4(x_{1j}) \cdot \eta_j \]
\[ = f_1^4(x_{11}) \cdot \eta_1 + f_2^4(x_{12}) \cdot \eta_2 + f_3^4(x_{13}) \cdot \eta_3 + f_4^4(x_{14}) \cdot \eta_4 + f_5^4(x_{15}) \cdot \eta_5 \]
\[ = f_1^4(90) \cdot 0.24 + f_2^4(85) \cdot 0.18 + f_3^4(90) \cdot 0.16 + f_4^4(95) \cdot 0.22 + f_5^4(75) \cdot 0.20 \]
\[ = 1 \times 0.24 + 0.75 \times 0.18 + 1 \times 0.16 + 1 \times 0.22 + 1 \times 0.25 \times 0.20 \]
\[ = 0.67 \]

So we have the coefficient of observing object 1 “Solna” in four grey classes showing as: \( \sigma_1 = (\sigma_1^1, \sigma_1^2, \sigma_1^3, \sigma_1^4) = (0.90, 0.87, 0, 0.67) \)

Similarly we can get \( \sigma_2, \sigma_3, \sigma_4 \) in the same way, we have as follows:
\( \sigma_2 = (\sigma_2^1, \sigma_2^2, \sigma_2^3, \sigma_2^4) = (0.36, 0.54, 0.27, 0.31) \) which is showed the coefficient of Sollentuna in different grey classes.

\( \sigma_3 = (\sigma_3^1, \sigma_3^2, \sigma_3^3, \sigma_3^4) = (0.58, 0.69, 0, 0.435) \)

\( \sigma_4 = (\sigma_4^1, \sigma_4^2, \sigma_4^3, \sigma_4^4) = (0.69, 0.67, 0, 0.49) \)

Accordingly, we have
\[
\max_{1 \leq k \leq 4} \{ \sigma^k \} = 0.90 = \sigma_1^1 \\
\max_{1 \leq k \leq 4} \{ \sigma^k \} = 0.54 = \sigma_2^2 \\
\max_{1 \leq k \leq 4} \{ \sigma^k \} = 0.69 = \sigma_3^3 \\
\max_{1 \leq k \leq 4} \{ \sigma^k \} = 0.69 = \sigma_4^4
\]

Where observing objects 1 and 4 belong to grey class “Excellent”, and the other two objects 2 and 3 belong to grey class “Good”, in addition, because \( \sigma_1^1 = 0.90 > 0.69 = \sigma_4^4 \), so we finally choose Solna as investment district to make investment plan.

### 4.3 Decision on property type

After deciding the investment district, XXX real estate developing company is planning to choose the property type from the following four projects according to the type of property available in current market, they are ① Residence House ② Shops ③ Office Building ④ Business Housing. Again by looking through the literature concerning real estate investment, market analysis and discuss with professionals in real estate field, we finalized on four decision indicators to select property type, which are Rate of return, Construction costs, Expected variance of NPV and Payback period. In addition, by comparing the similar four projects in Solna district we assume the following data table concerning these four indicators in different project.

<table>
<thead>
<tr>
<th></th>
<th>Rate of return (%)</th>
<th>Construction costs (per square meter)</th>
<th>Expected variance of NPV (%)</th>
<th>Payback period (Years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Residence House</td>
<td>5</td>
<td>30000</td>
<td>5</td>
</tr>
<tr>
<td>2.</td>
<td>Retail</td>
<td>6.5</td>
<td>20000</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td></td>
<td>Office building</td>
<td>6</td>
<td>25000</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Business house</td>
<td>6</td>
<td>27500</td>
<td>7</td>
</tr>
</tbody>
</table>

The four indicators in the table above compose of evaluation matrix of these four projects. So we can have it as $X=(x_{ij})_{4\times4}$, where $x_{ij}$ is the value of project $i$ concerning decision indicator $j$.

$$X = \begin{bmatrix}
5 & 30000 & 5 & 20 \\
6.5 & 20000 & 6 & 15 \\
6 & 25000 & 8 & 18 \\
6 & 27500 & 7 & 17 \\
\end{bmatrix}$$

(1) Establish standard pattern

Each indicator in the evaluation matrix above has different polarity. Rate of return is maximum indicators, which means the bigger its value is, the better the indicator represents. Conversely Construction costs, Expected variance of NPV and Payback period are minimum indicators, which mean that the smaller its value is, the better the indicator represent.

