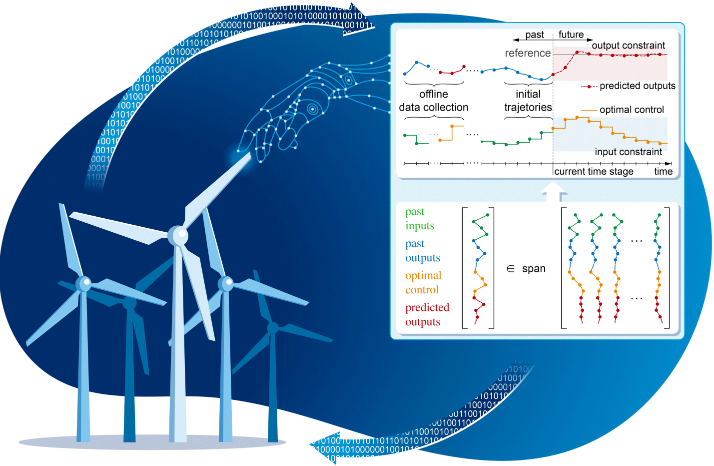
Data-Driven Control Based on Behavioral Systems Theory



Florian Dörfler

ETH Zürich

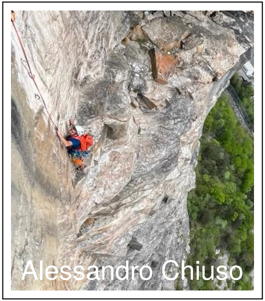
ERNSI 2023

Stockholm

Science & rope partners

Jeremy Coulson

Linbin Huang



Alessandro Chiuso

Ivan Markovsky & Alberto Padoan

John Lygeros

\+ many others

Roy Smith

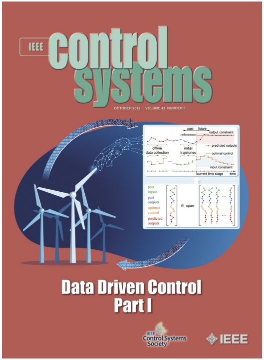
Thoughts on data in control systems

increasing role of ***data-centric methods*** in science / engineering / industry due to

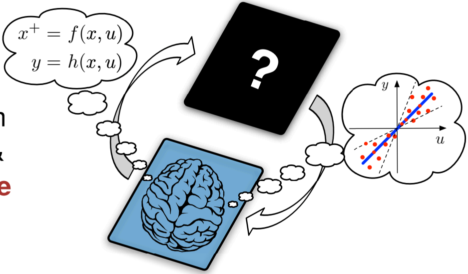
- ***methodological advances*** in statistics, optimization, & machine learning (ML)
- unprecedented availability of ***brute force***: deluge of data & computational power
- …and ***frenzy*** surrounding big data & ML

Make up your own opinion, but ML works too well to be ignored – also in control **?!?**

“One of the major developments in control over the past decade – & one of the most important moving forward – is the interaction of ML & control systems.” [CSS roadmap]



Scientific landscape



long & rich history (auto-tuning, system identification, adaptive control, RL, ...) & **vast & fragmented research landscape**

⟶ useful direct / indirect classification

direct data-driven control

minimize control cost (u, y)

subject to trajectory (u, y) compatible with data (u^d, y^d)

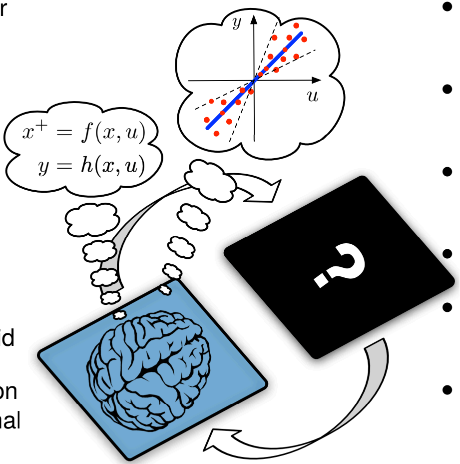
indirect (model-based) ***data-driven control***

model-based design {
minimize control cost (u, y)
subject to trajectory (u, y) compatible with the model

system identification {
where model $\in$ argmin fitting criterion (u^d, y^d)
subject to model belongs to certain class

Indirect vs. direct

- models are useful for design & beyond
- modular → easy to debug & interpret
- id = projection on model class
- id = noise filtering
- harder to propagate uncertainty through id
- no (robust) separation principle → suboptimal
- …



- some models are too complex to be useful
- end-to-end → suitable for non-experts
- harder to inject side info but no bias error
- noise handled in design
- transparent: no unmodeled dynamics
- possibly optimal but often less tractable
- …

lots of pros, cons, counterexamples, & ***no universal conclusions*** [`discussion`]

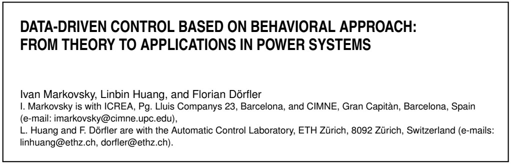
Today's menu

1. {behavioral systems} ∩ {subspace ID}: ***fundamental lemma***
2. potent direct method: data-enabled predictive control ***DeePC***
3. salient ***regularizations*** for robustification & inject side info
4. ***case studies*** from robotics & energy domain + tomatoes ☺

blooming literature (2-3 ArXiv / week)

