ON THE UNCERTAINTY MODELING AND VALIDATION FOR EXTERNAL STORES AERODYNAMICS

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Abstract. A wind tunnel model representing a generic delta wing configuration with external stores is considered for flutter investigations. Complex eigenvalues are estimated for the wind tunnel model for airspeeds up to the flutter limit, and compared to eigenvalues predicted by a numerical model. The impact of external stores mounted to the wing tip is investigated both experimentally and numerically, and besides the impact of the stores as such, the capability of the numerical model to account for increasing model complexity is investigated. An uncertainty approach based on robust flutter analysis is demonstrated to account for modeling imperfections. Uncertainty modeling issues and the reliability of uncertainty models are discussed. Provided that the present uncertainty mechanism can be determined, it is found that available uncertainty tools may be used to efficiently compute robust flutter boundaries.

1 INTRODUCTION

Flutter testing is rarely performed on full scale aircraft due to the high risk for structural damage and failure. Instead, flutter boundaries are computed based on a numerical model of the aircraft, and flight testing under less critical conditions can be performed to collect data for validation of the numerical model. Clearly, problems arise when the flight test data shows deviation from the numerically predicted data. Deviations are likely to occur for any aircraft configuration, since model imperfections and simplifications always lead to some uncertainty in the numerical model, and the question arises what impact these uncertainties have on the predicted flutter boundaries.

In most cases, some parts of the numerical model, such as the mass properties of different components, are well known, whereas other elements, like the aerodynamic loads in some region of a wing with complex geometry, are known to be subject to uncertainty. Earlier studies\(^1,2,3,4\) have shown how aerodynamic uncertainties can be introduced in the numerical model based on physical reasoning and known modeling difficulties. This study aims at demonstrating how uncertainties can be introduced
in the numerical model, and how data points collected at subcritical conditions may be used to establish an uncertainty model that is capable of producing reliable flutter boundaries.

In the present study, a wind tunnel model was designed to demonstrate and evaluate uncertainty modeling approaches. A fairly simple wing geometry and structure are chosen to minimize the errors introduced by modeling simplifications. The model complexity is then increased gradually, and analyses and experiments are performed for each configuration to detect modeling difficulties. An external store in the form of a wing tip missile is used for this purpose, and the impact on the flutter behavior is investigated, as well as the capability of flutter analysis tools to provide correct results for increasing model complexity. In cases where the numerical predictions deviate from the experimental results, robust analysis is applied and uncertainty modeling approaches are evaluated.

2 EXPERIMENTAL SETUP

The considered model is a generic delta wing configuration as shown in Figure 1 with a semi-span of 0.88 m. The wing is mounted vertically on the floor in the low-speed wind tunnel L2000 at the Royal Institute of Technology (KTH). Due to the high risk involved in flutter testing, it was found convenient to design a low-cost wind tunnel

Figure 1: Delta wing mounted in the low-speed wind tunnel L2000 at KTH.
model with a low level of complexity. The wing consists of a glass fiber plate with carbon fiber stiffeners to obtain the desired structural properties. The wing can easily be equipped with external stores, such as an underwing missile and a wing tip missile. In the present study, the impact of a wing tip store will be investigated.

2.1 STRUCTURAL DESIGN

The wind tunnel model is meant to represent a generic delta wing configuration of a fighter aircraft. Due to the length and velocity scales imposed by the wind tunnel environment, some properties such as the Mach number of the model will not represent the full scale model. The model was however scaled in terms of the structural properties with respect to the aeroelastic behavior. The objective was to obtain eigenfrequencies $f$ such that the reduced frequencies $k = 2\pi f \cdot L/V$ are the same for both model and full scale aircraft, where $L$ is a reference length, and the velocity $V$ is chosen at a typical flight condition.

In the present case, it was found that both the length scaling and the velocity scaling factors between model and full-scale aircraft are in the order of 10, making the frequency scaling factor equal to one. Therefore, structural eigenfrequencies of the model were chosen to be equal to the eigenfrequencies of a representative full-scale structure. Since flutter behavior typically is governed by the lower eigenfrequencies of a structure, the first bending and the first torsional eigenfrequencies were emphasized.