Now we construct the optimal results vector, namely is standard pattern: $X_0= (6.5, 20000, 5, 15)$, which stands for the optimal value in each indicator. Take Rate of Return for example, the bigger the better, so we chose 6.5 here. Add this optimal vector $X_0$ to the evaluation matrix above, and we get a new $5\times4$ evaluation matrix : $X'=(x_{ij})_{5\times4}$

$$X' = \begin{bmatrix}
6.5 & 20000 & 5 & 15 \\
5 & 30000 & 5 & 20 \\
6.5 & 20000 & 6 & 15 \\
6 & 25000 & 8 & 18 \\
6 & 27500 & 7 & 17 \\
\end{bmatrix}$$

(2) Gray target transformation
In the valuation matrix above, each indicator has different physical meaning; the
dimension of indicators are different and have great disparity in number among each
indicator. We have to normalize them to make them comparable by doing grey target
transformation.

We introduce one way of removing dimension in the following:

Maximize indicator: \( t_{ij} = \frac{x_{ij} - \min x_{ij}}{\max x_{ij} - \min x_{ij}} \),

Minimize indicator: \( t_{ij} = \frac{\max x_{ij} - x_{ij}}{\max x_{ij} - \min x_{ij}} \)

where \( 1 \leq i \leq m+1 \)

After transferring, we change previous valuation indicators matrix \( X' = (x_{ij})_{5 \times 4} \) to the
new one \( T_{ij} = (t_{ij})_{5 \times 4} \). And at the same time, standard pattern \( X_0 = (8.3, 88750, 6.7, 5.1) \) is converted to Standard bull's-eye: \( X_0 = (1 1 1 1) \).

\[
\begin{bmatrix}
1 & 1 & 1 & 1 \\
(5 - 5)/(6.5 - 5) & (30000 - 20000)/(30000 - 20000) & 1 & 0 \\
(6.5 - 5)/(6.5 - 5) & (30000 - 30000)/(30000 - 20000) & 0.667 & 1 \\
(6 - 5)/(6.5 - 5) & (30000 - 25000)/(30000 - 20000) & 0 & 0.4 \\
(6 - 5)/(6.5 - 5) & (30000 - 27500)/(30000 - 20000) & 0.333 & 0.6 \\
\end{bmatrix}
\]

So finally we get

\[
T = \begin{bmatrix}
1 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 \\
1 & 0 & 0.667 & 1 \\
0.667 & 0.5 & 0 & 0.4 \\
0.667 & 0.25 & 0.333 & 0.6 \\
\end{bmatrix}
\]

(3) Calculate the difference between two poles. And get new matrix.

After grey target transformation, we get difference information matrix \( R = (r_{ij})_{5 \times 4} \) by
dealing with \( T_{ij} = (t_{ij})_{(m+1) \times n} \) according to the formula \( r_{ij} = |1 - t_{ij}| \).
So difference information space is \( \Delta_{CR} = (\Delta, 0.5, 1, 0) \). Resolution coefficient \( \rho \) is 0.5; the biggest difference of the two poles \( \Delta_{max}=1 \); the smallest difference of the two poles \( \Delta_{min}=0 \).

(4) Calculate Bull's-eye coefficient and bull's-eye-degree to optimize decision-making

Then we take Bull's-eye coefficient formula mentioned in the introduction of grey system theory to calculate Bull's-eye coefficient,

\[
\varphi_{oi}(j) = (\Delta_{min} + \rho \Delta_{max}) / (\rho_{ij} + \rho \Delta_{max})
\]

Take \( \varphi_{oi}(j) \) as element making Bull's-eye coefficient matrix \( \varphi_{oi}(j)_{4*4} \), so we have

\[
\varphi_{oi} = \begin{bmatrix}
0.333 & 1 & 1 & 0.333 \\
1 & 0.333 & 0.6 & 1 \\
0.6 & 0.5 & 0.333 & 0.455 \\
0.6 & 0.4 & 0.428 & 0.556
\end{bmatrix}
\]

And then according to the formula below calculate bull's-eye-degree,

\[
y_{0i} = \frac{1}{n} \sum_{j=1}^{n} \varphi_{oi}(j)
\]

So \( y_{0i} = 1/4(0.333+1+1+0.333) = 0.67 \)

Similarly we can get \( y_{02} = 0.73 \), \( y_{03} = 0.472 \), \( y_{04} = 0.496 \)

Apparently we have \( y_{02} > y_{03} > y_{04} > y_{03} \). Hence in the four available projects project 2 which is retail is the best, investing in residence building is the second best and following is business house and office building.