→ tutorial [`link`] to get started

- [`link`] to graduate school material
- [`link`] to survey
- [`link`] to related bachelor lecture
- [`link`] to related publications



DATA-DRIVEN CONTROL BASED ON BEHAVIORAL APPROACH:
FROM THEORY TO APPLICATIONS IN POWER SYSTEMS

Ivan Markovsky, Linbin Huang, and Florian Dörfler
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Behavioral view on dynamical systems

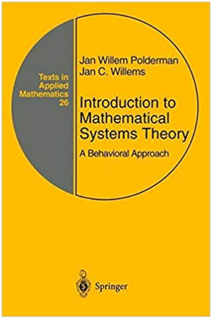
Definition: A discrete-time ***dynamical system*** is a 3-tuple $(\mathbb{Z}_{\geq 0}, \mathbb{W}, \mathscr{B})$ where

(i) $\mathbb{Z}_{\geq 0}$ is the *discrete-time* axis,

(ii) $\mathbb{W}$ is the *signal space*, &

(iii) $\mathscr{B} \subseteq \mathbb{W}^{\mathbb{Z}_{\geq 0}}$ is the *behavior*.

$\mathscr{B}$ is the set of all trajectories



Texts in Applied Mathematics 26

Jan Willem Polderman
Jan C. Willems

Introduction to Mathematical Systems Theory

A Behavioral Approach

Springer

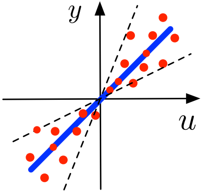
Definition: The dynamical system $(\mathbb{Z}_{\geq 0}, \mathbb{W}, \mathscr{B})$ is

(i) ***linear*** if $\mathbb{W}$ is a vector space & $\mathscr{B}$ is a subspace of $\mathbb{W}^{\mathbb{Z}_{\geq 0}}$

(ii) & ***time-invariant*** if $\mathscr{B} \subseteq \sigma\mathscr{B}$, where $\sigma w_t = w_{t+1}$.

LTI system $=$ shift-invariant subspace of trajectory space

$\longrightarrow$ abstract perspective suited for ***data-driven control***



LTI systems & matrix time series

foundation of subspace system identification & signal recovery algorithms

$\big(u(t), y(t)\big)$ satisfy LTI ***difference equation***

$$b_0 u_t + b_1 u_{t+1} + \ldots + b_n u_{t+n} + a_0 y_t + a_1 y_{t+1} + \ldots + a_n y_{t+n} = 0$$

(ARX / kernel representation)

$$\Rightarrow$$

$$\Leftarrow$$

under assumptions

$\begin{bmatrix} 0 & b_0 & a_0 & b_1 & a_1 & \ldots & b_n & a_n & 0 \end{bmatrix}$ in left nullspace of ***trajectory matrix*** (collected data)

$$\mathscr{H}\begin{pmatrix} u^d \\ y^d \end{pmatrix} = \begin{array}{c} \begin{bmatrix} \begin{pmatrix} u^d_{1,1} \\ y^d_{1,1} \end{pmatrix} & \begin{pmatrix} u^d_{1,2} \\ y^d_{1,2} \end{pmatrix} & \begin{pmatrix} u^d_{1,3} \\ y^d_{1,3} \end{pmatrix} & \cdots \\ \begin{pmatrix} u^d_{2,1} \\ y^d_{2,1} \end{pmatrix} & \begin{pmatrix} u^d_{2,2} \\ y^d_{2,2} \end{pmatrix} & \begin{pmatrix} u^d_{2,3} \\ y^d_{2,3} \end{pmatrix} & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ \begin{pmatrix} u^d_{T,1} \\ y^d_{T,1} \end{pmatrix} & \begin{pmatrix} u^d_{T,2} \\ y^d_{T,2} \end{pmatrix} & \begin{pmatrix} u^d_{T,3} \\ y^d_{T,3} \end{pmatrix} & \cdots \end{bmatrix} \\ \begin{matrix} \underbrace{\qquad\qquad}_{\text{1st experiment}} & \underbrace{\qquad\qquad}_{\text{2nd}} & \underbrace{\qquad\qquad}_{\text{3rd} \ldots} \end{matrix} \end{array}$$

Fundamental Lemma

Given: data $\begin{pmatrix} u_i^d \\ y_i^d \end{pmatrix} \in \mathbb{R}^{m+p}$ & LTI complexity parameters $\left\{ \begin{array}{l} \text{lag } \ell \\ \text{order } n \end{array} \right.$

