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# Crystal-field effects and temperature dependence of magnetic anisotropy in rare-earth-based hard magnets

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# OUTLINE

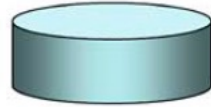
- Rare-earth (RE) based permanent magnets: performance and cost challenges
- Two-sublattice model of magnetic transition-metal-RE intermetallics
- Brief summary of crystal-field theory
- DFT+DMFT with quasiautomatic (Hubbard-I) treatment of RE  $4f$
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- High-rank crystal field, magnetization and CF excitations of  $\text{NdCo}_5$
- Complex 2-14-1 magnets: CF, anisotropy, effect of RE substitutions

# Performance of permanent magnets

Before advent of rare-earth magnets (pre 1950)



Rare-earth permanent magnets (after 1950)



**Demagnetizing field**

$$H_d = -NM \quad (0 \leq N \leq 1)$$

**vs coercivity  $H_c$**

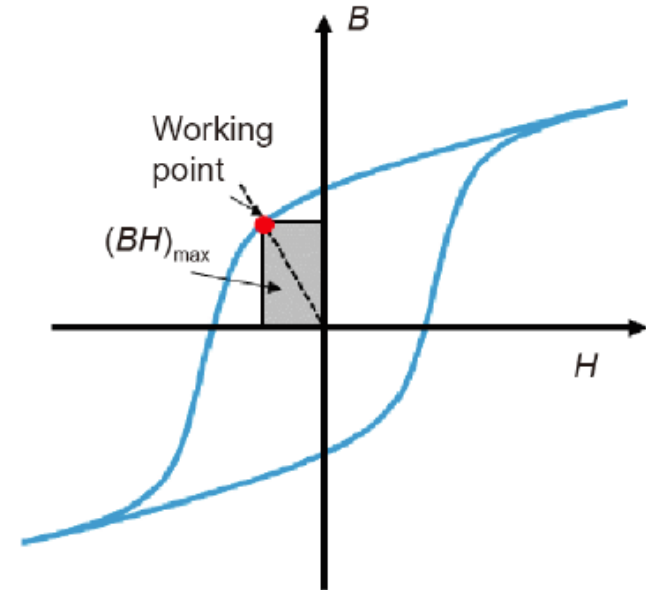
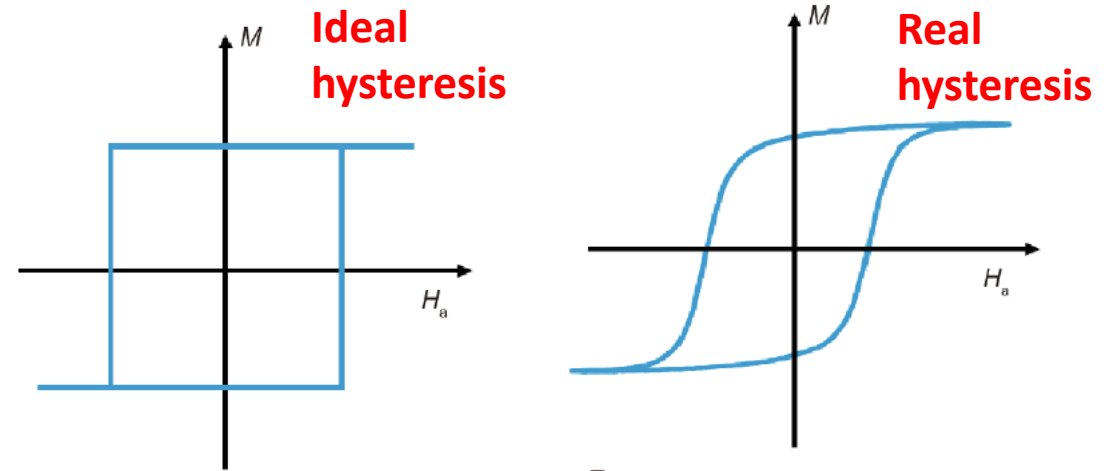
$H_c < H_d$  unless  $N \ll 1$   
shaped to minimize  $N$

$H_c > H_d$  at any  $N$   
shaped to maximize  
stray field

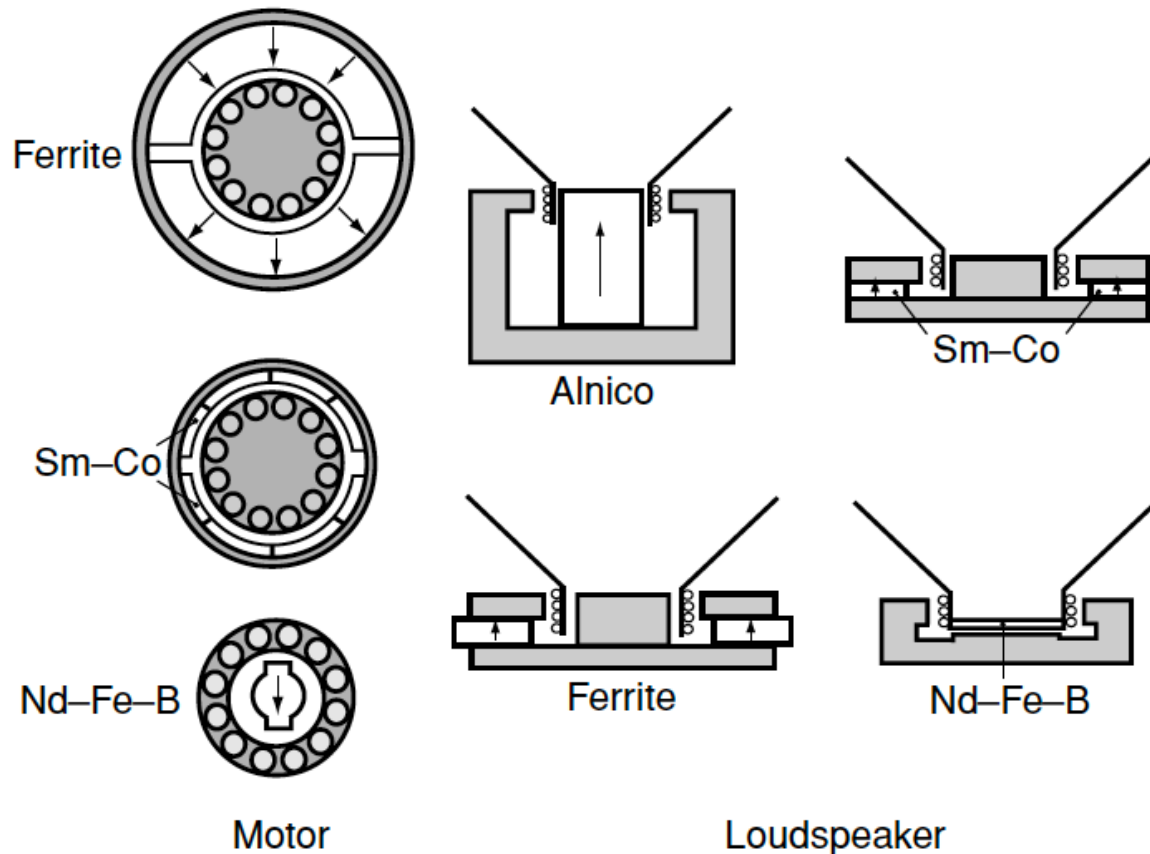
Energy stored in the stray field of ideal magnet

$$BH = \mu_0(H + M)H = \mu_0(1 - N)NM_s^2$$

Ideal magnet ( $N = 1/2$ )  $|BH|_{MAX} = \mu_0 M_s^2 / 4$



# Permanent magnets for applications

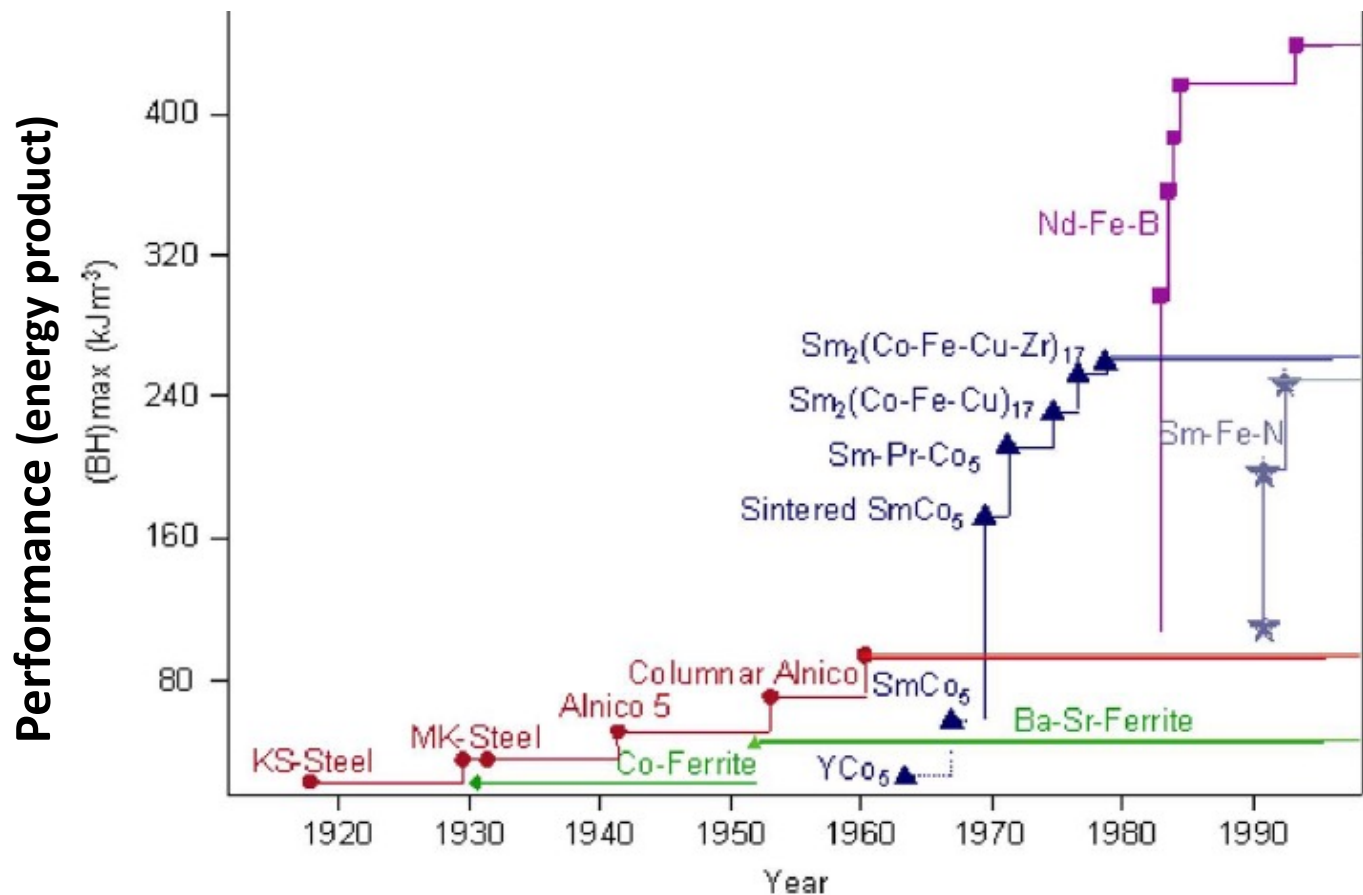


Applications need  
maximum energy product  $|BH|_{MAX}$   
at finite T!

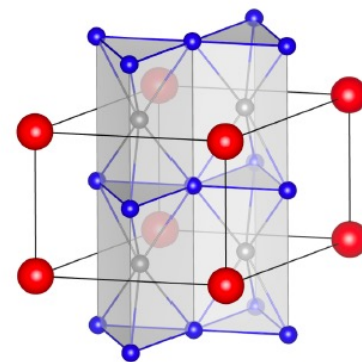
- **High magnetic density ( $M_s$ )**  
intrinsic property
- **High coercivity ( $H_c$ )**  
defined by intrinsic magnetic hardness  
(anisotropy field  $H_A$ ) and microstructure.  
Maximum  $H_c \approx 20-30\% H_A$
- **High Curie temperature ( $T_c$ )**  
intrinsic property

# Rare-earth-based permanent magnets

## Increasing performance of hard magnets in XX century

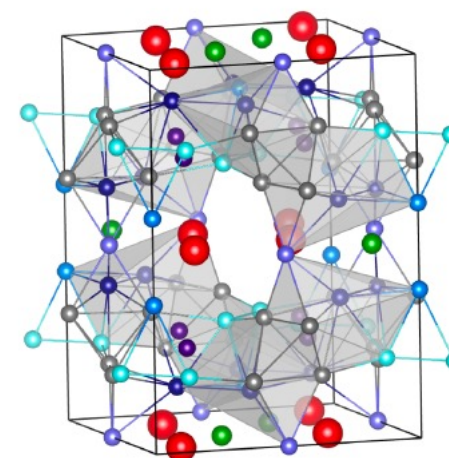
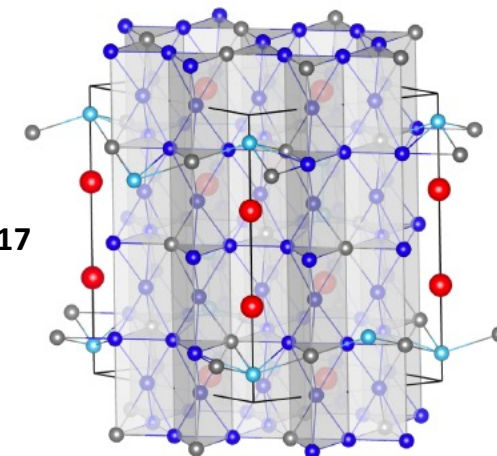


Coey IEEE Trans. Magn. 47, 4671 (2011)  
Myake & Akai JPSJ 87, 041009 (2018)