To obtain specific eigenfrequencies as determined by the aeroelastic scaling, a few design parameters were defined in the structural design. In general, both eigenfrequencies and eigenmode shapes can be controlled by the mass and stiffness distribution. In the present study, a fixed wing geometry was assumed, and a glass fiber composite plate was considered as a baseline structure. Carbon fiber composite spars were attached to the composite plate to control the structural properties of the wing. Two spars were mounted on both the upper and lower side of the wing, and the spar dimensions and positions were chosen to obtain the desired eigenfrequencies. The mode shapes were not designed specifically.

2.2 EXTERNAL STORE

The main focus of the study is to investigate the aerodynamic impact of an external store on the flutter behavior. A realistic wing tip missile was therefore manufactured and mounted to the wing tip as shown in Figure 2. The missile is mounted to a launcher beam that is assumed to be rigidly connected to the wing tip. Different configurations were considered to identify the aerodynamic impact of different missile components on the flutter behavior of the wing. Figure 3 shows the five considered configurations. In order to reduce the effects on the flutter speed due to structural differences between the different configurations, mass balancing was used to compensate for different components once removed from the wind tunnel model. The mass balancing for the fins was located within the missile body, whereas the mass balancing for the missile was
located within the launcher beam to maintain the aerodynamic shape of the wing tip region.

3 NUMERICAL MODEL

Numerical models of the delta wing were generated in both Nastran\textsuperscript{5} and ZAERO\textsuperscript{6}. The model is composed of shell elements for the wing, mass elements for the missile, and aerodynamic panels for both wing and missile. The missile assembly was assumed to be rigid.

3.1 STRUCTURAL MODELING

The geometry of the structural elements of the wing plate was defined such that the location of the carbon fiber spars coincides with the location of structural grid points in order to allow for accurate modeling of the different material properties. Nastran CQUAD4 elements were selected for connecting the structural grid points of the wing plate. Figure 4 shows the discretization of the structural elements of the wing plate. Note that the location of the carbon fiber stiffeners is indicated in the Figure. The irregular discretization in the wing tip region is chosen to allow for accurate modeling of the launcher beam attachment.

Material properties were derived in several steps, starting by the glass fiber plate with-
out the carbon fiber spars. Vibration testing of the plate was performed to obtain material data by matching the measured eigenfrequencies to the eigenfrequencies predicted by the numerical model. The wing was freely supported to exclude effects due to imperfections of the boundary conditions once mounted to the wind tunnel floor.

Material data from the manufacturer were used to determine material properties of the carbon fiber spars. Once attached to the glass fiber wing, another vibration test was performed to validate the material data. After that, the wing was mounted into
the wind tunnel, and yet another vibration test was performed. The measured eigen-
frequencies appeared to be somewhat lower than expected for a rigidly clamped wing, 
and it was found that introducing some flexibility in the numerical model of the clamp-
ing improved the frequency matching. The flexibility was chosen such that the first 
and second eigenfrequencies predicted by the numerical model were within 0.1 Hz 
from the measured eigenfrequencies from the vibration testing.

The missile was assumed to be rigid, and only the mass properties of the missile were 
modeled. A grid point representing the degrees of freedom of the missile assembly was 
defined and attached to the wing tip. In the experimental setup, the missile launcher is 
rigidly connected to the wing tip along the entire tip chord, thus eliminating chordwise 
bending deformations of the wing tip region. Masses for the various components of 
the missile were then rigidly attached to that grid point.

The structural damping was neglected in the structural model, and it was assumed 
that this simplification results in slightly conservative flutter predictions without any 
significant impact on the general flutter behavior. The numerical approaches presented 
in the following sections are based on a model without structural damping, but could 
easily be modified to include damping.