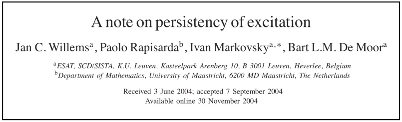
$$\underbrace{\text{set of all } T\text{-length trajectories} = \left\{ (u, y) \in \mathbb{R}^{(m+p)T} : \exists x \in \mathbb{R}^{nT} \text{ s.t. } x^+ = Ax + Bu\,,\ y = Cx + Du \right\}}_{\text{parametric state-space model}} = \underbrace{\text{colspan} \begin{bmatrix} \begin{pmatrix} u_{1,1}^d \\ y_{1,1}^d \end{pmatrix} & \begin{pmatrix} u_{1,2}^d \\ y_{1,2}^d \end{pmatrix} & \begin{pmatrix} u_{1,3}^d \\ y_{1,3}^d \end{pmatrix} & \cdots \\ \begin{pmatrix} u_{2,1}^d \\ y_{2,1}^d \end{pmatrix} & \begin{pmatrix} u_{2,2}^d \\ y_{2,2}^d \end{pmatrix} & \begin{pmatrix} u_{2,3}^d \\ y_{2,3}^d \end{pmatrix} & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ \begin{pmatrix} u_{T,1}^d \\ y_{T,1}^d \end{pmatrix} & \begin{pmatrix} u_{T,2}^d \\ y_{T,2}^d \end{pmatrix} & \begin{pmatrix} u_{T,3}^d \\ y_{T,3}^d \end{pmatrix} & \cdots \end{bmatrix}}_{\text{raw data (every column is an experiment)}}$$

if and only if the trajectory matrix has rank $m \cdot T + n$ for all $T \geq \ell$

$$\text{set of all } T\text{-length trajectories } = \left\{ (u, y) \in \mathbb{R}^{(m+p)T} : \exists x \in \mathbb{R}^{nT} \text{ s.t. } x^+ = Ax + Bu\,,\; y = Cx + Du \right\} \quad = \quad \text{colspan} \begin{bmatrix} \begin{pmatrix} u^d_{1,1} \\ y^d_{1,1} \end{pmatrix} & \begin{pmatrix} u^d_{1,2} \\ y^d_{1,2} \end{pmatrix} & \begin{pmatrix} u^d_{1,3} \\ y^d_{1,3} \end{pmatrix} & \dots \\ \begin{pmatrix} u^d_{2,1} \\ y^d_{2,1} \end{pmatrix} & \begin{pmatrix} u^d_{2,2} \\ y^d_{2,2} \end{pmatrix} & \begin{pmatrix} u^d_{2,3} \\ y^d_{2,3} \end{pmatrix} & \dots \\ \vdots & \vdots & \vdots & \vdots \\ \begin{pmatrix} u^d_{T,1} \\ y^d_{T,1} \end{pmatrix} & \begin{pmatrix} u^d_{T,2} \\ y^d_{T,2} \end{pmatrix} & \begin{pmatrix} u^d_{T,3} \\ y^d_{T,3} \end{pmatrix} & \dots \end{bmatrix}$$

all trajectories constructible from finitely many previous trajectories

- ***standing on the shoulders of giants***: classic Willems' result was only "if" & required further assumptions: Hankel, persistency of excitation, controllability



A note on persistency of excitation

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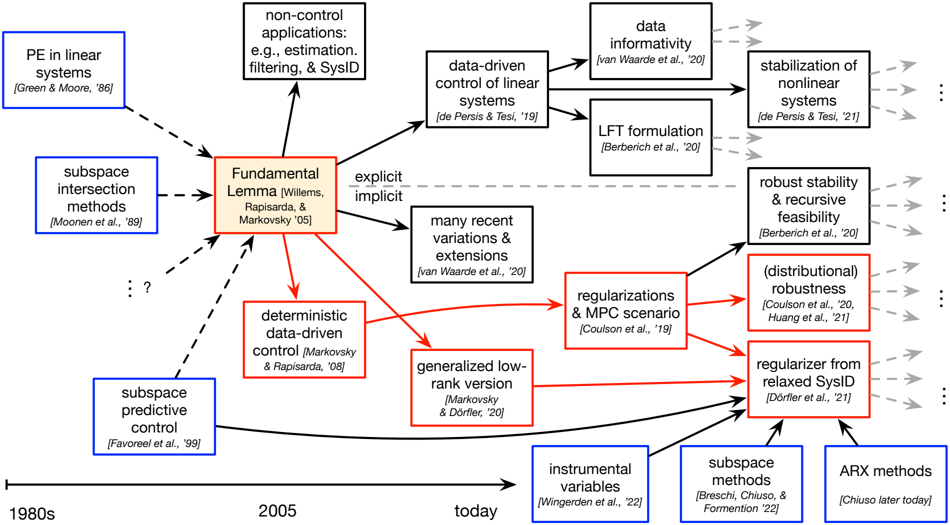
- terminology ***fundamental*** is justified : motion primitives, subspace SysID, dictionary learning, (E)DMD, . . . all implicitly rely on this equivalence

- many recent ***extensions*** to other ***system classes*** (bi-linear, descriptor, LPV, delay, Volterra series, Wiener-Hammerstein, . . .), other ***matrix data structures*** (mosaic Hankel, Page, . . .), & other ***proof methods***

Bird’s view: SysID & today’s path



PE in linear systems *[Green & Moore, ’86]*

subspace intersection methods *[Moonen et al., ’89]*

⋮ ?