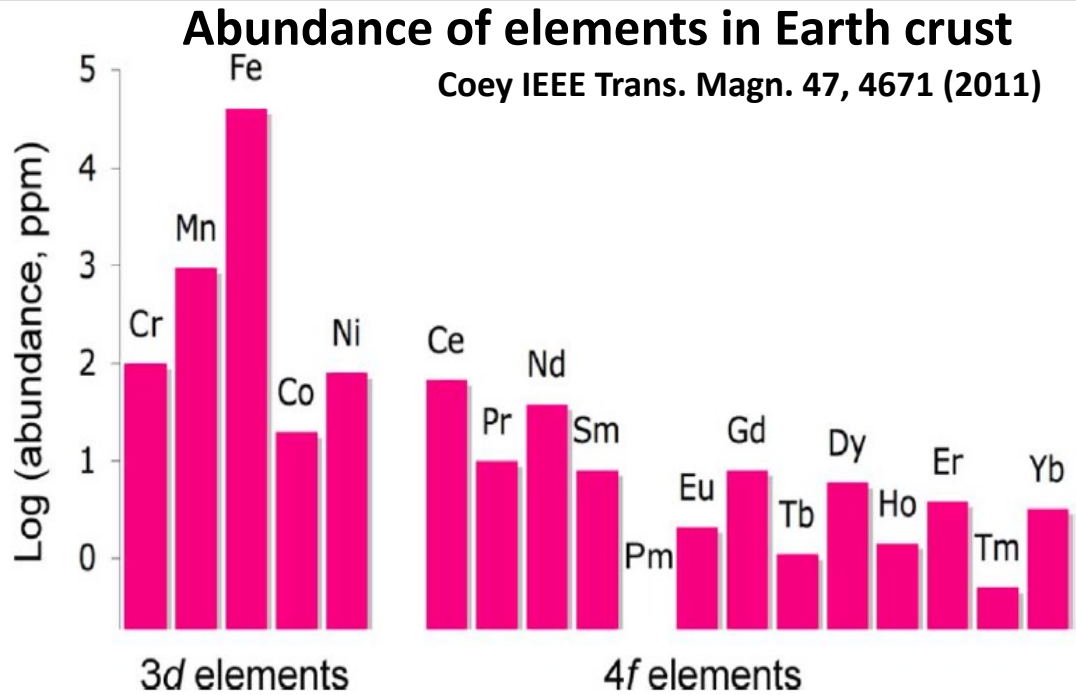
1-5 family (1960s)  
**SmCo<sub>5</sub>**

2-17 family (1970s)  
**Sm<sub>2</sub>(Co, Fe, Cu Zr)<sub>17</sub>**

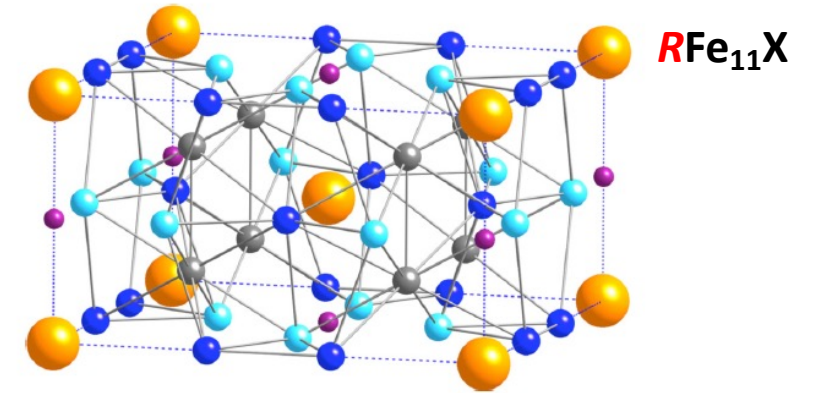


2-14-1 family (1980s)  
**Nd<sub>2</sub>Fe<sub>14</sub>B**  
**(Nd,Dy)<sub>2</sub>Fe<sub>14</sub>B**

# Rare-earth-base permanent magnets: current challenges



“1-12” family:



(i) low concentration of RE; (ii) good intrinsic properties, but (iii) poor thermodynamic stability and microstructure challenges

- Heavy REs are rare and expensive
- Light RE (Ce) do not provide sufficient magnetic hardness
- No new industry relevant RE-based permanent magnets family discovered since 1980s

**Goals: predict intrinsic hardness (magnetocrystalline anisotropy)**

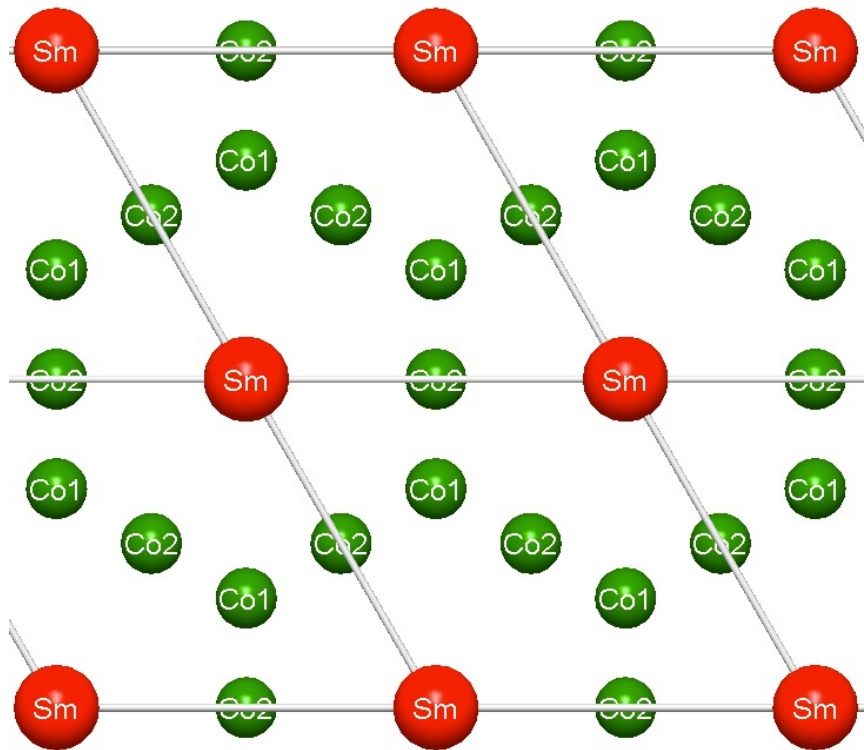
**in perspective TM-RE systems to search for new RE permanent magnets**

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# Hard-magnetic intermetallics: a two-sublattice picture

SmCo<sub>5</sub>: view along hex c axis



- **Dense TM lattice (Fe, Co):**  
high magnetization density  
high Curie temperature
- **Rare-earth sublattice:**  
large spin-orbit → high single-ion anisotropy

**Hierarchy of intersite interactions:**

$$(3d-3d) \gg (3d-4f) \gg \cancel{(4f-4f)}$$

- **TM: strong spin-polarization, no dynamical correlations**



**magnetic GS in DFT**

- **RE: atomic 4f multiplets perturbed by environment (crystal and exchange fields)**



**beyond standard DFT**



# TM magnetism and TM-RE exchange coupling

## Total TM moments ( $\mu_B$ ) per site

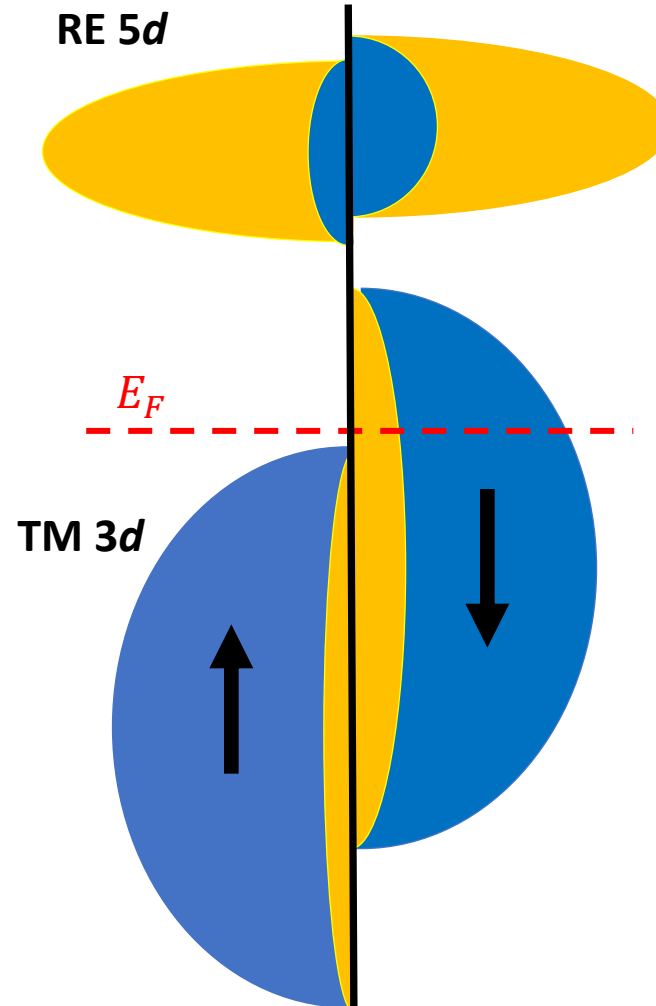
$\text{Nd}_2\text{Fe}_{14}\text{B}$	bcc-Fe	$\text{SmCo}_5$	hcp-Co
2.2	2.22	1.79	1.72

similar to elemental itinerant 3d ferromagnets

## Curie temperature (K)

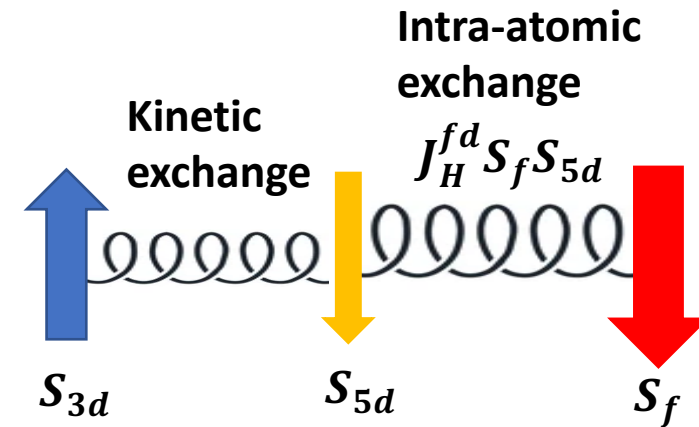
$\text{Nd}_2\text{Fe}_{14}\text{B}$	bcc-Fe	$\text{SmCo}_5$	hcp-Co
585	1044	997	1360

lower than in elemental 3d ferromagnets



$$M_S(3d) > 0$$

Hybridization-admixed 5d  
below  $E_F$   $M_S(5d) < 0$



Coey "Magnetism and magnetic materials" (Cambridge 2009)