3.2 AERODYNAMIC PANELS

The aerodynamic modeling was performed in both Nastran and ZAERO. Nastran 
CAERO1 elements were used to model aerodynamic panels for computation of doublet-
lattice\(^7\) aerodynamic loads. The wing surface was covered by 12 spanwise times 14 
chordwise lifting surfaces equally distributed on the wing area. The launcher beam 
and missile body were modeled as flat panels parallel to the wing surface, since it was 
assumed that in-plane airloads generated by panels perpendicular to the wing sur-
face can be neglected in the flutter analysis. Depending on the configuration, between 
200 (Config. 5) and 402 (Config. 1) aerodynamic panels were defined. In Figure 4, the 
Nastran aerodynamic panels for the full missile configuration are shown.

The ZAERO aerodynamic model using CAERO7 elements utilizes the same discretiza-
tion as the Nastran model for the wing, the missile launcher beam, and the missile fins. 
The missile body, however, was modeled using BODY7 elements.

To connect the structure and the aerodynamic panels, a surface spline was defined for 
the wing plate using a SPLINE1 in both Nastran and ZAERO, and a beam spline was 
defined for connecting the missile panels to the grid point defining the missile motion, 
using SPLINE5 in Nastran and ATTACH in ZAERO.

4 FLUTTER RESULTS

In most applications, flutter testing of real aircraft is restricted to subcritical airspeeds 
due to the high risk involved in operating the aircraft in flutter conditions. As the air-
speed approaches the flutter stability limit, the aeroelastic damping of the structure
is reduced, leading to weakly damped oscillations once the wing is subject to external excitation. The oscillations correspond to complex eigenvalues in a linear stability analysis, where the real and imaginary part represent the damping and frequency of a particular mode, respectively. Rather than comparing the flutter speed from numerical predictions and experiments, eigenvalues can be compared instead to validate the numerical model without the need to operate the aircraft at the flutter limit. The wind tunnel model, however, was designed to operate at and even beyond the flutter limit, and eigenvalues representing both stable and unstable conditions can be measured and compared to numerical predictions.

4.1 EXPERIMENTAL RESULTS

Experiments were performed at different airspeeds and for different missile configurations. To estimate eigenvalues from oscillations, the wind tunnel model was equipped with an accelerometer in the wing tip region to monitor the structural response to an external excitation. Figure 5 shows the measured oscillation for an airspeed below the flutter speed, where the oscillations decay due to the aeroelastic damping.

![Figure 5: Stable response for excitation at a subcritical airspeed.](image)

The data was first collected for the most simple configuration without the missile at-
tached to the wing. There is some beating in the wave during the first two seconds after the excitation, indicating that there are several modes present in the response. This can hardly be avoided when using impulse excitation as in the present case, since many different modes will be excited simultaneously. The other modes are however significantly more damped and thus only one mode dominates the motion after the first few seconds. Increasing the airspeed beyond the flutter limit, the oscillations increase instead, as shown in Figure 6, indicating that one mode is unstable.

![Figure 6: Increasing oscillation at a supercritical airspeed.](image)

In this case, there was no need for external excitation since any disturbance in the airflow can initiate the oscillations. Despite the wing being designed for large deflections, wind tunnel testing above the flutter speed is dangerous due to the steadily increasing amplitudes, and would eventually cause damage to the model.

Experiments were primarily performed at subcritical airspeeds, and only a few experiments were run above the flutter speed in order to assure that the mode actually becomes unstable. From the measurements, complex experimental eigenvalues $p_{exp} = \sigma_{exp} + i\omega_{exp}$ were estimated by identifying a state-space system based on the measured time series. Frequency $f_{exp} = \omega_{exp}/(2\pi)$ and damping defined as $2\sigma_{exp}/\omega_{exp}$.
are then extracted from the eigenvalues. Figure 7 shows the resulting damping and frequency versus airspeed. The flutter speed lies between 25 and 26 m/s, where the damping crosses zero, corresponding to a purely imaginary eigenvalue. Similar experiments were performed for all configurations. Note that numerical predictions, that were obtained as described in the next Section, are included in the Figure.