4.4 Prediction on investment scale and property price

After deciding what kind of property is the most profitable and the least risk to invest
in Solna--retail, now we have a prediction on property sales and prices by looking through the historic data from 2005 to 2009 with method of grey forecasting model.

The following two tables show statistic data concerning the retail property prices in Solna from 2005 to 2009 and total retail property sales in Solna from 2005 to 2009 respectively.

### The prices in Solna 2006-2009 (kSEK per square meter)

<table>
<thead>
<tr>
<th>year district</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solna</td>
<td>6.6</td>
<td>7.4</td>
<td>10.3</td>
<td>13.2</td>
</tr>
</tbody>
</table>

### The sales in Solna 2006-2009 (10k square meter)

<table>
<thead>
<tr>
<th>year district</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solna</td>
<td>5.1</td>
<td>8.1</td>
<td>10.3</td>
<td>6.6</td>
</tr>
</tbody>
</table>

In the following part, we will first have a prediction on retail prices in year 2010. According to the first table above, we build original data column as follows,

\[ x^{(0)} = \{ 6.6, 7.4, 10.3, 13.2 \} \]

And use \((1-AGO)\) to do a first-order accumulated generating a new column by use of original data above as,

\[ x^{(1)} = \{ 6.6, 14, 24.3, 37.5 \} \]

Then we try to solve the parameter vector of the differential equation by least-squares
method

\[ A = \begin{bmatrix} a \\ u \end{bmatrix} = (B^T B)^{-1} B^T Y \]

Where

\[ B = \begin{bmatrix} -10.3 \\ -19.15 \\ -30.9 \end{bmatrix} \]

\[ B^T = \begin{bmatrix} -10.3 & -19.15 & -30.9 \\ 1 & 1 & 1 \end{bmatrix} \]

\[ Y = (7.4, 10.3, 13.2)^T \]

So after calculation we get \[ A = \begin{bmatrix} -0.31 \\ 6.14 \end{bmatrix} \]

And then put parameter \( a \) and \( u \) to property prices forecasting model:

\[ \hat{x}^1(t+1) = \begin{bmatrix} x^0(1) - \frac{u}{a} \\ \frac{u}{a} \end{bmatrix} e^{-at} + \frac{u}{a}^t (t = 0, 1, \ldots, n - 1) \]

\[ = [6.6 + 6.14/0.35] e^{0.31t} - 6.14/0.35 \]

\[ = 24.14 e^{0.31t} - 17.54 \]

Do (r-IAGO) to get grey forecasting model of original data

\[ \hat{x}^0 (i) = \hat{x}^1 (i) - \hat{x}^1 (i - 1), (i = 2, 3, \cdots, n) \]

Finally we have the following table showing the real property prices and forecasting prices from 2006 to 2009, and 2010 in Solna. (kSEK per square meter)

<table>
<thead>
<tr>
<th></th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real price</td>
<td>6.6</td>
<td>7.4</td>
<td>10.3</td>
<td>13.2</td>
<td></td>
</tr>
<tr>
<td>Forecasting price</td>
<td>6.6</td>
<td>8.77</td>
<td>11.96</td>
<td>16.31</td>
<td>22.23</td>
</tr>
<tr>
<td>Residual errors</td>
<td>0</td>
<td>1.37</td>
<td>1.66</td>
<td>3.11</td>
<td></td>
</tr>
<tr>
<td>-----------------</td>
<td>-----</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td></td>
</tr>
</tbody>
</table>

**Model testing**

Average of residual errors

\[ \bar{q} = \frac{1}{n} \sum_{k=1}^{n} q(k) = 1/4(0+1.37+1.66+3.11) = 1.535 \]