Herbst, Rev. Mod. Phys. 1991

Givord et al. J. Appl. Phys. 1979

Campbell J. Phys. F: Met. Phys. 1972

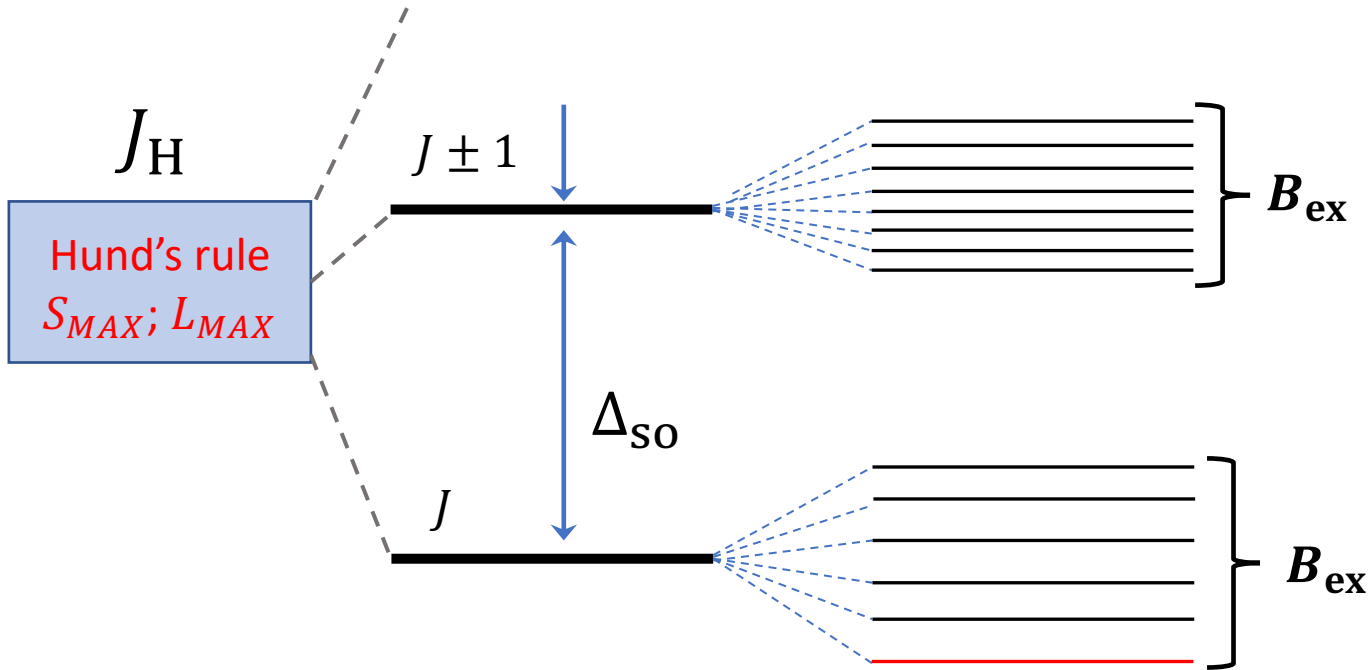
Brooks et al. J. Phys.: Cond. Mat. 1989

# Rare-earth single ion anisotropy: the origin

Hierarchy of interaction on the 4f shells in RE permanent magnets

$$U \gg J_H \gg \Delta_{so} \gg B_{ex} > \Delta_{cf}$$

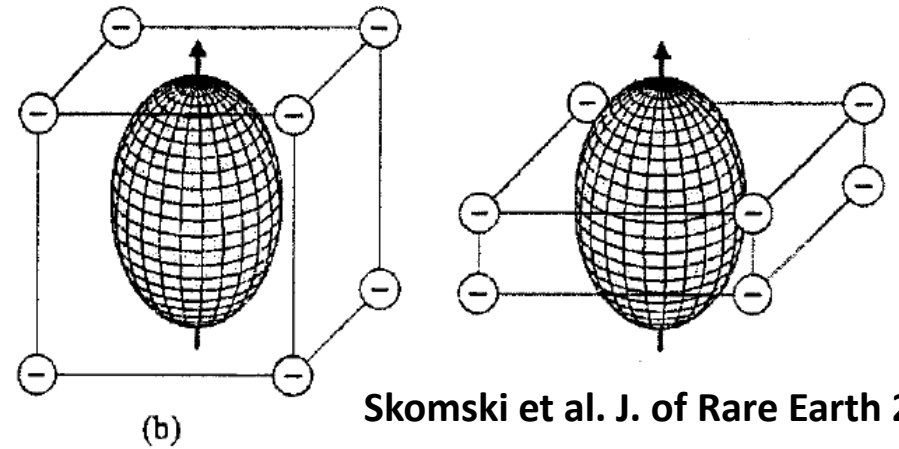
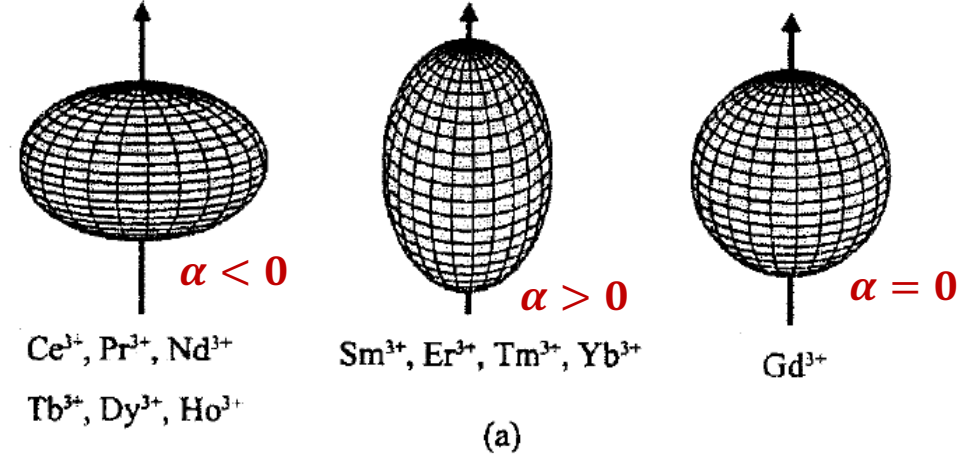
Coulomb
Hund's rule
Spin-orbit
Exchange field
Crystal field



Ground state:  $J_z = J; S_z = (g - 1)J$

4f charge density coupled to spin by SO

Charge density clouds for RE 3+ ions



Point-charge model: intuitive, but not quantitative!

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# Crystal-field formalism

e.g. Kuz'min & Tishin, in  
Handbook of Magnetic Materials,  
Vol. 17 (2007)

4f shell Hamiltonian: 
$$H_{4f} = E_0 + H_U + H_{SO} + 2\mu_B \mathbf{B}_{ex} \hat{\mathbf{S}}_f + H_{cf}$$

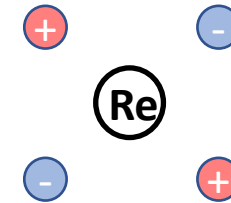
Crystal field within 4f shell 
$$\langle \varphi_m^f \sigma | H_{cf} | \varphi_{m'}^f \sigma' \rangle = \delta_{\sigma\sigma'} \langle \varphi_m^f | V_{ns}(\mathbf{r}) | \varphi_{m'}^f \rangle$$

Non-spherical 1-electron potential expanded in real spherical tensors

$$V_{ns}(\mathbf{r}) = \sum_{k=1}^{\infty} \sum_{q=0}^k A_k^q(r) T_k^q(\hat{r})$$

$$T_k^q(\hat{r}) = \begin{cases} Y_k^0(\hat{r})/N_k & q = 0 \\ [Y_k^q(\hat{r}) + Y_k^{-q}(\hat{r})]/N_k & q > 0 \end{cases}$$

[in point-charge model  $\langle \varphi^f | A_k^q(r) | \varphi^f \rangle = \langle \varphi^f | A_k^q r^k | \varphi^f \rangle = A_k^q \langle r^k \rangle \rightarrow$  popular CFP notation]



CF is a one-electron operator: 
$$H_{cf} = \sum_{kq} A_k^q \langle r^k \rangle \hat{T}_k^q ; \quad \hat{T}_k^q = \sum_{mm'\sigma} \langle Y_3^m | T_k^q | Y_3^{m'} \rangle f_{m\sigma}^\dagger f_{m'\sigma}$$

Only even  $k = 2, \dots, 2l (= 2, 4, 6$  for  $f$ -electrons) contribute  $\Rightarrow$  max. 15 crystal-field parameters (CFP) in total  
 $0 \leq q \leq k$  in a system with inversion  $[3 A_2^q + 5 A_4^q + 7 A_6^q]$

Reduced by point-group symmetry: only 2 independent CFP in cubic symmetry  
 $A_4^0$  and  $A_6^0$

4 CFPs in hexagonal case  
 $A_2^0, A_4^0, A_6^0$  and  $A_6^6$

# Full two-sublattice model at finite temperature: our approach

$$F = F_{3d} + F_{4f} + F_Z$$

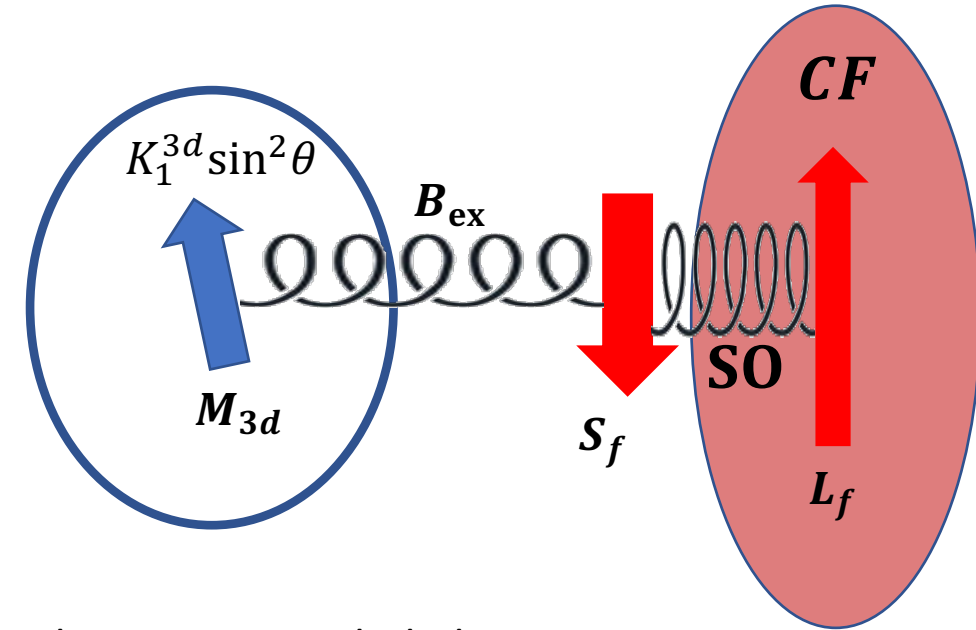
$$F_{3d} = K_1^{3d}(T) \sin^2 \theta \quad \text{TM anisotropy}$$

$$F_{4f} = -T \ln \text{Tr}[\exp(-H_{4f}/T)] \quad \text{RE ions in quasiatomic picture}$$

$$H_{4f} = H_{SO} + H_{ex} + H_{cf}$$

$$= \lambda_{SO} \mathbf{S}_f \mathbf{L}_f + 2\mu_B B_{ex}(T) \mathbf{n}_S^{3d} \mathbf{S}_f + \sum_{kq} A_k^q \langle r^k \rangle \hat{T}_k^q \quad \text{multiplet mixing is included}$$

$$F_Z = -H_{ext}[M_{4f}(T) + M_{3d}(T)] \quad \text{external field}$$



- $F_{4f} \Rightarrow$  all parameters are from ab initio: CFPs  $A_k^q \langle r^k \rangle$ , exchange field  $B_{ex}$ , spin-orbit  $\lambda_{SO}$
- $F_{3d} \Rightarrow$  zero-temperature ab initio ( $M_{3d}, K_1^{3d}$ ), semi-empirical temperature dependence

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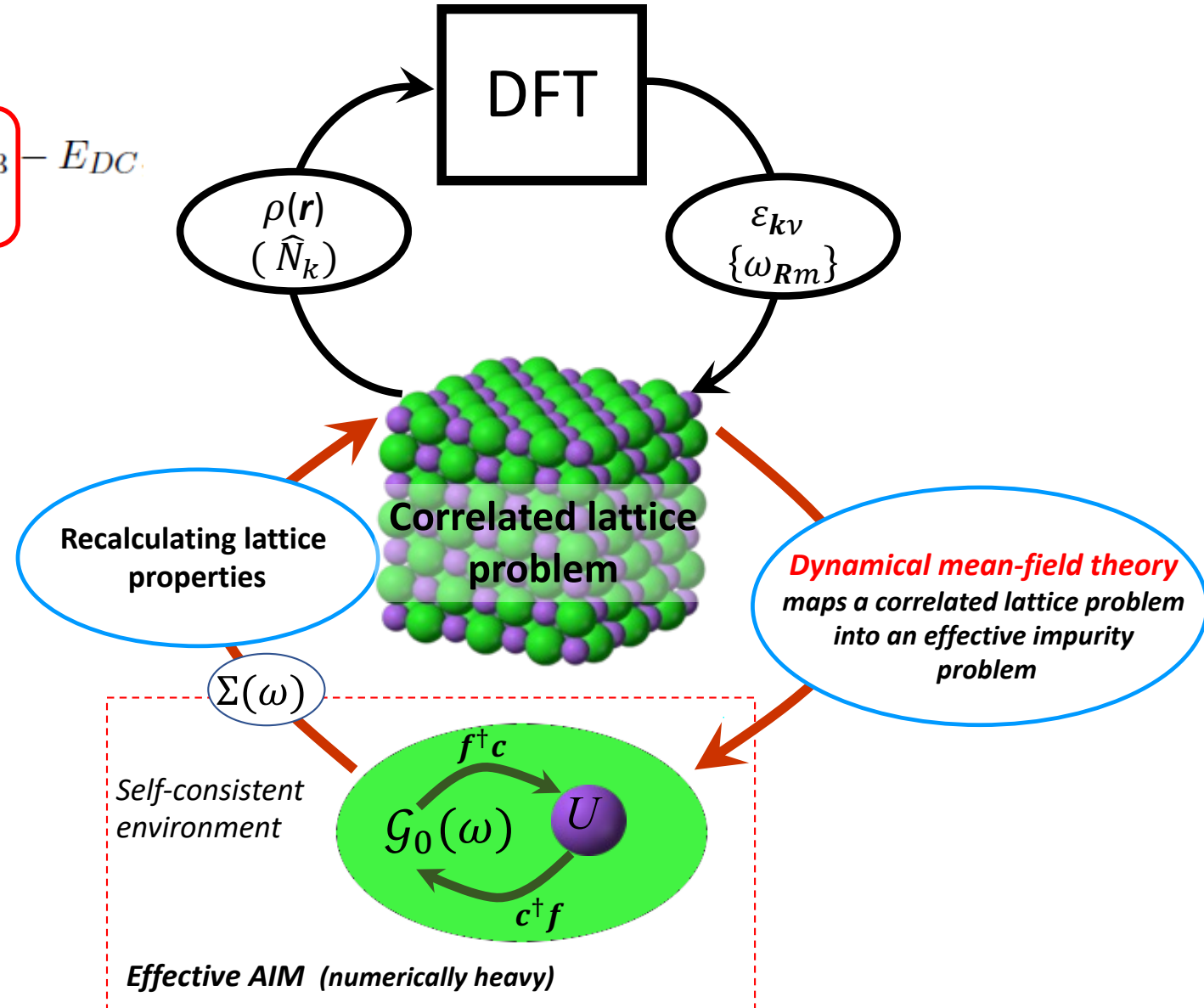
# Ab initio approach to correlated states: DFT+DMFT

Correlated lattice problem:

$$\hat{H}_{DFT+U} = \sum_{k\nu} \varepsilon_{k\nu} c_{k\nu}^\dagger c_{k\nu} + \sum_{i, 1,2,3,4} \langle 12|U|34 \rangle f_{i1}^\dagger f_{i2}^\dagger f_{i4} f_{i3} - E_{DC}$$

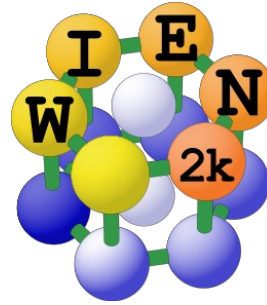
Metzner & Vollhardt PRL 1989  
 Georges & Kotliar PRB 1992  
 Anisimov et al. J. Phys. Cond.Mat. 1997  
 Lichtenstein & Katsnelson PRB 1998

some reviews:  
 Georges et al. Rev. Mod. Phys. 1996  
 Kotliar et al. Rev. Mod. Phys. 2006  
 Georges AIP Conf. Proc. 2004  
 arXiv:0403123



# DFT+DMFT: details of implementation and calculations

- Wien2k electronic structure code  
<http://susi.theochem.tuwien.ac.at/>



- “TRIQS” implementation of DMFT and its interface with Wien2k  
<https://triqs.github.io/>



M. Aichhorn et al. Computer Physics Communications 204, 200 (2016)

- $U$  and  $J_H$ : have little impact on results as long as they are sufficiently large

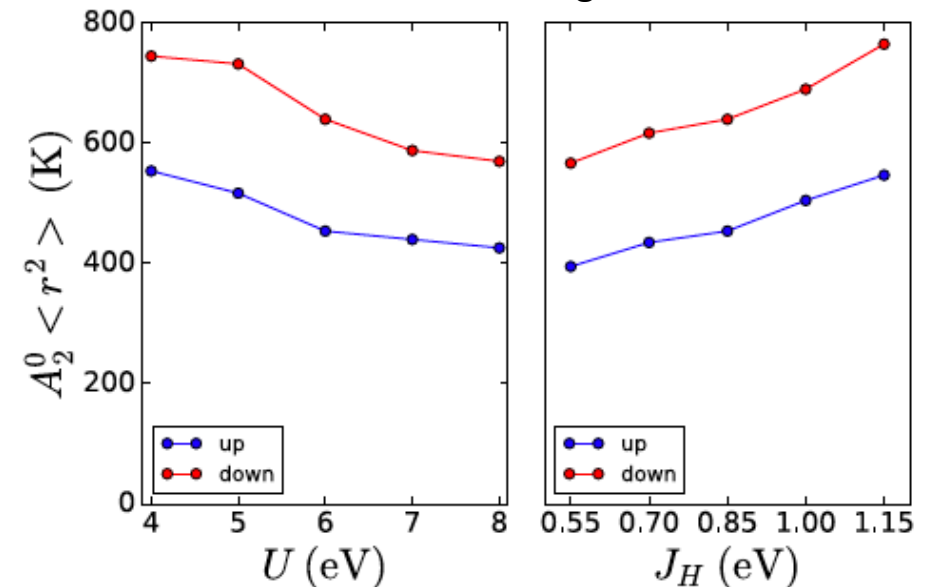
$U=6$  to  $8$  eV

$J_H$  [independent from crystalline environment for RE]  
from optical measurements on  $RF_3$