4.2 NOMINAL FLUTTER ANALYSIS

The nominal analysis is performed by solving the flutter equation

\[ F_0(p, M) \eta = \left[ M_0 p^2 + \left( \frac{L}{V} \right)^2 K_0 - \rho L^2 Q_0(p, M) \right] \eta = 0 \]

where \( M_0, K_0, \) and \( Q_0(p, M) \) are the nominal mass, stiffness, and aerodynamic matrices that define the nominal flutter matrix \( F_0 \). The airspeed, air density and Mach number are denoted \( V, \rho, \) and \( M, \) respectively, and \( L \) is the reference length used to make the equation nondimensional. The flutter equation is a nonlinear eigenvalue problem with complex eigenvalues \( p = g + ik \) and vectors of modal coordinates \( \eta \). The flutter stability limit is found as the real part of the eigenvalue becomes zero. The imaginary part of the eigenvalue is the reduced frequency \( k = \omega \cdot L/V \), and the real part \( g = \sigma \cdot L/V \) is a measure of the damping of the system. As for the eigenvalues estimated by the experimental time series, the damping is then defined as \( 2g/k \). Note that \( \omega \) and \( \sigma \) used in the analysis correspond to \( \omega_{exp} \) and \( \sigma_{exp} \) from the experiment, respectively.

In Figure 7, the numerical results are shown along with the measurements. The comparison shows that the flutter speed is predicted accurately by the numerical model. Note that the experimental data is more noisy for lower velocities. This is due to the higher aeroelastic damping, making the oscillations decay faster, which leads to shorter sampling times and therefore less accurate values for frequency and damping. In general, however, the analysis predicts both frequency and damping very well.

There is also a slight offset in the frequency, which may be due to an inaccurate structural model. It was for example found that the frequency drops by 0.1 Hz if the temperature in the tunnel increases by 3 degrees Celsius. Therefore, it is expected that the measured frequencies are slightly lower as the closed-loop wind tunnel is under operation and increases the air temperature.

4.3 INFLUENCE OF EXTERNAL STORES

When attaching the missile to the wing, the flutter behavior is expected to change both in the analysis and in the experiment. Since mass balancing was used to account for the missile components, however, all configurations should have similar structural properties, and the main differences are expected to be due to different aerodynamic loads. The missile was attached to the wing in several steps, beginning with the missile body
Figure 7: Comparison of numerical and experimental results for the configuration without the missile.

(Conf. 4). Flutter results from this configuration are shown in Figure 8. The Figure shows that the quality of the damping data is somewhat lower than in the case before. Nevertheless, the damping obtained in the experiments agrees with the prediction fairly well. Again, there is a constant offset in the frequency, but the frequency trend is captured well. Nastran and ZAERO produce very similar results for this configuration.

In the next step, the canard fins were attached to the model. The frequency and damping curves for this configuration are shown in Figure 9. Compared to the case without canard fins, the flutter speed was reduced by 1.5 m/s in the analysis. There is some scatter in the measured damping, but it seems that the mean value fits reasonably well to the numerical results. The predicted frequency follows the measured values nicely, although the slope of the predicted curve is slightly less than in the experiments.

Next, the canard fins were removed again and the rear fins attached to the missile. Results from these investigations are shown in Figure 10. The rear fins increase the flutter speed significantly, which is predicted both in the analysis and found in the experimen-
tal investigations. The Nastran and ZAERO damping predictions deviate slightly from each other, with the experimental values being located between these curves. Nastran is slightly conservative, whereas ZAERO overpredicts the flutter speed. The frequency trend is predicted well, but again there is an offset between analysis and experiment.

