Multidimensional spectral analysis using rational models with application to radar and medical imaging.

Mathematical models provide important tools to extract, compare, and fuse the information content of signals. Recently a new theoretic framework based on convex optimization and analytic functions was developed and have lead to new analysis methods based on rational models. This research leverages these recent advances to develop methods for analysis of multidimensional systems.

Consider the multidimensional, discrete-time, zero-mean, and homogeneous stochastic process $y(t)$ defined for $t \in \mathbb{Z}^d$. Homogeneity implies that covariances $c(k) := E(y(t+k)\overline{y(t)})$ are invariant with "time" t. The power spectrum, $\Phi(\theta) : \mathbb{T}^d \to \mathbb{R}$ where $\mathbb{T} := [-\pi, \pi]$, represents the energy distribution across frequency of the signal, and is defined as the non-negative function whose Fourier coefficients are the covariances:

$$c(k) = \frac{1}{(2\pi)^d} \int_{\mathbb{T}^d} e^{-ik^T\theta} \Phi(\theta) dm(\theta) \tag{1}$$

for $k := (k_1, \dots, k_d)^T \in \mathbb{Z}^d$, where $dm(\theta) := \prod_j^d d\theta_j$, and $\theta := (\theta_1, \dots, \theta_d)^T \in \mathbb{T}^d$. The *spectral estimation problem* is to determine $\Phi(\theta)$ based on a finite sample $y(t_1), \dots, y(t_N)$. This is a key component in many signal processing techniques and plays a fundamental role in prediction, analysis, and modelling of signals. This research include the following four topics related to spectral estimation.

- Given a finite covariance sequence $\{c(k)\}_{k\in\Lambda}$ where $\Lambda \subset \mathbb{Z}^d$, the *rational covariance extension problem* (RCEP) is the problem of parametrizing all spectra $\Phi(\theta)$ that satisfy the covariance constraints (1) for $k \in \Lambda$, and that are non-negative rational trigonometric functions:

$$\Phi(\theta) = \frac{P(\theta)}{Q(\theta)}, \quad \text{where } P(\theta), Q(\theta) \in \mathfrak{P}_+ \backslash \{0\}, \tag{2}$$

  and $\mathfrak{P}_+ = \{P(\theta) = \sum_{k\in\Lambda} p_k e^{ik^T\theta} \,:\, P(\theta) \geq 0 \text{ for } \theta \in \mathbb{T}^d\}$. This problem is important in systems theory for estimation and realization of low degree systems, and is well understood in the one-dimensional setting. In this work we consider the multidimensional setting as well as generalizations to general basis functions.

- An important step in spectral estimation is the estimation of the covariances. The estimation of a covariance $c(k)$ requires time samples whose differences (lags) are equal to k. When the sampling points are few or where the pattern is irregular there is often only a few lags (or even no lags) in the data samples that matches a desired lag, which makes this a challenging problem. This often results in covariance estimates with large variance or considerable bias. Here we will study and design methods for such sampling patterns and seek to quantify the uncertainty in such estimates.

- In order to achieve reliability in signal analysis it is important to quantify uncertainty and bound the estimation error of methods. In previous work we show that the metrics that are appropriate for quantifying uncertainty in spectral estimation are those continuous with respect to the weak* topology. Here we study the uncertainty in specific spectral estimation methods with the aim to develop a framework for quantitative assessment applicable for comparisons and evaluations of estimation methods.

- We also study the application of low order spectral analysis methods for radar and medical imaging. In particular we will consider the use rational models for compression and representation of images. We will also use methods based on optimal mass transport for tracking, automatic target recognition, and detection of anomalies in images.