Carnall et al. J. Chem. Phys. 90, 3443 (1989)

CFP in  $NdFe_{11}N$  vs  $U$  and  $J_H$

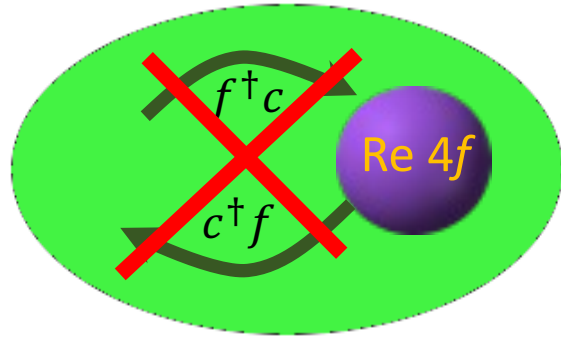
Delange et al. PRB 96, 155132





# Quasi-atomic (Hubbard-I) approximation

J. Hubbard, Proc. R. Soc. A 1963



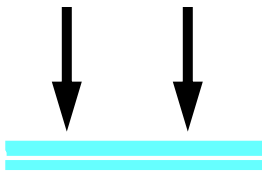
DFT 4f levels

$E_F$



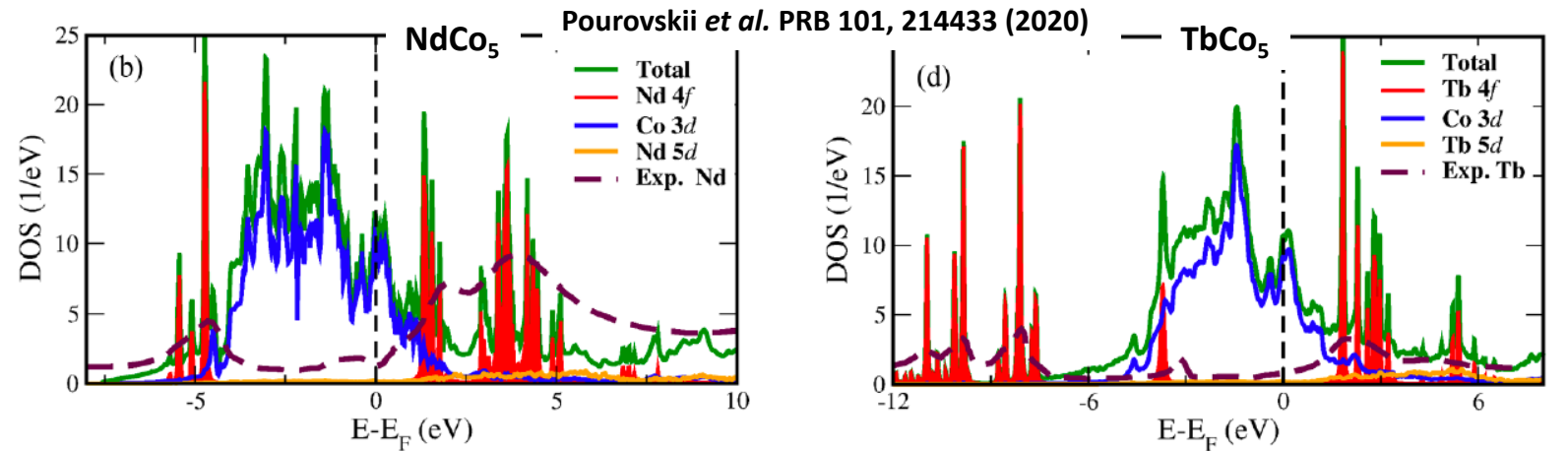
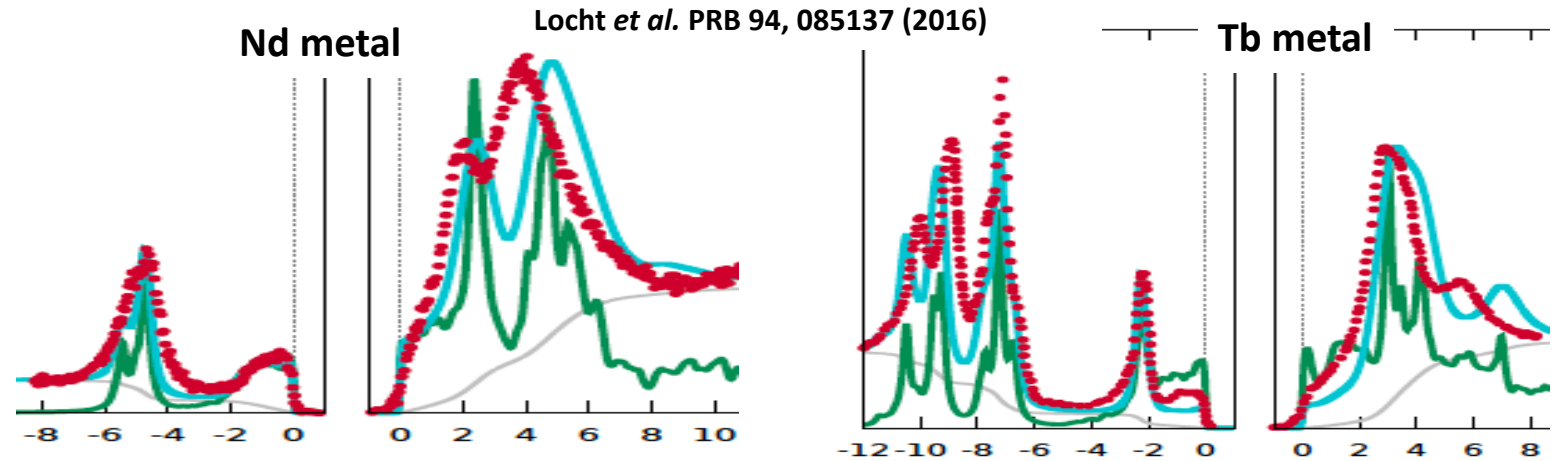
$$H_{at} = \epsilon_{ab} f_a^\dagger f_b + U_{abcd} f_a^\dagger f_b^\dagger f_d f_c$$

$$\epsilon = E0 + CF + EX + SO$$



DFT+Hubl cannot describe

- Kondo/intermediate valence effects



**BUT can be very useful to describe:**

- quasi-atomic electronic structure 4f
- crystal field (CF) and the resulting RE single-ion magnetic anisotropy

- RE  $4f$  crystal fields from DFT+Hubbard-I

PHYSICAL REVIEW B **96**, 155132 (2017)

**Crystal-field splittings in rare-earth-based hard magnets: An *ab initio* approach**

Pascal Delange,<sup>1</sup> Silke Biermann,<sup>1,2</sup> Takashi Miyake,<sup>3</sup> and Leonid Pourovskii<sup>1,2,4</sup>

# Crystal-field from DFT+Hubbard-I: a straightforward naive approach

1. Obtain one-electron levels for RE 4f shell by self-consistent DFT+Hubbard-I:

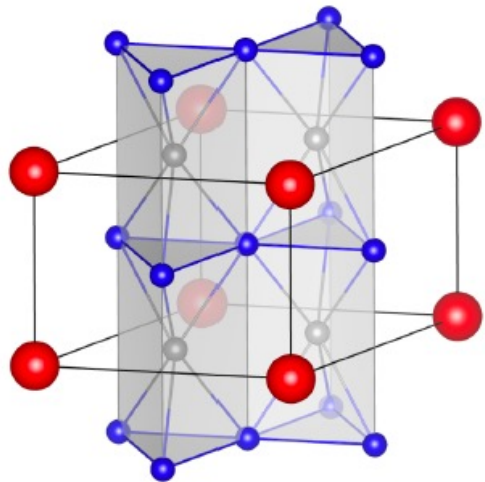
$$\epsilon = -I\mu + \sum_{\mathbf{k}} P_{\mathbf{k}} H_{\text{KS}}^{\mathbf{k}} P_{\mathbf{k}}^{\dagger} - \Sigma_{\text{dc}}$$

2. Fit them to the expected form  $H_{4f} = E_0 + 2\mu_B \mathbf{B}_{\text{ex}} \hat{S}_z + H_{\text{so}} + H_{\text{cf}} + H_U$

$$\epsilon_{mm'}^{\sigma\sigma'} = \langle \varphi_m^f \sigma | H_{4f} | \varphi_{m'}^f \sigma' \rangle = (E_0 + \mu_B B_{\text{ex}}) \delta_{mm'} \delta_{\sigma\sigma'} + \langle \varphi_m^f \sigma | \lambda_{\text{so}} \sum_i \mathbf{l}_i \mathbf{s}_i | \varphi_{m'}^f \sigma' \rangle + \sum_{kq} A_k^q \langle r^k \rangle \hat{T}_k^q$$

by a least-square  $\Rightarrow$  SO coupling  $\lambda_{\text{so}}$ , CF parameters  $A_k^q \langle r^k \rangle$  and exchange field  $B_{\text{ex}}$

Test:  $\text{SmCo}_5$



$A_2^0 \langle r^2 \rangle$ (K)	$A_4^0 \langle r^4 \rangle$ (K)	$A_6^0 \langle r^6 \rangle$ (K)	$A_6^6 \langle r^6 \rangle$ (K)	$B_{\text{ex}}$ (Tesla)
305	34	32	875	235

Estimating anisotropy =  $K_1 \sin^2 \theta$ :

$$K_1 = -3J(J - \frac{1}{2}) \alpha A_2^0 \langle r^2 \rangle$$

$$\text{Sm}^{3+}: J=5/2 \quad \alpha > 0$$

$$A_2^0 \langle r^2 \rangle > 0 \Rightarrow K_1 < 0$$

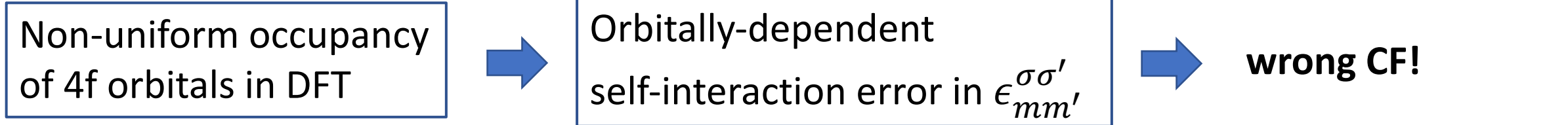
easy-plane anisotropy  
Wrong!!

# Crystal field from DFT+Hubbard-I: correcting self-interaction

**Exact XC:**  $\langle \varphi_m^f \sigma | H_H + H_{XC} | \varphi_m^f \sigma \rangle = 0$  but

with approximate XC functionals (LDA, GGA...)  $\langle \varphi_m^f \sigma | H_H + H_{XC} | \varphi_m^f \sigma \rangle \neq 0 \Rightarrow$  self-interaction!

**LDA self-interaction error impacts CF splitting** [Brooks *et al.*, PRL 79, 2546 (1997)] :

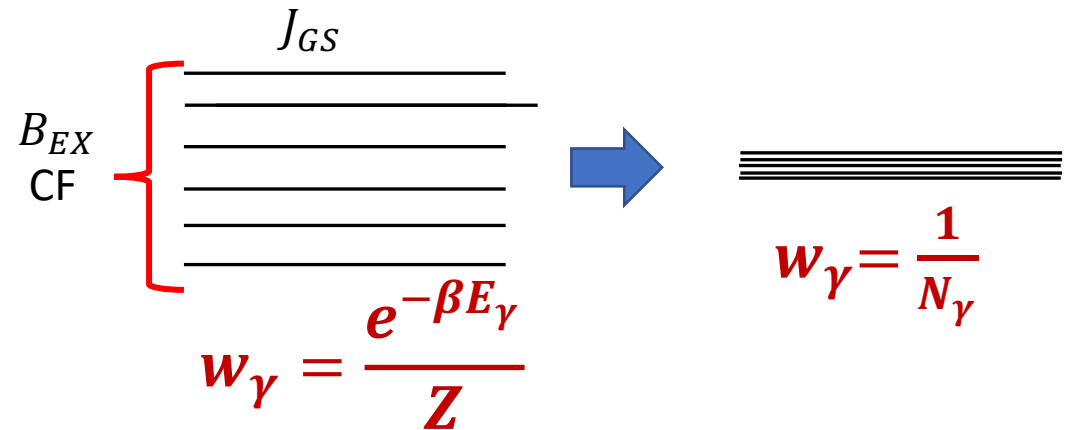


**Solution:**

**spherically averaging the ground state multiplet**  
in self-consistent DFT+Hub-I

**equal occupancy of one-electron orbitals**

**orbitally-independent self-interaction error (simple shift)**



Delange *et al.* PRB 96, 155132 (2017)

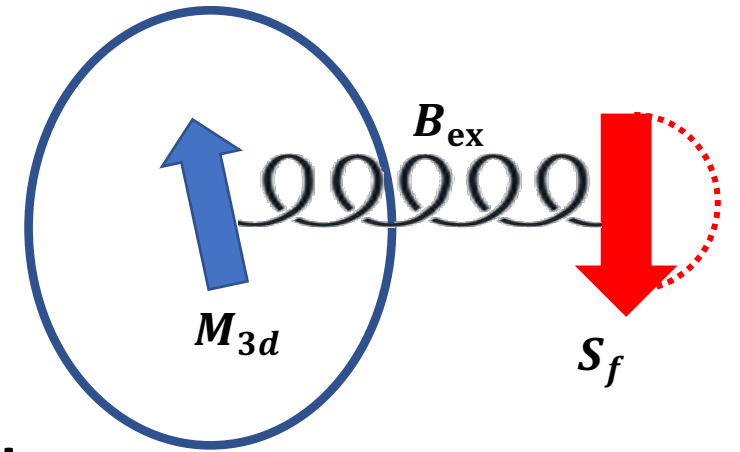
# Exchange field from DFT+Hubbard-I: suppressing spurious intra-4f contribution