Finally, the complete configuration was considered by attaching the canard wings, with results as shown in Figure 11. As before, the canard wings lead to a destabilization of the wing. In the analysis, this can be seen by a reduction of the predicted flutter speed from 34 to 32 m/s for the Nastran model and from 36 to 34 m/s for ZAERO. The experiment is again in between the predictions by Nastran and ZAERO, with Nastran being slightly conservative. There is also a more pronounced scatter in the experimental damping data. The frequency is again predicted well with a slight constant offset.

The investigations of the different configurations show that the numerical model is fairly accurate. As the rear fins are included, the predictions by Nastran and ZAERO deviate from each other, with the experimental data lying in between these predictions. In the following Section, a robust approach will be presented where uncertainty
modeling will be used to account for modeling imperfections in order to derive robust flutter bounds that capture the experimental data entirely. Due to the similar behavior of the Nastran and the ZAERO models, the robust approach is based on the Nastran model only. It is assumed that a robust approach based on the ZAERO model would produce similar results.

5 ROBUST APPROACH

Comparison between the experimental and predicted eigenvalues indicates that the numerical model captures the flutter behavior fairly well. As the missile is attached to the wing, however, the analysis is not as accurate, in particular as the rear fins are included. It is therefore assumed that the error is due to modeling imperfections that can be accounted for by introducing uncertainties in the numerical model. Instead of computing nominal eigenvalues from Equation (1), the objective is to compute eigenvalue bounds by considering an uncertain flutter equation according to
Figure 10: Flutter results with missile body and rear fins.

\[
\begin{bmatrix}
M(\delta)p^2 + \left(\frac{L}{V}\right)^2 K(\delta) - \frac{\rho L^2}{2} Q(p, M, \delta)
\end{bmatrix} \eta = 0
\]

where the system matrices \(M(\delta), K(\delta)\) and \(Q(p, M, \delta)\) now depend on a set \(\delta\) of uncertainty parameters \(\delta_i\), defined such that the uncertain system matrices equal the nominal system matrices in Equation (1) for \(\delta = 0\). By assembly of an uncertainty matrix \(\Delta\) containing the uncertainty parameters \(\delta_i\), Equation (2) can be posed in a form where \(\mu\) analysis can be applied to investigate possible solutions to the eigenvalue problem. In previous studies, this approach has been used in order to find robust stability limits, where the \(\mu\) value was computed to determine if the uncertainty can lead to a critical eigenvalue \(p = ik\). In the present study, the focus is not only on the flutter boundary, but on a range of flight conditions, where robust analysis is used to determine the possible variation of the eigenvalues at a given flight condition.
5.1 Uncertainty Modeling

Uncertainty modeling based on uncertain system matrices as discussed above is convenient for introducing uncertainties based on physical reasoning. In many cases, some parts of the nominal model are known to contain uncertainty due to modeling difficulties, whereas other parts may be known to be accurately modeled. In the present case, the structural model has been tuned to fit experimental data from vibration testing experiments, and is considered accurate.

Uncertainty is only introduced in the aerodynamic matrix, where a modeling approach as presented in Refs.\(^1\)\(^,\)\(^2\) is used to allow for uncertainties in selected aerodynamic panels only. The aerodynamic matrix \(Q_0\) is therefore partitioned into left and right partitions \(Q_0(k, M) = L \cdot R(k, M)\), where \(R(k, M)\) computes the pressure coefficients in each aerodynamic panel as a function of the modal coordinates, and \(L\) computes modal forces from the pressure coefficients.

Aerodynamic uncertainty is introduced such that the pressure coefficients for a subset
of aerodynamic panels $i$ is allowed to vary according to $c_{pi} = c_{pi0}(1 + w_i \delta_i)$, where $c_{pi0}$ is
the nominal pressure coefficient, and $w_i$ is a real valued uncertainty bound that is chosen such that the uncertainty parameters $|\delta_i| \leq 1$. As shown in Ref.\textsuperscript{2}, the aerodynamic matrix can then be written
\[ Q = Q_0 + Q_L \Delta Q_R \]  
(3)
where $\Delta$ is a block diagonal matrix containing the uncertainty parameters $\delta_i$, and $Q_L$ and $Q_R$ are scaling matrices based on $R, L$ and $w_i$. Due to the bounds on $\delta_i$ and the structure of $\Delta$, the uncertainty matrix fulfills $\Delta \in S\Delta$ with
\[ S\Delta = \{ \Delta : \Delta \in \Delta \text{ and } \bar{\sigma}(\Delta) \leq 1 \} \]  
(4)
where $\Delta$ denotes the block structure of the uncertainty matrix, and $\bar{\sigma}(\cdot)$ denotes the maximum singular value.