subspace predictive control *[Favoreel et al., ’99]*

non-control applications: e.g., estimation. filtering, & SysID

Fundamental Lemma [Willems, Rapisarda, & Markovsky ’05]

explicit

implicit

deterministic data-driven control *[Markovsky & Rapisarda, ’08]*

data-driven control of linear systems *[de Persis & Tesi, ’19]*

many recent variations & extensions *[van Waarde et al., ’20]*

generalized low-rank version *[Markovsky & Dörfler, ’20]*

data informativity *[van Waarde et al., ’20]*

LFT formulation *[Berberich et al., ’20]*

regularizations & MPC scenario *[Coulson et al., ’19]*

stabilization of nonlinear systems *[de Persis & Tesi, ’21]*

robust stability & recursive feasibility *[Berberich et al., ’20]*

(distributional) robustness *[Coulson et al., ’20, Huang et al., ’21]*

regularizer from relaxed SysID *[Dörfler et al., ’21]*

⋮

⋮

⋮

⋮

instrumental variables *[Wingerden et al., ’22]*

subspace methods *[Breschi, Chiuso, & Formention ’22]*

ARX methods *[Chiuso later today]*

1980s

2005

today

Output **M**odel **P**redictive **C**ontrol (**MPC**)

$$
\begin{aligned}
\underset{u,x,y}{\text{minimize}} \quad & \sum_{k=1}^{T_{\text{future}}} \|y_k - r_k\|_Q^2 + \|u_k\|_R^2 \\
\text{subject to} \quad & \left.\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k + Du_k \end{aligned}\right\} \quad \forall k \in \{1, \ldots, T_{\text{future}}\} \\
& \left.\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k + Du_k \end{aligned}\right\} \quad \forall k \in \{-T_{\text{ini}} - 1, \ldots, 0\} \\
& \left.\begin{aligned} u_k &\in \mathcal{U} \\ y_k &\in \mathcal{Y} \end{aligned}\right\} \quad \forall k \in \{1, \ldots, T_{\text{future}}\}
\end{aligned}
$$

quadratic cost with $R \succ 0, Q \succeq 0$ & ref. r

model for **prediction** with $k \in [1, T_{\text{future}}]$

model for **estimation** with $k \in [-T_{\text{ini}} - 1, 0]$ & $T_{\text{ini}} \geq$ lag (many flavors)

hard operational or safety **constraints**

> *"[MPC] has perhaps too little system theory and too much* ***brute force*** *[...], but MPC is an area where all aspects of the field [...] are in synergy." – Willems '07*

Elegance aside, for an LTI plant, deterministic, & with known model, MPC is the ***gold standard of control***.

Data-**e**nabl**e**d **P**redictive **C**ontrol (**DeePC**)

$$\begin{aligned}
\underset{g,u,y}{\text{minimize}} \quad & \sum_{k=1}^{T_{\text{future}}} \|y_k - r_k\|_Q^2 + \|u_k\|_R^2 \\
\text{subject to} \quad & \mathscr{H}\begin{pmatrix} u^{\mathsf{d}} \\ y^{\mathsf{d}} \end{pmatrix} \cdot g = \begin{bmatrix} u_{\text{ini}} \\ y_{\text{ini}} \\ \hline u \\ y \end{bmatrix} \\
& \left.\begin{array}{l} u_k \in \mathcal{U} \\ y_k \in \mathcal{Y} \end{array}\right\} \quad \forall k \in \{1, \dots, T_{\text{future}}\}
\end{aligned}$$

quadratic cost with $R \succ 0, Q \succeq 0$ & ref. r

non-parametric model for **prediction** and **estimation**

hard operational or safety **constraints**

- real-time measurements $(u_{\text{ini}}, y_{\text{ini}})$ for estimation — updated **online**
- trajectory matrix $\mathscr{H}\begin{pmatrix} u^{\mathsf{d}} \\ y^{\mathsf{d}} \end{pmatrix}$ from past experimental data — collected **offline** (could be adapted online)

→ ***equivalent to MPC*** in deterministic LTI case . . .

but needs to be robustified in case of noise / nonlinearity !

Regularizations make it work

$$\underset{g,\,u,\,y,\,\sigma}{\text{minimize}} \quad \sum_{k=1}^{T_{\text{future}}} \|y_k - r_k\|_Q^2 + \|u_k\|_R^2 + \lambda_y \|\sigma\|_p + \lambda_g h(g)$$

$$\text{subject to} \quad \mathscr{H}\begin{pmatrix} u^{\mathsf{d}} \\ y^{\mathsf{d}} \end{pmatrix} \cdot g = \left[\begin{array}{c} u_{\text{ini}} \\ y_{\text{ini}} \\ \hline u \\ y \end{array}\right] + \begin{bmatrix} 0 \\ \sigma \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{matrix} u_k \in \mathcal{U} \\ y_k \in \mathcal{Y} \end{matrix}\right\} \quad \forall k \in \{1, \dots, T_{\text{future}}\}$$

measurement noise

→ infeasible y_{ini} estimate

→ estimation slack σ

→ moving-horizon least-square filter

noisy or nonlinear (offline) **data matrix**

→ any $\binom{u}{y}$ feasible

→ add regularizer $h(g)$

Bayesian intuition: regularization ⇔ prior, e.g., $h(g) = \|g\|_1$ sparsely selects {trajectory matrix columns} ∼ low-order basis ∼ low-rank surrogate