TM magnetization  $\Rightarrow$  Exchange field  $\mathbf{B}_{\text{ex}} \Rightarrow$  4f spin polarization  $\langle \mathbf{S}_f \rangle$

but  $\langle \mathbf{S}_f \rangle \Rightarrow$  DFT XC potential  $\Rightarrow$  4f own exchange field ( $B_{\text{ex}}^{ff}$ )

4f Hund's rule included in  $\hat{H}_U \Rightarrow B_{\text{ex}}^{ff}$  is double counting (DC)

removing it by a DC correction is hard:  $\frac{B_{\text{ex}}}{B_{\text{ex}}^{ff}} \sim \frac{\mu_B B_{\text{ex}}}{\langle \mathbf{S}_f \rangle J_H} \sim \frac{100 \text{ to } 300 \text{ K}}{1 \text{ to } 3 \text{ eV}} < 1\% !$

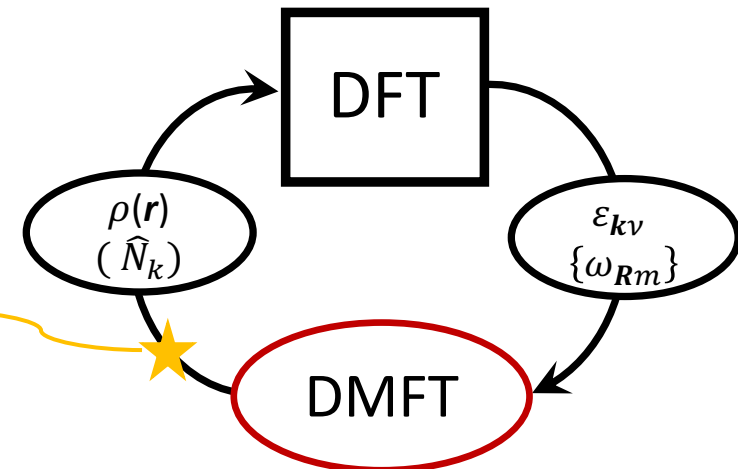


Solution: removing 4f spin polarization from output DMFT density

Delange et al. PRB 96, 155132 (2017)

$$\tilde{N}^{\mathbf{k}} = N^{\mathbf{k}} + \frac{1}{2} P^\dagger(\mathbf{k}) [T n^{ff}(\mathbf{k}) T^\dagger - n^{ff}(\mathbf{k})] P(\mathbf{k})$$

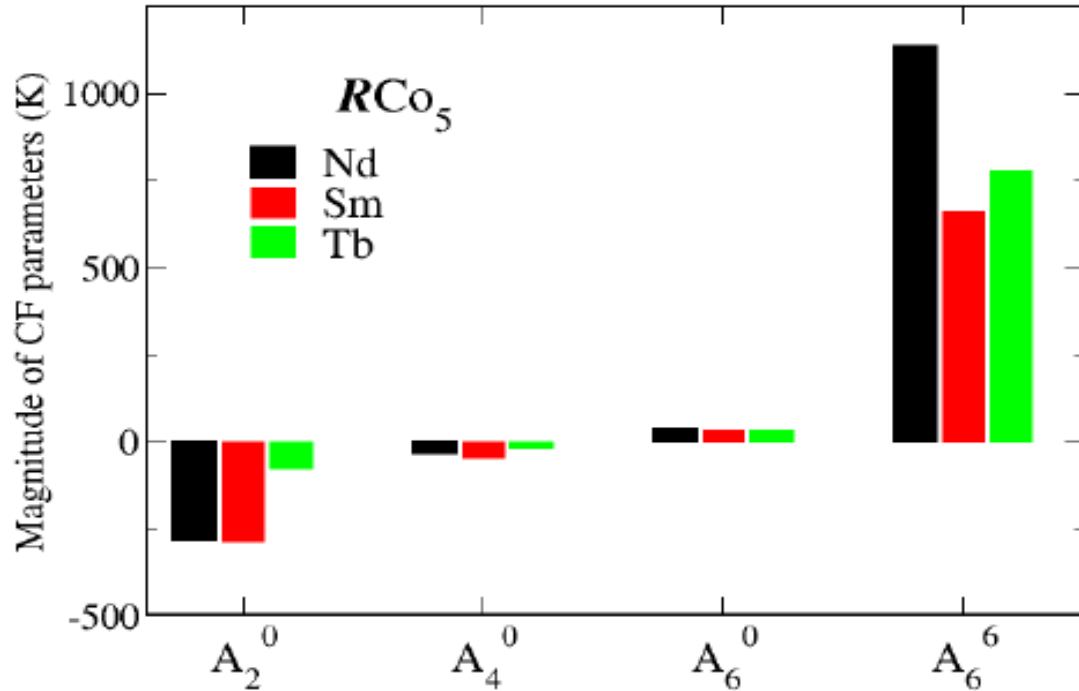
where  $T$  is the time-reversal operator



# Benchmarking on $R\text{Co}_5$ : CF parameters and states in $\text{SmCo}_5$

## Calculated crystal-field parameters ( $A_k^q \langle r^k \rangle$ ) in K

Delange et al. PRB 96, 155132 ; Pourouvkii et al. PRB 101, 214433



$$K_1 = -3J(J - \frac{1}{2}) \alpha A_2^0 \langle r^2 \rangle. \quad \text{Sm}^{3+}: \quad J=5/2 \quad \alpha > 0$$

Calculated  $A_2^0 \langle r^2 \rangle = 290 \text{ K} < 0 \Rightarrow K_1 > 0$  ! easy-axis anisotropy

Experimental estimates for  $A_2^0 \langle r^2 \rangle$  in  $\text{SmCo}_5$ : from 180 to 330 K

## Sm CF states in $\text{SmCo}_5$ : theory

1097	$+0.989 5/2; +5/2\rangle - 0.122 7/2; +5/2\rangle$
1000	$+0.977 5/2; +3/2\rangle - 0.211 7/2; +3/2\rangle$
826	$+0.977 5/2; +1/2\rangle - 0.209 7/2; +1/2\rangle$
606	$+0.973 5/2; -1/2\rangle - 0.225 7/2; -1/2\rangle$
380	$+0.983 5/2; -3/2\rangle - 0.181 7/2; -3/2\rangle$
0	$+0.984 5/2; -5/2\rangle - 0.171 7/2; -5/2\rangle$

## Exp. (magnetic form-factor) Givord et al. JAP 1979

1060	$.956 \begin{vmatrix} 5 \\ 2 \end{vmatrix} \begin{vmatrix} -3 \\ 2 \end{vmatrix} - .294 \begin{vmatrix} 7 \\ 2 \end{vmatrix} \begin{vmatrix} -3 \\ 2 \end{vmatrix}$
852	$.951 \begin{vmatrix} 5 \\ 2 \end{vmatrix} \begin{vmatrix} -1 \\ 2 \end{vmatrix} - .300 \begin{vmatrix} 7 \\ 2 \end{vmatrix} \begin{vmatrix} -1 \\ 2 \end{vmatrix}$
541	$.942 \begin{vmatrix} 5 \\ 2 \end{vmatrix} \begin{vmatrix} 1 \\ 2 \end{vmatrix} - .329 \begin{vmatrix} 7 \\ 2 \end{vmatrix} \begin{vmatrix} 1 \\ 2 \end{vmatrix}$
329	$.973 \begin{vmatrix} 5 \\ 2 \end{vmatrix} \begin{vmatrix} 3 \\ 2 \end{vmatrix} - .232 \begin{vmatrix} 7 \\ 2 \end{vmatrix} \begin{vmatrix} 3 \\ 2 \end{vmatrix}$
0	$.978 \begin{vmatrix} 5 \\ 2 \end{vmatrix} \begin{vmatrix} 5 \\ 2 \end{vmatrix} - .205 \begin{vmatrix} 7 \\ 2 \end{vmatrix} \begin{vmatrix} 5 \\ 2 \end{vmatrix}$
T(K)	

- High-rank crystal field, magnetization and CF excitations in  $\text{NdCo}_5$

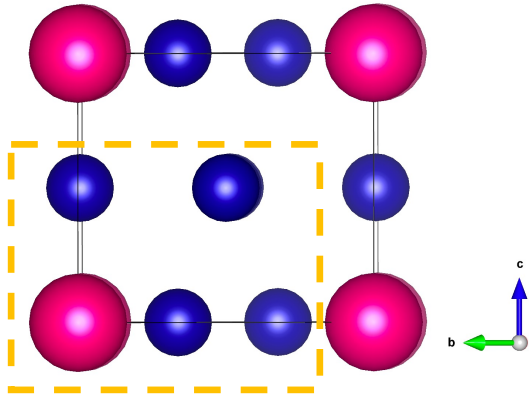
PHYSICAL REVIEW B **101**, 214433 (2020)

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**Higher-order crystal field and rare-earth magnetism in rare-earth- $\text{Co}_5$  intermetallics**

L. V. Pourovskii <sup>1,2</sup> J. Boust,<sup>1</sup> R. Ballou <sup>3</sup> G. Gomez Eslava <sup>3</sup> and D. Givord<sup>3,\*</sup>

# NdCo<sub>5</sub>: the puzzle of unsaturated moment



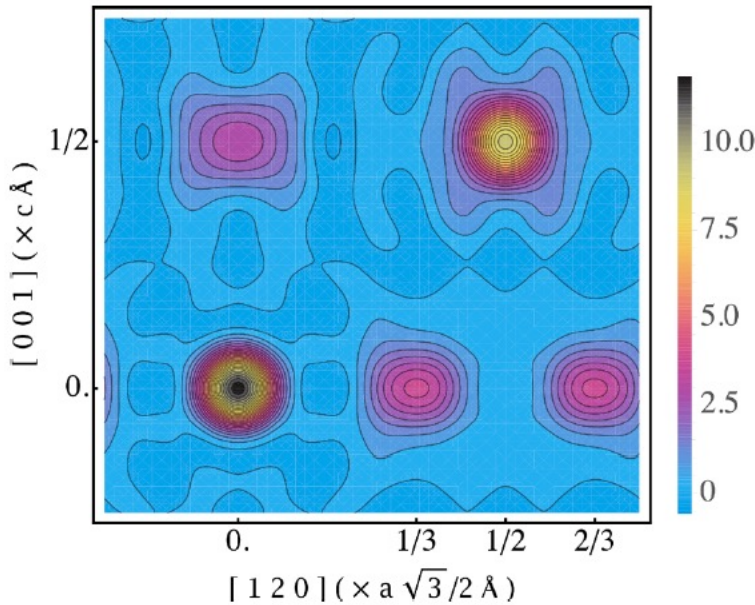
- Isostructural to SmCo<sub>5</sub>
- Spin-reorientation transition at  $T_{SR} \sim 280$  K
- $T < T_{SR}$  Nd anisotropy forces  $M_{tot} \parallel [100]$

**4f ground state from magn. form-factor exp.:**

$$|\Psi_G\rangle = .830 |9/2, -9/2\rangle - .558 |9/2, -5/2\rangle$$

Alameda et al. J. de Physique Colloque 43(C7), 133 (1981)

Projected magnetic density from measured magnetic structure factors



Pourovskii et al. PRB 101, 214433 (2020)

**GS  $M_{Nd} = 2.82 \mu_B < M_{Nd}^{Sat} = 3.27 \mu_B$**

**puzzling unsaturated Nd moment under large exchange field  $\sim 300$  Tesla**

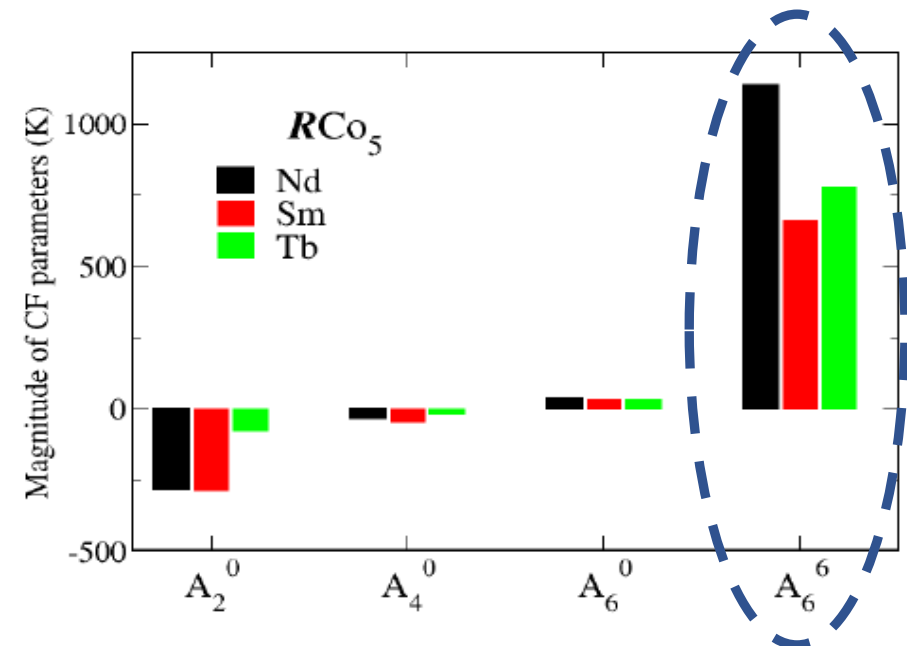
## Theoretical Nd 4f GS

(1st exited  $\sim 220$  K above)

$$0.83 |9/2; -9/2\rangle - 0.54 |9/2; -5/2\rangle$$

**with mag. mom =  $2.67 \mu_B$**

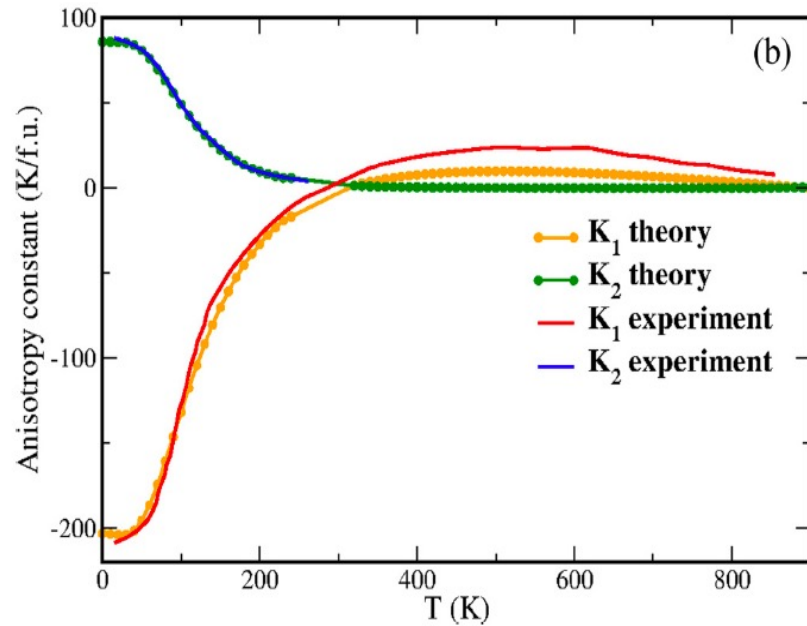
**with  $A_6^6 \langle r^6 \rangle$  set to 0  $\rightarrow$   
Nd 4f GS =  $|9/2; -9/2\rangle$**