5.2 $\mu$-p ANALYSIS

This uncertainty description is then inserted in the uncertain flutter equation (2) and the flutter equation can be written
\[ \left\{ I + \frac{pL^2}{2} Q_R F_0^{-1} Q_L \Delta \right\} \eta = 0. \]  
(5)
With the uncertain flutter equation in the form $(I - F(p) \Delta) w = 0$, so-called $\mu$-analysis\textsuperscript{10} can be applied to Equation (5). For a given system matrix $F(p)$, the structured singular value
\[ \mu(p) = \mu[F(p)] = \min_{\Delta} \left\{ \frac{1}{\bar{\sigma}(\Delta)} : \Delta \in \Delta, \det(I - F(p) \Delta) = 0 \right\} \]  
(6)
is defined based on the smallest norm $\bar{\sigma}(\Delta)$ that makes $p$ an eigenvalue to the uncertain flutter equation for a given flight condition. Note that there are no tools for computing the $\mu$ value exactly for any structured uncertainty $\Delta$. Instead, upper and lower bounds of the $\mu$ value can be computed, for example using the $\mu$ Toolbox in Matlab\textsuperscript{11}. In order to obtain a robust flutter boundary, the upper bound of the $\mu$ value will be considered in the following analysis.

Figure 12 shows an example of the variation of the upper-bound $\mu$ value in the complex plane for a region around the first two eigenvalues of the nominal flutter equation. The Figure shows that there are peaks at the locations of the nominal eigenvalues. For fixed values of $\mu$, elliptic areas can be identified around these eigenvalues, where the Figure indicates the areas corresponding to $\mu = 1$. From Equation (6) and the bounds on $\Delta$
stated in Equation (4), the following criteria hold:

\[ \mu(p) \geq 1 \iff p \text{ is an eigenvalue of (2) for some } \Delta \in S_\Delta \]  
(7)

\[ \mu(p) < 1 \iff p \text{ is not an eigenvalue of (2) for any } \Delta \in S_\Delta \]  
(8)

As described in Ref.8, the criterion (7-8) can be used to compute robust eigenvalues with minimum or maximum damping, from which damping bounds for a particular mode can be extracted and compared to experimental data. As described in the next Section, the \(\mu\)-\(p\) formulation also allows for straightforward model validation based on experimental estimates of flutter eigenvalues.

5.3 UNCERTAINTY VALIDATION

As uncertainty is introduced into the nominal system, not only the structure of the uncertainty, but also the bounds \(w_i\) have to be specified. Naturally, the bounds have significant impact on the robust flutter results. A too small magnitude may lead to results not capable of capturing the behavior of the system, whereas a too large bound
leads to flutter results being too conservative. In the present study, uncertainty validation will be used to estimate the uncertainty bound. For a given uncertainty structure, the minimum bound that is required to include the experimental results into the range of the robust predictions is selected. Note that in this study, only the most critical mode was considered with the eigenvalue measured at different velocities. Figure 13 demonstrates different approaches for the uncertainty validation based on a measured eigenvalue. The Figure shows the regions that bound the uncertain eigenvalue for

\[
\mu = 1 \text{ for different uncertainty norms. In the present study, it was found that the aero-
}\text{dynamic uncertainty allows for a circular variation of the eigenvalue. Validation based on eigenvalues (p-validation) results in a bound that assures that measured eigenvalue is a solution to the uncertain flutter Equation (2). According to Equations (7) and (8), the least possible uncertainty bound for a set of measured eigenvalues } p_{exp} \text{ is found when } \mu(p_{exp}) \geq 1 \text{ for all considered } p_{exp}, \text{ and when } \mu(\hat{p}_{exp}) = 1 \text{ for at least one measured eigenvalue } \hat{p}_{exp}, \text{ which in this case would be the decisive eigenvalue for the bound.}
\]