Robustness intuition: regularization ⇔ robustifies, e.g., in a simple case

$$\min_x \max_{\|\Delta\| \leq \rho} \|(A+\Delta)x - b\| \underset{\text{tight}}{\leq} \min_x \max_{\|\Delta\| \leq \rho} \|Ax - b\| + \|\Delta x\| = \min_x \|Ax - b\| + \rho\|x\|$$

regularization

$\Updownarrow$

incorporating priors
+ implicit SysID

Regularization = relaxing low-rank approximation in pre-processing

$$\begin{aligned} \text{minimize}_{u,y,g} \quad & \text{control cost}\left(u,y\right) \\ \text{subject to} \quad & \begin{bmatrix} u \\ y \end{bmatrix} = \mathscr{H}\begin{pmatrix} \hat{u} \\ \hat{y} \end{pmatrix} g \\ \text{where} \quad & \begin{pmatrix} \hat{u} \\ \hat{y} \end{pmatrix} \in \operatorname{argmin} \left\| \begin{pmatrix} \hat{u} \\ \hat{y} \end{pmatrix} - \begin{pmatrix} u^{\mathrm{d}} \\ y^{\mathrm{d}} \end{pmatrix} \right\| \\ & \text{subject to } \operatorname{rank}\left(\mathscr{H}\left(\begin{smallmatrix} \hat{u} \\ \hat{y} \end{smallmatrix}\right)\right) = mL + n \end{aligned}$$

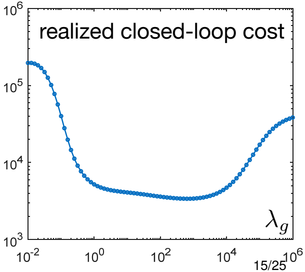
} optimal control

} low-rank approximation

↓ sequence of convex relaxations ↓

$$\begin{aligned} \text{minimize}_{u,y,g} \quad & \text{control cost}\left(u,y\right) + \lambda_g \cdot \|g\|_1 \\ \text{subject to} \quad & \begin{bmatrix} u \\ y \end{bmatrix} = \mathscr{H}\begin{pmatrix} u^{d} \\ y^{d} \end{pmatrix} g \end{aligned}$$

ℓ_1-regularization $=$ relaxation of low-rank approximation & smoothened order selection



Regularization ⇔ reformulate subspace ID

partition data as in subspace ID:

$$\mathscr{H}\begin{pmatrix}u^{\mathsf{d}}\\ y^{\mathsf{d}}\end{pmatrix} \sim \begin{bmatrix}U_{\mathsf{p}}\\ Y_{\mathsf{p}}\\ U_{\mathsf{f}}\\ Y_{\mathsf{f}}\end{bmatrix} \begin{array}{l}\left.\vphantom{\begin{matrix}U\\ Y\end{matrix}}\right\}(m+p)T_{\mathsf{ini}}\\ \left.\vphantom{\begin{matrix}U\\ Y\end{matrix}}\right\}(m+p)T_{\mathsf{future}}\end{array}$$

ID of optimal multi-step predictor

as in SPC: $K^{\star} = Y_F \begin{bmatrix}U_{\mathsf{p}}\\ Y_{\mathsf{p}}\\ U_{\mathsf{f}}\end{bmatrix}^{\dagger}$

→ ***indirect SysID + control*** problem

$$\begin{aligned} &\underset{u,y}{\text{minimize}} && \text{control cost}(u,y)\\ &\text{subject to} && y = K^{\star}\begin{bmatrix}u_{\mathsf{ini}}\\ y_{\mathsf{ini}}\\ u\end{bmatrix}\\ &\text{where} && K^{\star} = \underset{K}{\text{argmin}}\left\| Y_F - K\begin{bmatrix}U_{\mathsf{p}}\\ Y_{\mathsf{p}}\\ U_{\mathsf{f}}\end{bmatrix}\right\| \end{aligned}$$

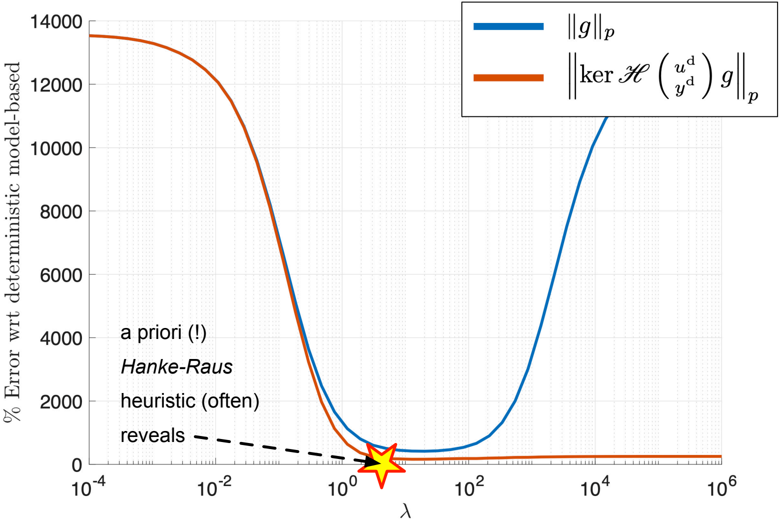
The above is ***equivalent to regularized DeePC***