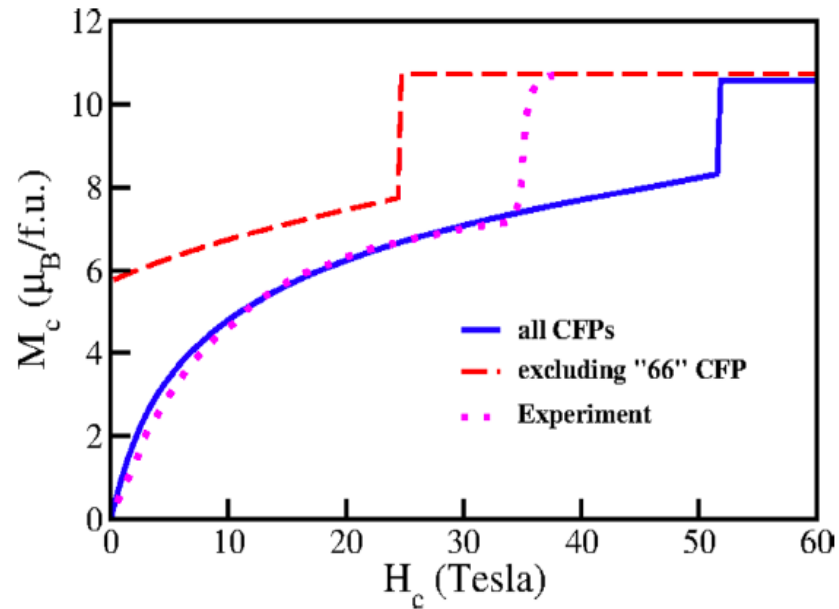


# NdCo<sub>5</sub>: theory vs experiment

Anisotropy constants  $K_1, K_2$

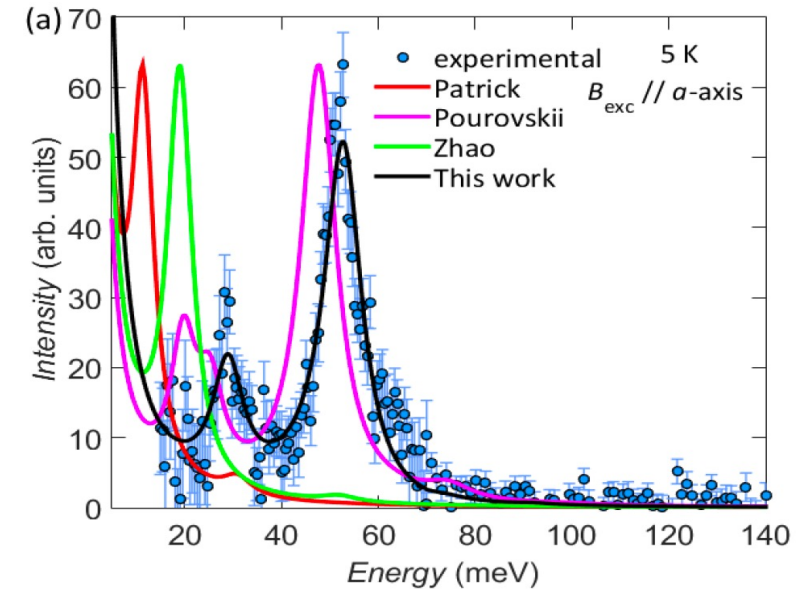


Magnetization curves along hard [001] direction



Inelastic neutron scattering

de Almeida Passos et al. arXiv:2306.00821



- $K_1(T)$  of YCo<sub>5</sub> is used for Co sublattice
- Anisotropy constants extracted from theoretical magnetization curves [Suscksmith-Thompson(ST) method]

Ab initio theories and magnetization curves fits vs INS

	$A_2^0 \langle r^2 \rangle$ (K)	$A_6^6 \langle r^6 \rangle$ (K)	$B_{exc}$ (T)
custom python code	-300	900	535
SPECTRE	-460±170	1026±60	506±5
PyCrystalField	-240±100	1150±60	470±30
Patrick and Staunton PRMat (2019)	-415	146	252
Pourovskii <i>et al.</i> [3]	-285	1134	292
Zhao <i>et al.</i> PRB 43, 8593 (1991)	-510	143	558

# Origin of large “66” crystal field in $R\text{Co}_5$

Pourovskii et al. PRB 101, 214433 (2020)

## Projective construction of Wannier basis

Amadon et al. PRB 2008  
Aichhorn et al. PRB 2009

$$|\chi_{m\sigma}^{\mathbf{k}}\rangle = \sum_{\nu \in \mathcal{W}} |\psi_{\mathbf{k}\nu}\rangle \langle \psi_{\mathbf{k}\nu} | \chi_{m\sigma} \rangle$$

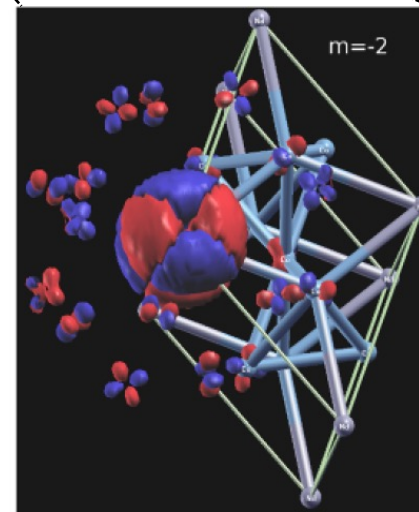
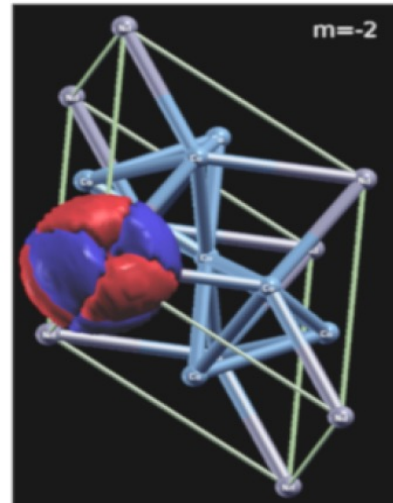
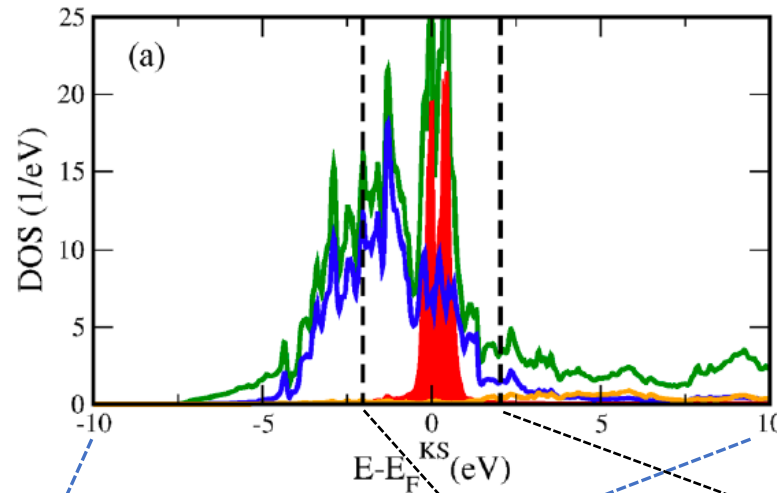
orthonormalization

Wannier set  $\{\omega_{m\sigma}^{\mathbf{k}}\}$

larger  $W$

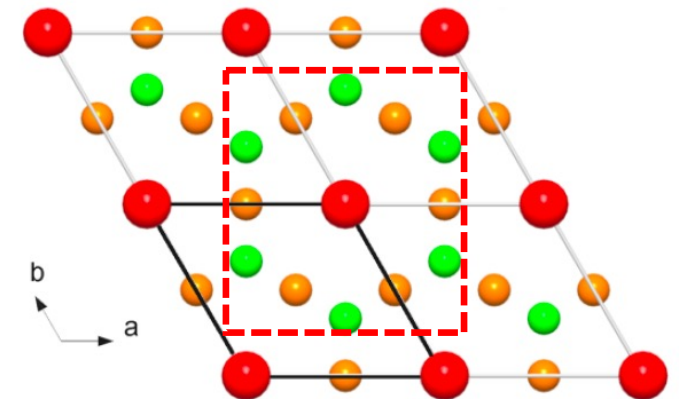
more localized orbitals  $\omega_{m\sigma}^{\mathbf{k}}$

## NdCo<sub>5</sub> Kohn-Sham DOS



## CFP & $B_{\text{ex}}$ vs energy window

Energy window (eV)	$A_2^0 \langle r^2 \rangle$	$A_6^6 \langle r^6 \rangle$	$B_{\text{ex}}$
$[-10 : 10], (\mathcal{W}_l)$	-198	45	326
$[-2 : 2], (\mathcal{W}_s)$	-285	1134	292






- Complex 2-14-1 magnets: crystal-field, anisotropy, effect of RE substitutions

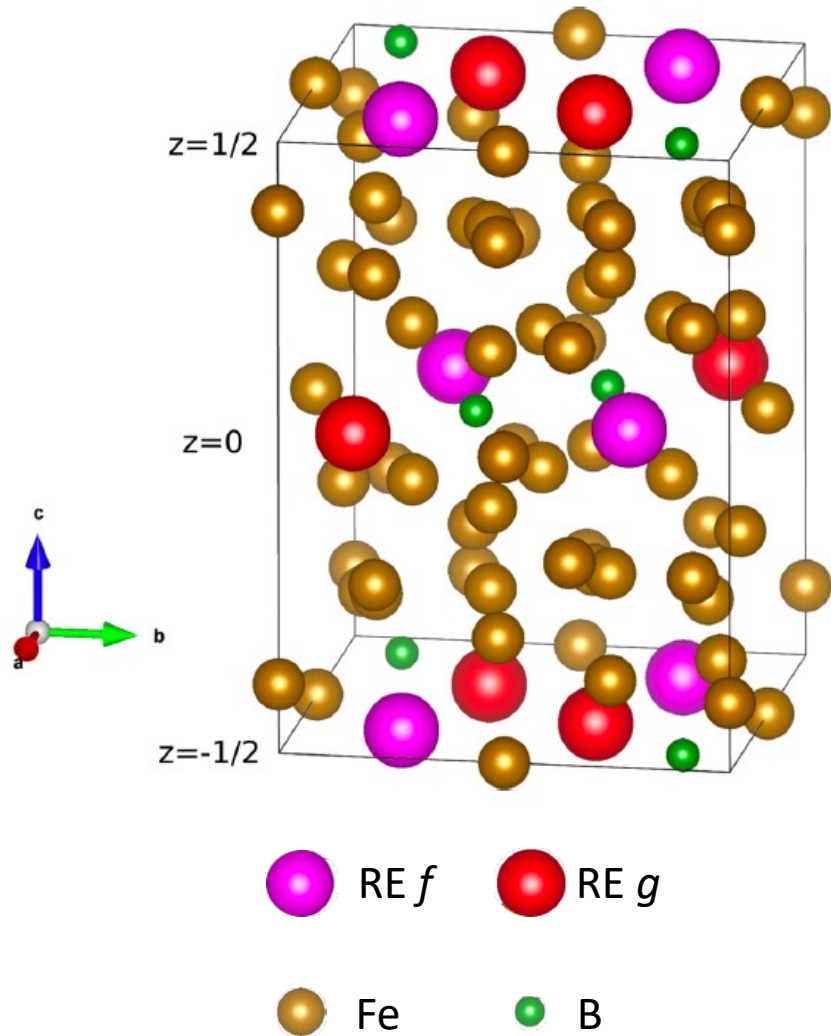
PHYSICAL REVIEW MATERIALS **6**, 084410 (2022)

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**Ce and Dy substitutions in  $\text{Nd}_2\text{Fe}_{14}\text{B}$ : Site-specific magnetic anisotropy from first principles**

James Boust,<sup>1</sup> Alex Aubert ,<sup>2</sup> Bahar Fayyazi,<sup>2</sup> Konstantin P. Skokov,<sup>2</sup> Yurii Skourski,<sup>3</sup>  
Oliver Gutfleisch ,<sup>2</sup> and Leonid V. Pourovskii <sup>1,4</sup>

# Nd<sub>2</sub>Fe<sub>14</sub>B: crystal structure, composition and RE substitutions



- Complex structure, 68 atoms/cell, 2 inequivalent RE sites
- Low-symmetry of RE sites → many CF parameters:

$$H_{cf} = \sum_{kq} A_k^q \langle r^k \rangle \hat{T}_k^q$$

$A_k^q \neq 0$  for  $kq = 20, 2\bar{2}, 40, 4\bar{2}, 44, 60, 6\bar{2}, 64, 6\bar{6}$   
 and different on the two RE sites

for review on “2-14-1” magnets see  
Herbst Rev. Mod. Phys. 63, 819 (1991)

- RE substitution: Nd → cheaper and abundant Ce, La or Nd → expensive heavy RE (Dy)
  - impact on intrinsic properties?
  - effect of substitution site preference?