The variance of the original data is

\[ S_1^2 = \frac{1}{n} \sum_{k=1}^{n} (x^{(0)}(k) - \bar{x})^2, \bar{x} = \frac{1}{n} \sum_{k=1}^{n} x^{(0)}(k) \]

\[ \bar{x} = 1/4(6.6+7.4+10.3+13.2) = 9.375 \]

\[ S_1^2 = 1/4[(6.6 - 9.375)^2 + (7.4 - 9.375)^2 + (10.3 - 9.375)^2 + (13.2 - 9.3752)^2] = 6.77 \]

The variance of residuals is following the same way:

\[ S_2^2 = \frac{1}{n} \sum_{k=1}^{n} (q(k) - \bar{q})^2 \]

\[ = 1/4[(0-1.535)^2+(1.37-1.535)^2+(1.66-1.535)^2+(3.11-1.535)^2] \]

\[ = 1.22 \]

So we have mean variance ratio

\[ C = \frac{S_2}{S_1} = 1.22/6.77 = 0.18 < 0.35 \]

The small error probability is

\[ P = P[|q(k) - \bar{q} < 0.6745S_1|] \]

After calculation, every p is smaller than 4.566, hence P<0.95, which lies in the best accuracy testing class. This proves that the forecasting model is reliable. So the grey forecasting model we built can reflect the real property prices situation in Solna.
And similarity, we can have sales prediction in the same procedure:

Where we have \( x^{(0)} = \{ 5.1, 8.1, 10.3, 6.6 \} \)

\[
x^{(1)} = \{ 5.1, 13.2, 23.5, 30.1 \}
\]

In the parameter vector

\[
A = \begin{bmatrix} 3 \\ u \end{bmatrix} = (B^T B)^{-1} B^T Y
\]

Where

\[
B = \begin{bmatrix} -9.15 & 1 \\ -18.35 & 1 \\ -26.8 & 1 \end{bmatrix}
\]

\[
B^T = \begin{bmatrix} -9.15 & -18.35 & -26.8 \\ 1 & 1 & 1 \end{bmatrix}
\]

\[
Y = (8.1, 10.3, 6.6)^T
\]

Finally we have \( A = \begin{bmatrix} -0.158 \\ 9.97 \end{bmatrix} \)

By taking use of grey forecasting model we get real Property Sales and forecasting sales from 2006 to 2009 and 2010 in Solna.

<table>
<thead>
<tr>
<th>Year</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real sales</td>
<td>5.1</td>
<td>8.1</td>
<td>10.3</td>
<td>6.6</td>
<td></td>
</tr>
<tr>
<td>Forecasting sales</td>
<td>5.1</td>
<td>11.67</td>
<td>13.68</td>
<td>16</td>
<td>18.75</td>
</tr>
<tr>
<td>Residual errors</td>
<td>0</td>
<td>3.57</td>
<td>3.38</td>
<td>9.4</td>
<td></td>
</tr>
</tbody>
</table>

And in the model testing we have the variance of the original data \( S_1^2 = 3.69 \)

The variance of residuals is \( S_2^2 = 11.42 \)

Mean variance ratio \( C = \frac{S_2^2}{S_1^2} = \frac{11.42}{3.69} = 3.09 \), which lies in a very bad accuracy
testing class. This is showing the limitation of the application of grey forecasting model. Because grey forecasting model is only applicable into the situation where the original data has the trend of exponential growth, it can have accurate forecasting. However in the case above, the property sales had a great fall down because of the financial crisis, which does not meet the requirement of applying grey model. Hence the forecasting result can not really reflect the property sales. Here it is better to adopt another forecasting method.

4.5 Analysis

In this paper, I build clustering model to have a decision on investment district. This model has advantages of quantizing decision indicators that can not be realized in NPV method and in regression analysis. In the process of deciding investment decisions, I choose District economy, Location, Social environment, Traffic and Market demand these five decision indicators to have analysis on investment district.