where $\text{Proj}\begin{pmatrix}u^{\mathsf{d}}\\ y^{\mathsf{d}}\end{pmatrix}$ projects

orthogonal to $\ker\begin{bmatrix}U_{\mathsf{p}}\\ Y_{\mathsf{p}}\\ U_{\mathsf{f}}\end{bmatrix}$

$$\begin{aligned} &\underset{g,u,y}{\text{minimize}} && \text{control cost}(u,y) + \lambda_g \left\|\text{Proj}\begin{pmatrix}u^{\mathsf{d}}\\ y^{\mathsf{d}}\end{pmatrix} g\right\|_p\\ &\text{subject to} && \mathscr{H}\begin{pmatrix}u^{\mathsf{d}}\\ y^{\mathsf{d}}\end{pmatrix}\cdot g = \begin{bmatrix}u_{\mathsf{ini}}\\ y_{\mathsf{ini}}\\ u\\ y\end{bmatrix} \end{aligned}$$

Regularizations applied to stochastic LTI system & hyper-parameter selection



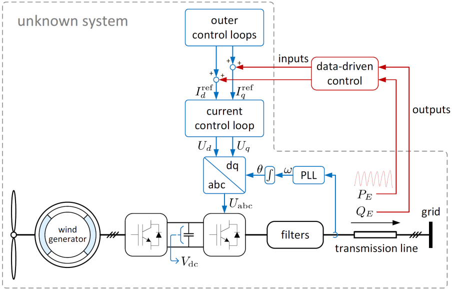
a priori (!)

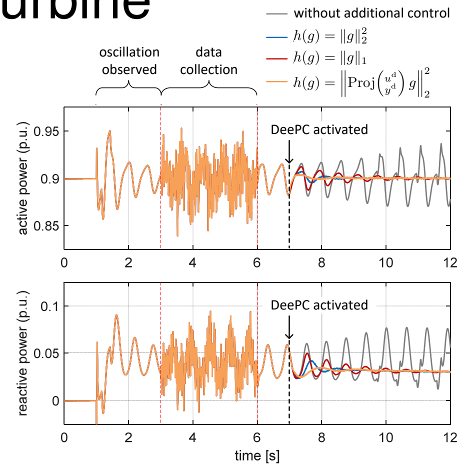
Hanke-Raus

heuristic (often)

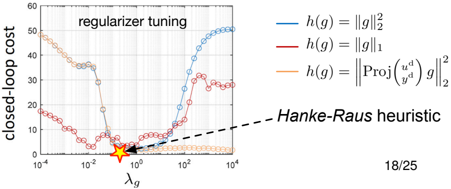
reveals

Case study: wind turbine

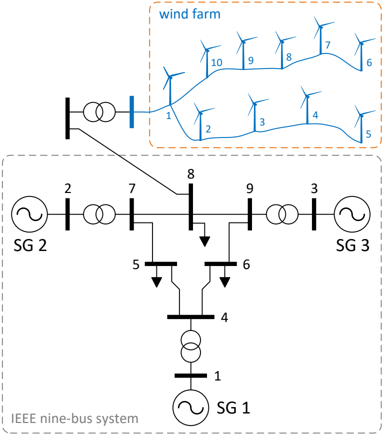


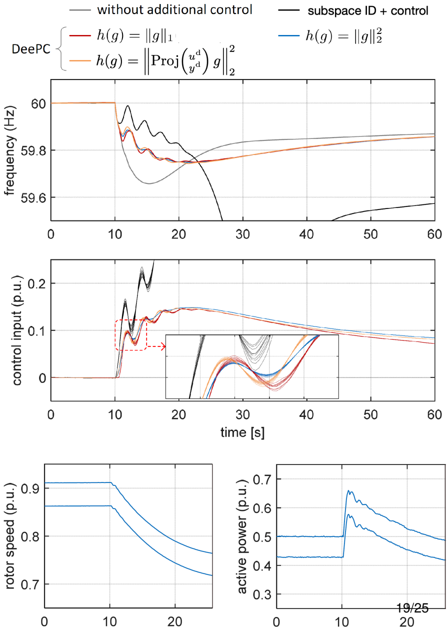


- turbine & grid model ***unknown*** to commissioning engineer & operator
- detailed ***industrial model***: 37 states & highly nonlinear (abc ↔ dq, MPTT, PLL, power specs, dynamics, etc.)
- weak grid → ***oscillations + sync loss***
- disturbance to be rejected by ***DeePC***



Case study +++ : wind farm





- ***high-fidelity models*** for turbines, machines, & IEEE-9-bus system
- ***fast frequency response*** via ***decentralized DeePC*** at turbines