# Generalized two-sublattice model: treatment of multiple RE sites and TM anisotropy

$$F(T) = \sum_{i\alpha} \omega_{i\alpha} F_{4f}^{(i\alpha)}(T) + F_{3d}(T)$$

$\omega_{i\alpha}$  : concentration of magnetic RE element  $\alpha$  on site  $i$

**Semi-empirical  $F_{3d}(T)$**  using ab initio  $M_{3d}$ , experimental  $T_c$  and the empirical Kuz'min formula:

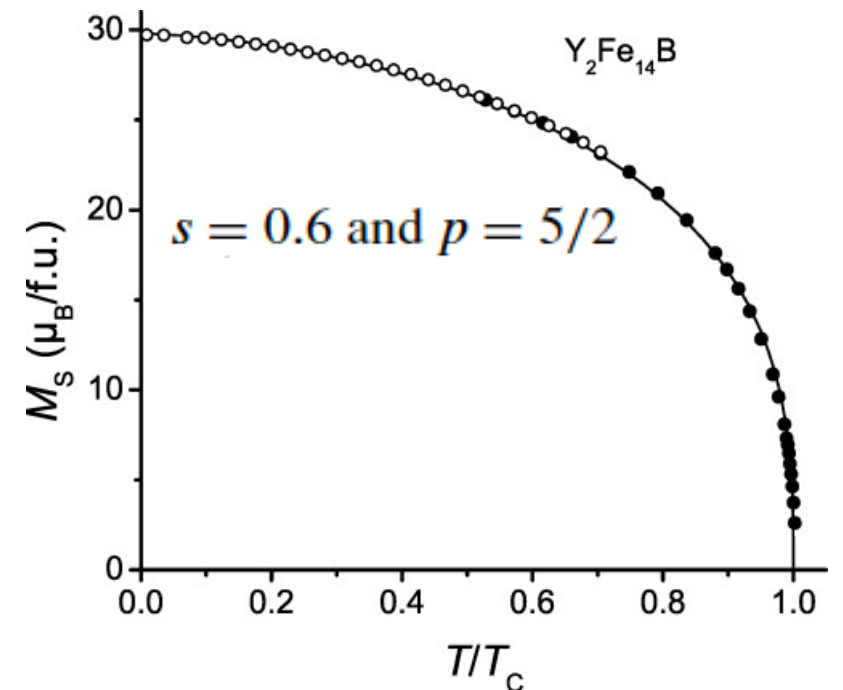
$$M_{3d}(T) = M_{3d}\alpha(T), \quad B_{\text{ex}}(T) = B_{\text{ex}}\alpha(T),$$

$$\alpha(T) = \left[ 1 - s \left( \frac{T}{T_c} \right)^{\frac{3}{2}} - (1-s) \left( \frac{T}{T_c} \right)^p \right]^{\frac{1}{3}}$$

Zener formula for  $T$ -dependence of  $K_1^{3d}(T)$ :

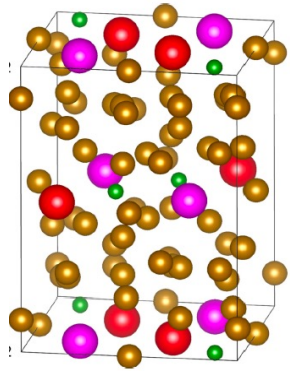
$$K_1^{3d}(T) = K_1^{3d} (M_{3d}(T)/M_{3d})^3 = K_1^{3d} \alpha(T)^3$$



$K_1^{3d}$  estimated from ab initio spin-orbital energy of Fe 3d in  $\text{La}_2\text{Fe}_{14}\text{B}$  [Liu et al. PRB 102, 205119 (2020)]

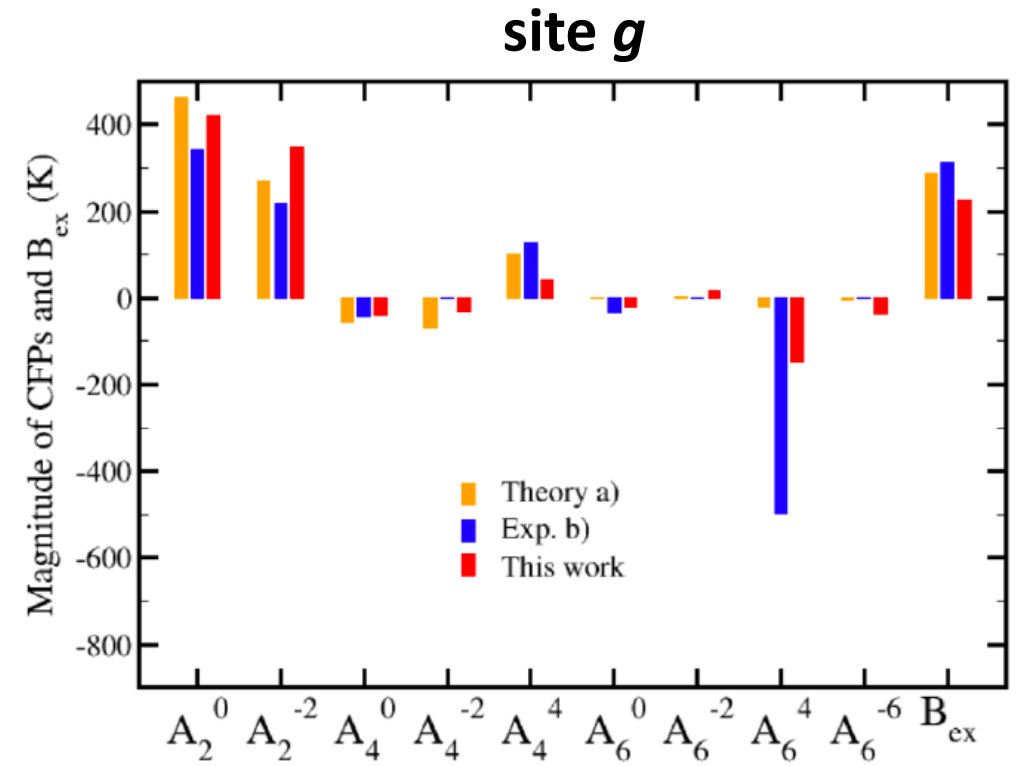
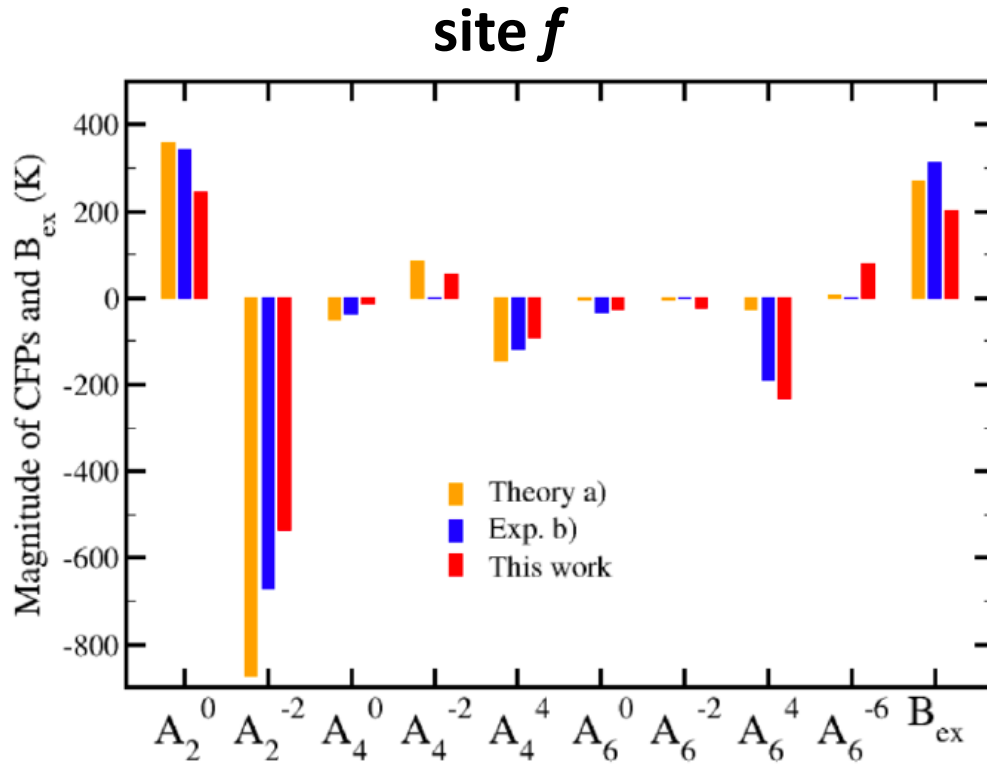


Kuz'min Phys. Rev. Lett. 94, 107204 (2005)  
Kuz'min et al. J. Appl. Phys. 107, 113924 (2010)

# $R_2Fe_{14}B$ : Nd crystal-field parameters



 Nd *f*  
 Nd *g*



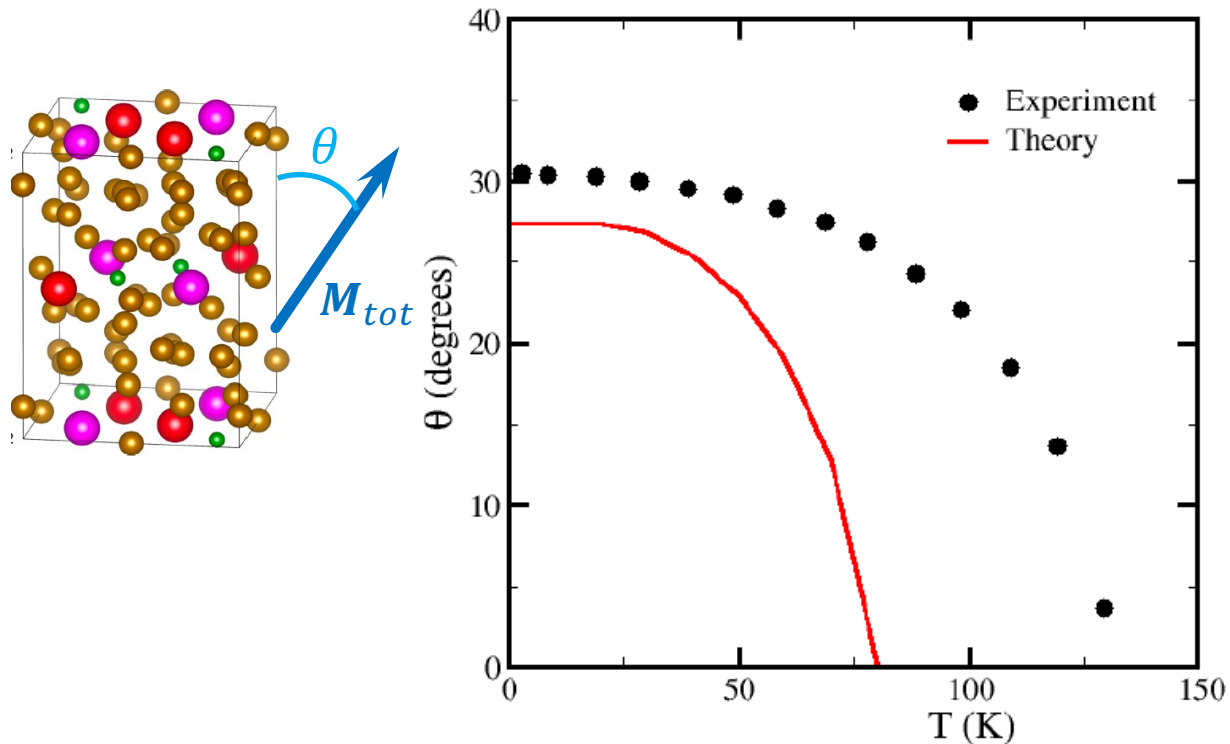
a) Sato et al. AIP Adv. 11, 025224 (2021).

b) Cadogan et al. J. Phys.F: Met. Phys. 18, 779 (1988); Gavigan et al. JMMM 72, L241 (1988)

- Site *g*: 70% larger “20” CFP → gives the leading contribution to anisotropy
- Effect of the large “22” CFP is canceled between RE planes having opposite signs of  $A_2^{-2}$
- Mixed systems  $R_2Fe_{14}B$  → substitution on one RE site does not affect CFPs on the second one

# $\text{Nd}_2\text{Fe}_{14}\text{B}$ : spin-reorientation transition (SRT) and magnetic anisotropy

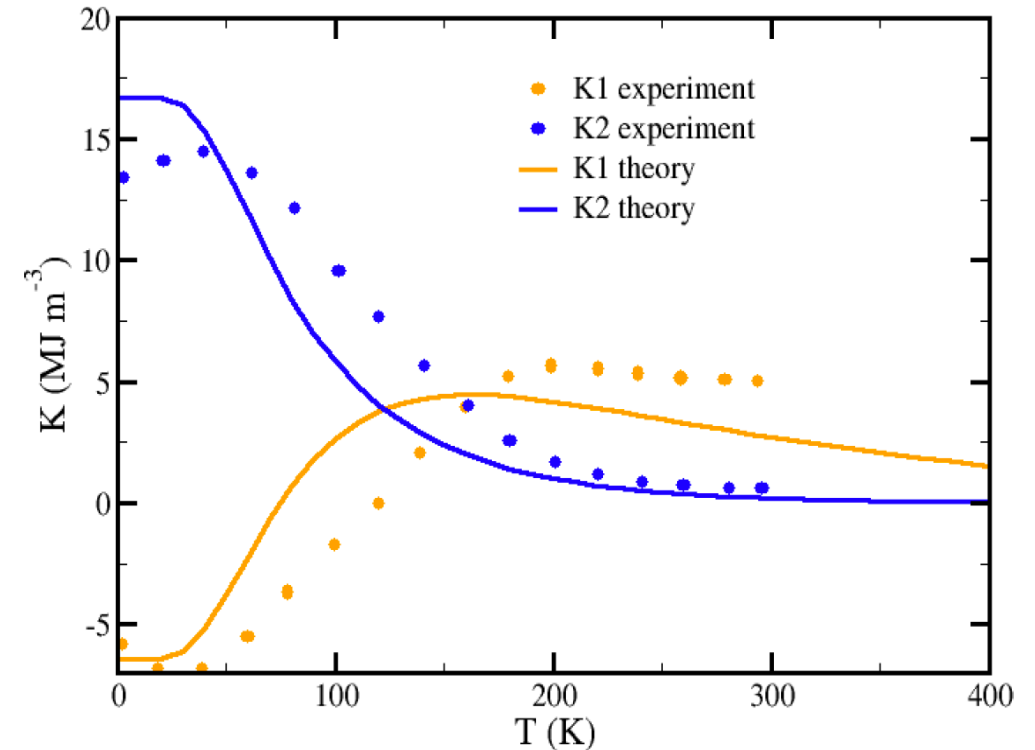
Azimuthal angle  $\theta$  of magnetization vs  $T$