By applying clustering model, it changes the whole investment district decision from qualitative assessment to quantitative assessment by quantizing indicators through quantitative comparison among the chosen four investment districts in terms of real situation, which make analysis more accurate and more intuitive. In addition, it does not need much data and data does not have to conform to a certain statistic distribution compared to regression analysis, so it enhances the objectivity in a certain level. In the paper, we have four grey classes as assessment basis to quantize indicators and then score each assessment indicator according to its real situation, and follow the specific mathematic process in clustering model to get the coefficient of each observing object in four different grey classes. From the application above we have

\[
\begin{align*}
\sigma_1 &= (\sigma_1^1, \sigma_1^2, \sigma_1^3, \sigma_1^4) = (0.90, 0.87, 0, 0.67) \\
\sigma_2 &= (\sigma_2^1, \sigma_2^2, \sigma_2^3, \sigma_2^4) = (0.36, 0.54, 0.27, 0.31) \\
\sigma_3 &= (\sigma_3^1, \sigma_3^2, \sigma_3^3, \sigma_3^4) = (0.58, 0.69, 0, 0.435)
\end{align*}
\]
\[ \sigma_4 = (\sigma_1^4, \sigma_2^4, \sigma_3^4, \sigma_4^4) = (0.69, 0.67, 0, 0.49) \]

And then choose the biggest coefficient of each observing object in these four grey classes, we get following

\[ \max_{1 \leq k \leq 4} \{ \sigma_1^k \} = 0.90 = \sigma_1^1 \]
\[ \max_{1 \leq k \leq 4} \{ \sigma_2^k \} = 0.54 = \sigma_2^2 \]
\[ \max_{1 \leq k \leq 4} \{ \sigma_3^k \} = 0.69 = \sigma_3^3 \]
\[ \max_{1 \leq k \leq 4} \{ \sigma_4^k \} = 0.69 = \sigma_4^4 \]

Where observing objects 1 and 4 belong to grey class” Excellent”, in addition, because \( \sigma_1^1 = 0.90 > 0.69 = \sigma_4^1 \), so we finally choose object 1 which represents for Solna as investment district to make investment plan.

As we talked above in chapter 3, real estate investment system is a grey system. In the investment and decision process, investors have to face huge of uncertainty factors, and grey system in this case will become a very effective investment decision method in the situation of poor information and strong uncertainty. Especially for grey clustering model and grey relation analysis model, they both are applicable whether sample data are regular or not.

In the decision on property type, I take use of grey relation analysis mode which has the same theory and idea as grey clustering model, they both give assessment on observing objects by virtue of applying decision indicators, which are Rate of return, Construction costs, Expected variance of NPV and Payback period. And follow four calculation steps in grey relation analysis model to get bull's-eye-degree for each observing object. In the paper we have bull's-eye-degree for residence house is 0.67; retail is 0.73; office building is 0.472 and business house is 0.496. Because the bigger the bull's-eye-degree is, the better the project is, we finally choose retail to investment, and the following is residential house, business house and the last one is office building. From the results above shows that applying grey relation analysis model it is
much easier to compare the advantages or disadvantages of each investment property type from the perspective of relative quantity instead of absolute quantity, which can guarantee a more intuitive and objective analysis and the calculation in grey relation analysis is simple and pragmatic. It also can remove dimensions of data resulting from different physical meaning, and transfer original data to comparable data columns (Shi, 1994). So in grey system theory, we do not need too much associated factors because of comparability among data. In another sense, it is much easier to find data with grey system theory. However, in both two grey clustering and grey relation analysis models, what we should pay attention is that we should adjust decision indicator according to the real case. The decision indicators taken in the paper are only from the concerns of real estate development not from the perspective of social and environmental benefits. And the indicators we have ignored may also have influence on accuracy of decision-making. It may cause low accuracy in investment decision because of the rough decision indicators. So we can appropriately narrow down investment decision indicators according to the real case to make it more accurate.

After deciding what kind of property is the most profitable and the least risk to invest in Solna--retail, then we have a prediction on property sales and prices by virtue of GM(1,1). I build grey forecasting model by taking use of historic data for property price and sales. And I get prediction on retail price in Solna in 2010 is 22230 per square meter. Model testing precision of price prediction lies in the best accuracy testing class, which reflects the forecasting model is reliable. And from the results we get, we can see that retail price in Solna is in the growth trend. But for prediction of retail sales, the precision of forecasting model is very low, because grey forecasting model is only applicable to the situation where data has the characteristic of exponential growth, which indicates that grey forecasting model has a great limitation in prediction future trend. In addition, the establishment of grey forecasting model is on the basis of the changes of time and I did not take the changes in space into
consideration. In this paper, I mainly adopt calculating average method to deal with the data changes in time, which has a certain limitation and we need further research and study in changes of space.