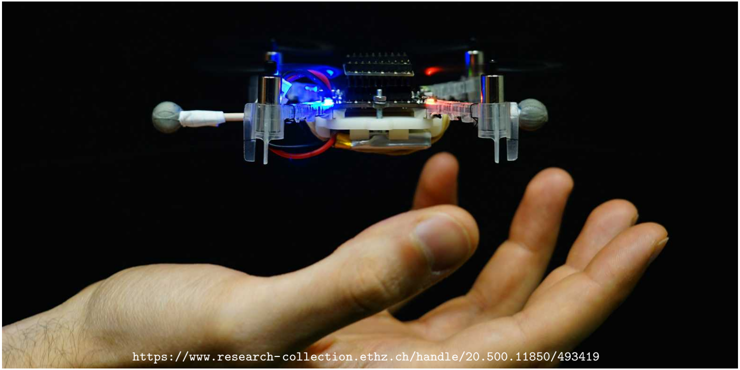
Towards a theory for nonlinear systems

naive idea: lift nonlinear system to large/∞-dim. bi-/linear system

→ Carleman, Volterra, Fliess, Koopman, Sturm-Liouville methods

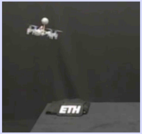
→ nonlinear dynamics can be approximated by LTI on finite horizon

regularization singles out relevant features / basis functions in data



https://www.research-collection.ethz.ch/handle/20.500.11850/493419

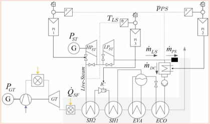
Works very well across case studies



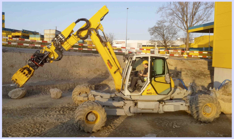
quad coptor fig-8 tracking

quadruped (by Fawcett, Afsari Amers, & Hamed)

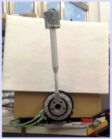
greenhouse automation (by Automatoes)



combined cycle power plant (by P Mahdavipour et. al)

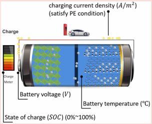


robotic excavator

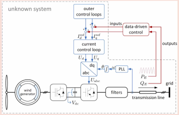


pendulum swing up

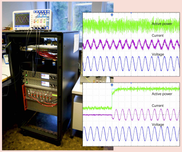
traffic coordination (by J. Wang et al.)



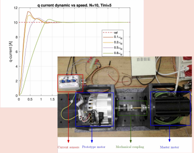
battery charging (by K. Chen et al.)



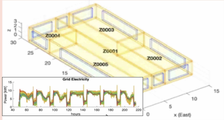
wind turbine control



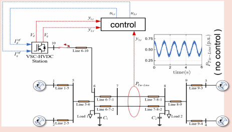
grid-connected converter



synchronous motor drive



energy hub & building automation



power system oscillation damping

regularization

$$\Updownarrow$$

robustification

Distributional robustification beyond LTI

- ***problem abstraction*** : $\min_{x \in \mathcal{X}} \; c\big(\widehat{\xi}, x\big) \;=\; \min_{x \in \mathcal{X}} \; \mathbb{E}_{\xi \sim \widehat{\mathbb{P}}}\big[c\left(\xi, x\right)\big]$

  where $\widehat{\xi}$ denotes *measured* data with *empirical distribution* $\widehat{\mathbb{P}} = \delta_{\widehat{\xi}}$

⇒ ***poor out-of-sample performance*** of above sample-average solution $x^\star$

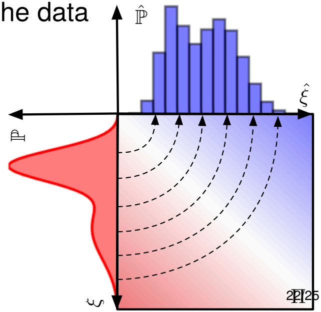
for real problem: $\mathbb{E}_{\xi \sim \mathbb{P}}\big[c\left(\xi, x^\star\right)\big]$ where $\mathbb{P}$ is the *unknown distribution* of ξ

- ***distributionally robust*** formulation accounting for all (possibly nonlinear) stochastic processes that could have generated the data

$$\inf_{x \in \mathcal{X}} \; \sup_{\mathbb{Q} \in \mathbb{B}_\epsilon(\widehat{\mathbb{P}})} \; \mathbb{E}_{\xi \sim \mathbb{Q}}\big[c\left(\xi, x\right)\big]$$

where $\mathbb{B}_\epsilon(\widehat{\mathbb{P}})$ is an ***ϵ-Wasserstein ball*** centered at empirical sample distribution $\widehat{\mathbb{P}}$:

$$\mathbb{B}_\epsilon(\widehat{\mathbb{P}}) = \left\{ \mathbb{P} \;:\; \inf_{\Pi} \int \left\| \, \xi - \widehat{\xi} \, \right\|_p d\Pi \;\leq\; \epsilon \right\}$$

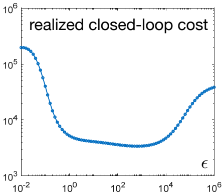


- ***distributionally robustness*** $\equiv$ ***regularization*** : under minor conditions