In the present study, comparison between the measured and nominal eigenvalues shows that the imaginary part corresponding to the frequency differs more than the real part corresponding to the damping. Due to the larger magnitude of the frequency, however, the relative error of the frequency is much less than the relative error of the damping and may possibly be neglected. It was considered inconvenient to perform p-validation, that aims on matching both the real and the imaginary part of the eigenvalue and therefore leads to unrealistically high uncertainty levels in order to capture the slight frequency offset observed in the nominal results. Since the damping is more critical than the frequency in flutter analysis, focus will be on possible deviations in the real part of the eigenvalues due to the uncertainty. In Figure 13, this approach is denoted g-validation. Another approach, k-validation, may for example be used for the validation of structural uncertainties based on data from ground vibration testing, where the structural damping (corresponding to g) may have minor importance, and

Figure 13: Different validation approaches for a representative case.
where uncertainties in the mass- and stiffness distribution have a more significant impact on the frequency.

In this study, validation with respect to the damping was performed ($g$-validation), aiming at finding the bound that allows an eigenvalue of the uncertain flutter equation to capture the measured damping. Using this validation technique, the measured frequency of the critical mode is allowed to differ from the robust predictions.

The goal of uncertainty validation is to establish a reliable uncertainty model by considering measured data points at airspeeds well below the flutter speed, and to use this model to predict the robust flutter boundary. This is particularly convenient in cases where flutter testing cannot be performed at the flutter speed, and where only test data from airspeeds below the flutter speed is available.

5.4 UNCERTAIN PANELS

The uncertainty modeling allows for specification of different aerodynamic panels to be subject to uncertainty, and it was found convenient to select panels in the region where the model is most likely to contain uncertainty that has an impact on the flutter results. As the nominal and experimental results have shown, the missile assembly has significant impact on the flutter behavior. The entire wing tip region was therefore considered for the uncertainty modeling. Patches containing aerodynamic panels were defined as shown in Figure 14. Twenty patches were defined for the configuration with the full missile attached to the wing. Fewer patches were defined for the other
configurations accordingly.

In general, increasing the number of uncertain panels will increase the computational effort required for computation of the $\mu$ value. It is therefore desirable to only consider those panels having significant impact on the flutter behavior. One approach for finding these panels is to perform uncertainty validation considering one uncertain patch at a time, and to compute the bound required to validate a set of measured data points. Performing the validation for the full missile configuration with respect to the measured damping of data points in Figure 11 yields the results shown in Figure 15. The decisive velocity was found to be the minimum airspeed of 25 m/s, where the measured damping deviates the most. The Figure shows that the first patch, located at the leading edge of the wing tip, would require an uncertainty bound of approximately 0.3, meaning that a variation of the pressure coefficients in this patch by 30% would be needed to explain the difference between the nominal analysis and the experimental data. It was found that selecting the most significant patches may establish an efficient and realistic uncertainty model. The Figure indicates the choice of the 7 most sensitive patches to obtain an uncertainty description of reasonable size. Note that the number

Figure 15: Required uncertainty norm for validation of experimental data using individual patches for the full missile configuration.
of considered patches is somewhat arbitrary, and the impact of this number on the robust flutter bounds is discussed below.

5.5 ROBUST RESULTS

The five presented configurations can be divided into two groups with significantly different behavior. Configurations 2, 4 and 5, where no rear fins are attached, show almost perfect agreement of the predicted and measured flutter speed, whereas configurations 1 and 3, where the rear fins are attached, show a slight deviation between analysis and experiment. Robust results are therefore presented for two configurations with different behavior. Note that the robust analysis was performed based on the nominal model generated in Nastran.