NPV and IRR these two methods both are widely used around the world, even today both of them are still used by most of decision makers. However, they both only work based on single objective investment decision-making and these methods have the disadvantages of more data needed, lots of calculations, long time taking and a significant share of human factor in application and these weak points have influence the accuracy of decision. In addition, these two methods don’t take risk factor into consideration. In the method of Monte Carlo Simulation, it introduces the analysis of risk factor. It can generate the probability distribution of risk level of a certain investment returns indicator such as NPV probability distribution or IRR probability distribution through statistical test and stochastic simulation in order to offer a further analysis and basis on investment decision (Li, 2005). However, Monte Carlo Simulation method is still based on single objective investment decision-making, and one of the most outstanding advantages of grey system theory over these methods above is that it is a multi-objective comprehensive evaluation method. Because the characteristics of real estate investment determine the complexity and multi-objectives in real estate investment decision making, for example, investment payback period, risk, economic benefits and social benefits etc. are all the factors concerning real estate investment when they make investment decision making. So real estate investment basically is a multi-objective decision process. The commonly used multi-objective optimization indicators evaluation method is to determine the weight of each indicator which are applied in real estate decision problem through scientific calculation and translate multi-objective problem to single objective problem. In this field, AHP (The analytic hierarchy process) is the multi-objective method researchers usually used. It breaks the elements of decision-making down into objectives, indicators and projects three levels, and based on three levels conduct
qualitative and quantitative analysis of decision making (Schwartz & Moon, 2000). However, the difficulty is that how to abstract the appropriate levels according to the real situation and how to give an appropriate and realistic quantitative treatment on each indicator. So AHP to a great extent relies on people's experience, it is influenced greatly by subjective factors and can not rule out the probable personal one-sidedness of decision makers. Conversely, grey system model is not only a multi-objective comprehensive evaluation method, which overcomes one-sidedness of conventional single objective valuation, but also it does not relate too much to personal experience and the information or data that needed in creating model is less than many other methods. So to some extent it improves the accuracy and objectivity of the projects and makes the decision process simpler.
5 CONCLUSION

In this paper, I have application of grey system theory based on the combination of a case study and its basic theory from the perspective of scientificity and practicality.

This paper is written on the basis of grey theory and takes both quantitative and qualitative analysis method to have an analysis and assessment on multi-objects decision in real estate decision-making. By virtue of the application in a real case to see how grey system theory works and hope to offer an alternative method to real estate developer in investment decision.

This paper focuses on the application of grey system theory in real estate investment decision-making. It mainly involves grey clustering decision model, grey relation decision model and grey forecasting model.

(1) Grey clustering decision model and grey relation decision model both work by assessing several different observed objects with different decision indicators in order to determine if observing objects satisfy given trade-off criteria. And finally rank these observed objects and choose the optimal one. In the case of this paper, it applies grey clustering model to choose investment district from Solna, Sollentuna, Sundbyberg and Taby these four projects and then have decision on investment property type based on grey relation decision model among four commercial property types in current property market.

(2) Grey forecasting model takes fully use of the information given by historic data. In the paper, it just uses four years historic data to have prediction on property sales and prices in 2010.
Disadvantages of this method

(1) In the prediction of Retail in terms of sales and prices, I only take historic data into consideration and no other information; it may have impact on prediction accuracy. And in the prediction of sales, grey forecasting model does not work here because of the historic data is not exponentially growth. It limits the application of grey forecasting model.

(2) In the application of grey clustering model and grey relation decision model, the decision indicators taken in the paper are only from the concerns of real estate development. And other indicators we have ignored also have influence on accuracy of decision-making.

(3) Grey system theory takes use of lots of advanced mathematic theory, though it can be resolved totally by hand, it still takes time in calculation. Grey theory will be more effective if it can be combined with computer program.

Till now, there are many different decision methods available for real estate decision maker, but it does not mean that these methods are perfect, grey system theory neither. We should choose decision method in accordance with the characteristic of the real situation. Even in some cases, we have different results by taking use of different decision method. In this paper, I just introduce another alternative decision method for real estate investment decision. We still need further studies and research on advantages and disadvantages of applying different method into different decision objects in order to gradually perfect real estate investment method.
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