Theorem: $$\underbrace{\inf_{x\in\mathcal{X}} \sup_{\mathbb{Q}\in\mathbb{B}_\epsilon(\widehat{\mathbb{P}})} \mathbb{E}_{\xi\sim\mathbb{Q}}\left[c\left(\xi,x\right)\right]}_{\text{distributional robust formulation}} \equiv \underbrace{\min_{x\in\mathcal{X}} c\left(\widehat{\xi},x\right) + \epsilon\,\mathrm{Lip}(c)\cdot\|x\|_p^\star}_{\text{previous regularized DeePC formulation}}$$

Cor : ℓ_∞-robustness in trajectory space

$\iff$ ℓ_1-regularization of DeePC

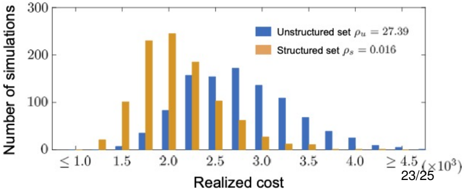


- similar for ***distributionally robust constraints***
- ***measure concentration***: average N i.i.d. data sets & $\epsilon \sim 1/N^{1/\dim(\xi)}$

  $\Longrightarrow \mathbb{P} \in \mathbb{B}_\epsilon(\widehat{\mathbb{P}})$ with high confidence
- more ***structured uncertainty sets*** :

  tractable reformulations (relaxations)

  & performance guarantees



white elephant: how does DeePC perform against SysID + control ?

surprise: **DeePC consistently beats** (certainty-equivalence) **identification & control** of LTI models across all real case studies !

why ?!?

Comparison: direct vs. indirect control

indirect* ID-based *data-driven control

minimize control cost (u, y)

subject to (u, y) satisfy parametric model

where $\left.\begin{array}{l}\text{model} \in \text{argmin id cost } (u^d, y^d) \\ \text{subject to model} \in \text{LTI}(n, \ell) \text{ class}\end{array}\right\} \text{ID}$

ID projects data on LTI class to ***learn predictor***

- with parameters (n, ℓ)
- removes noise & thus lowers variance error
- suffers bias error if plant is not in LTI(n, ℓ)

direct* regularized *data-driven control

minimize control cost (u, y) + $\lambda\cdot$ regularizer

subject to (u, y) consistent with (u^d, y^d) data

- no de-noising & no bias
- ***regularization robustifies prediction*** (not predictor)
- ***trade-off*** ID & control costs

take-away : ID wins when model class is known, noise is well behaved, & control task doesn't bias ID. Otherwise, ***DeePC can beat ID*** …it often does !

Conclusions

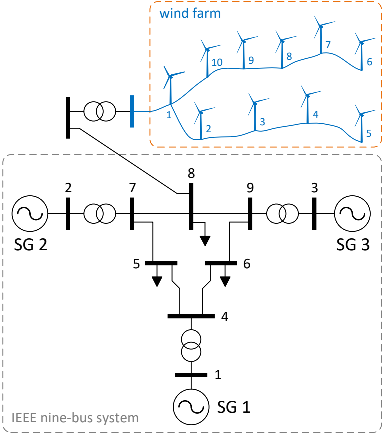
main take-aways

- matrix time series as predictive model
- robustness & side-info by regularization
- method that works in theory & practice
- focus is robust prediction not predictor ID

ongoing work

→ certificates for adaptive & nonlinear cases

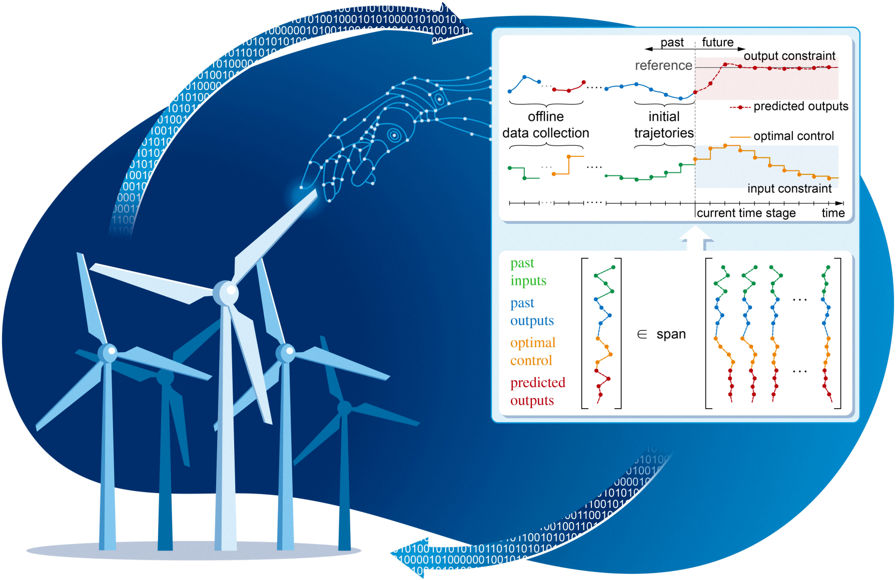
→ applications with a true “business case”, push TRL scale, & industry collaborations



questions we should discuss

- catch? violate no-free-lunch theorem ? → more real-time computation
- DeePC = subspace ID + robustification ? → more accessible & flexible
- when does direct beat indirect ? → Id4Control & bias/variance issues ?

Thanks !


past
future
reference
output constraint
predicted outputs
offline
data collection
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