- SRT conical  $\rightarrow$  uniaxial is reproduced ( $T_{\text{SRT}}$  underestimated)
- Zero- $T$  conical angle  $\theta$  is well reproduced

Exp.: Cadogan et al. J. Phys.F: Met. Phys. 18, 779 (1988)

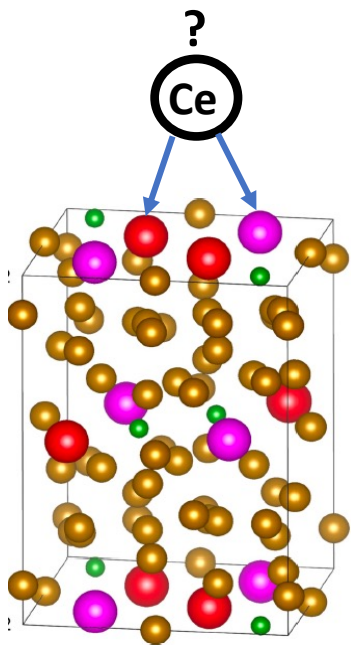
Anisotropy constants vs  $T$



- Change of  $K_1$  sign  $\rightarrow$  SRT
- Good agreement, though some underestimation of  $K_1$  at high- $T$

Exp.: Andreev et al. JETP 63, 608 (1986)

# Ce substitution: impact of preferential occupancy



$$r(\text{Ce } 4+) < r(\text{Nd } 3+)$$

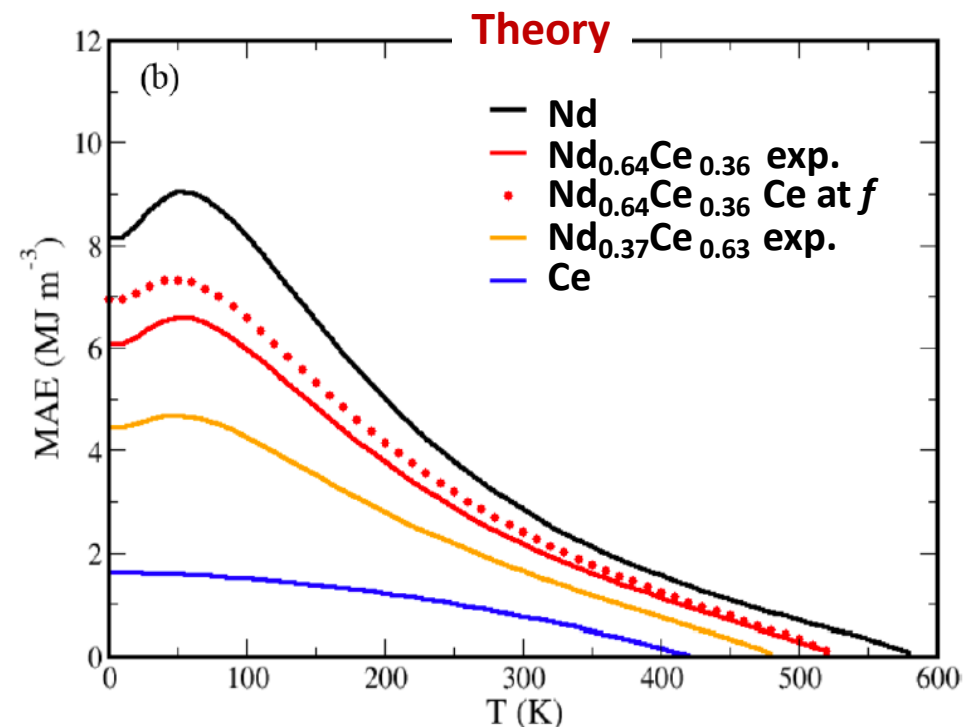
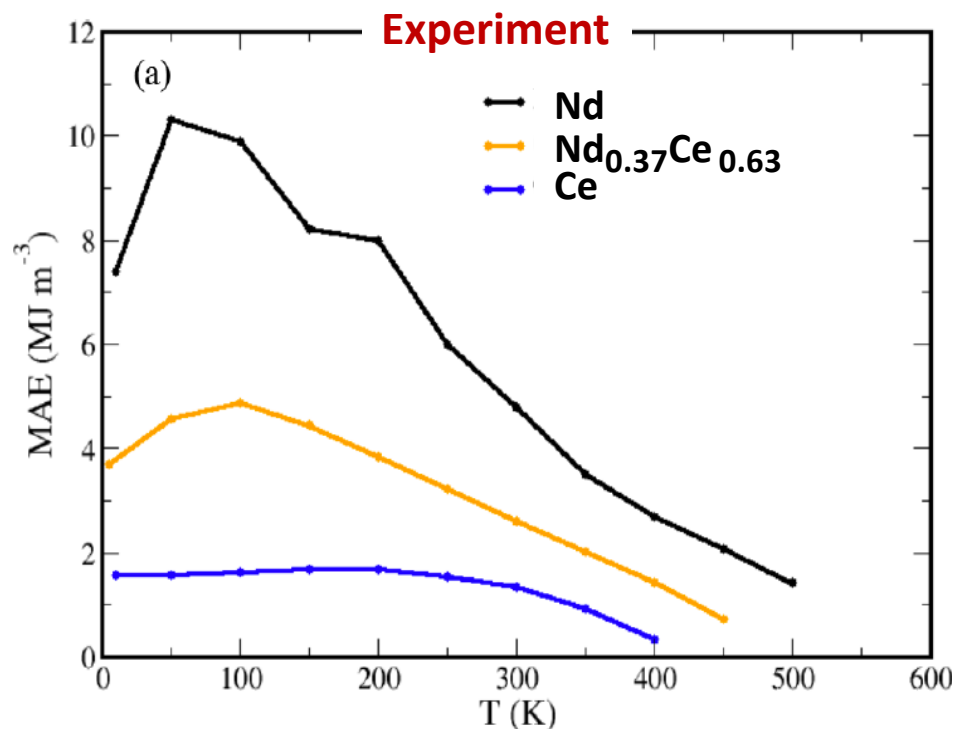


Ce likely to occupy smaller *f* site

[exp: neutron diffraction

Colin et al. APL 108, 242415 (2016)]

## Magnetocrystalline energy (MAE)



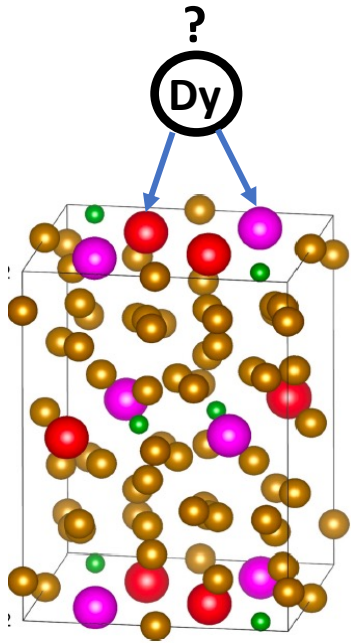
- Good agreement with experiment for  $\text{MAE} = F(M||a) - F(M||c)$
- Ce-at-*f* MAE by about 10% higher than for exp. occupancies

experiment: ~60% of Ce at *f*

Colin et al. APL 108, 242415 (2016)



# Preferential RE occupancy in $(\text{Nd,Dy})_2\text{Fe}_{14}\text{B}$

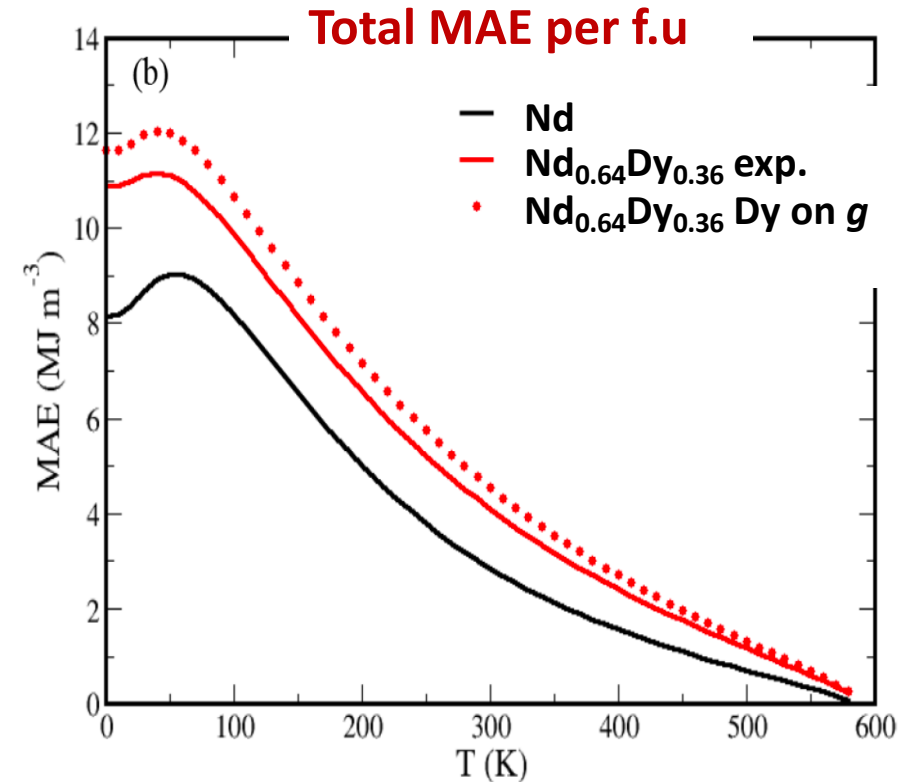
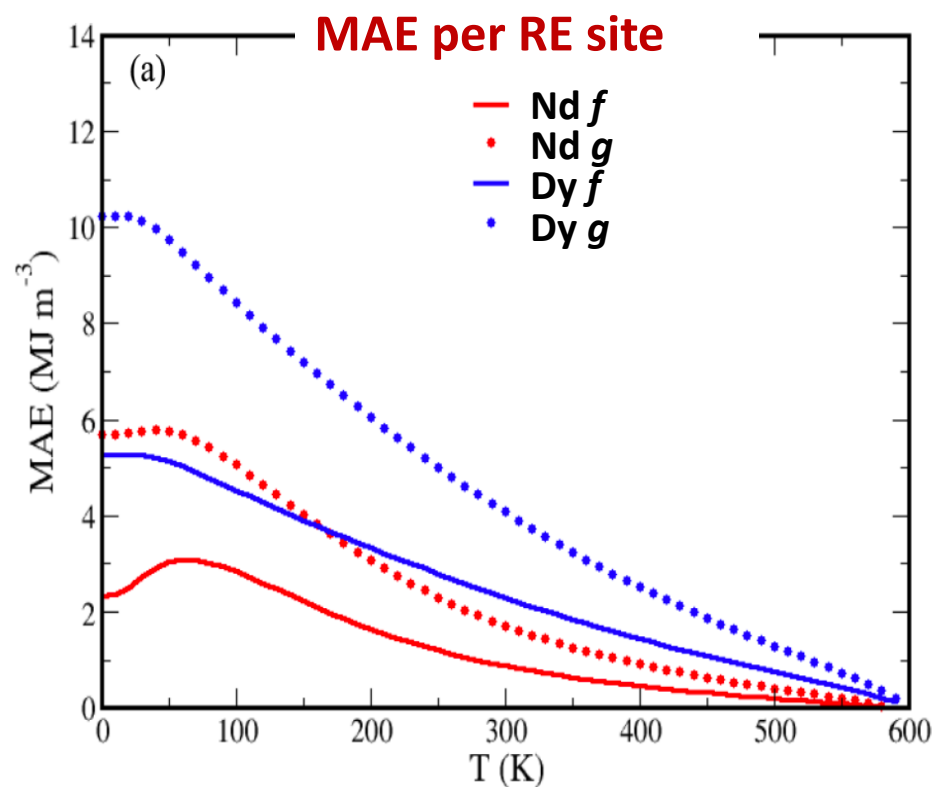


$r(\text{Dy } 3+) < r(\text{Nd } 3+)$   
[rare-earth contraction]



Dy likely go to *f* site  
[exp: 2/3 Dy goes to *f* site

Saito et al. J. Alloys Compd. 721, 476 (2017)]



- Much larger *g*-site MAE for both Nd and Dy cases (larger for Dy)
- $\text{MAE}(f) - \text{MAE}(g)$  is similar for Nd and Dy  $\rightarrow$  moderate effect on total MAE  
[~10% at 300 K]

# Challenges and perspectives

- **Improving treatment of RE-TM exchange coupling**  
[ $f$ - $f$  interaction, interplay of 4f-5d and 3d-5d couplings ?]

$B_{exc}$ (T)
535
$506 \pm 5$
$470 \pm 30$
252
292
558

NdCo<sub>5</sub> (from  
arXiv:2306.00821 )

exp.

theory

exp.

- **Ab initio approach to TM magnetization and TM anisotropy at finite T**  
[Force-theorem exchange interactions + Monte-Carlo?]

- **Ce beyond DFT: intermediate valence and its impact on anisotropy**

npj | Quantum Materials

npj Quantum Materials (2021)6:2

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ARTICLE OPEN

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Intrinsically weak magnetic anisotropy of cerium in potential hard-magnetic intermetallics

Anna Galler<sup>1</sup>✉, Semih Ener<sup>2</sup>, Fernando Maccari<sup>2</sup>, Imants Dirba<sup>2</sup>, Konstantin P. Skokov<sup>2</sup>, Oliver Gutfleisch<sup>2</sup>, Silke Biermann<sup>1,3,4,5</sup> and Leonid V. Pourovskii<sup>1,3</sup>

- **Realistic compositions, defects, integration with micromagnetic and machine-learning models**

# Coauthors and acknowledgments

## Theory

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Delange**



**James  
Boust**

**Silke Biermann**

*AIST, Tsukuba*

**Takashi Miyake**

## Experiment

*Institut Néel (Grenoble)*

**Dominique Givord**

**Rafik Ballou**

**Gabriel Gomez Eslava**

*Technical University of Darmstadt*

**Konstantin Skokov**

**Alex Aubert**

**Bahar Fayyazi**

**Yurii Skourski**

**Oliver Gutfleisch**



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