First, considering the most simple configuration without the missile (Config. 5), the validation was performed for one patch at a time, and it was found that the leading edge wing tip patch introduced the most significant uncertainty. Using this patch only, an uncertainty level of 24% was required to validate the uncertain model with respect to the measured damping, which was considered to be realistic. Figure 16 shows the measured and nominally predicted damping as a function of the velocity, as well as the robust boundaries based on the validated model. In this case, the decisive test case for the validation bound was found to be 21 m/s, where the measured damping touches the robust boundary. All other experimental points are within the robust boundaries. Due to the conservative nature of the nominal model, the worst-case prediction is even more conservative. The validation would have produced similar results even if no data points would have been available at and above the flutter speed. In this case, the validation procedure would thus be particularly useful in cases where no flutter data points can be collected, since the bounds established at lower airspeeds can be used for robust analysis at higher speeds.

Another approach is to increase the airspeed stepwise, performing an uncertainty validation and predicting updated flutter bounds at every test point. Figure 17 shows results for the configuration with the full missile, where the airspeed was increased from 25 m/s up to 35 m/s. Robust boundaries were computed based on uncertainty bounds obtained from $g$-validation performed at each test point. Note that the nominally predicted flutter speed is 32 m/s and independent of any uncertainty in the model.

Starting at 25 m/s, the uncertainty validation gives an uncertainty bound that results in a robust flutter speed between 28.6 and 35.6 m/s. In a flight test scenario where flutter conditions should be avoided, the airspeed could thus be increased to 28.6 m/s based on the current validation. Increasing the airspeed to 26 m/s and performing another validation gives new bounds for the robust flutter speed, allowing for airspeeds up to 28.9 m/s. Increasing the airspeed stepwise, the lower-bound flutter speed is reached at about 30 m/s, where both the actual testing speed and the lower-bound prediction coincide.
In the present study, it was possible to operate the wind tunnel model at flutter conditions, and the airspeed could therefore be increased further, and the uncertainty validation could be performed for airspeeds at and above the flutter speed. In general, it was found that the nominal model and the experimental data agree better for higher airspeeds, therefore reducing the bandwidth of the robust boundaries as increasing the airspeed.

Note that different uncertainty models were used for the current configuration. Between 1 and 20 uncertain panels as shown in Figure 14 were used, but it was found that due to the uncertainty validation adjusting the uncertainty bound, robust results only change marginally.

6 CONCLUSIONS

This paper investigated the impact of an external store on the flutter behavior of a delta wing configuration. A wind tunnel model was used to provide experimental results for evaluation of nominal and robust flutter results.
In order to identify modeling uncertainties, it was found convenient to model the missile in several steps, and to investigate how the flutter behavior changed gradually, both in the experiment and the analysis. It was found that the flutter behavior depends strongly on rather small missile details. This is mainly due to the location of the missile at the wing tip, where even small loads can have significant impact. Not only the flutter behavior as such, but also the performance and reliability of the numerical analysis may depend on rather small details, such as the missile rear fins.

In many cases, when differences between analysis and testing are detected, finding the modeling error is rather difficult. In this study, the aerodynamic loads were assumed to be the most significant source of uncertainty, and uncertainty in the pressure coefficients was introduced to account for the model imperfections. The uncertainty models chosen in this study do not guarantee that the correct numerical model is covered by the uncertainty description. Instead, the uncertain patches were defined by engineering judgment, and the most sensitive patches selected. This selection was found to increase the computational efficiency without compromising the quality of the robust flutter prediction.
It is clear that in cases with modeling errors in the structural data, a robust approach based on aerodynamic uncertainty may be very inefficient, since unreasonably large uncertainty bounds may be required in order to cover these errors. For this reason, it was here found unnecessary to adjust the uncertainty bound in order to capture the frequency error that most likely is due to errors in the structural model.

In the present case, it was found that aerodynamic uncertainty can be used to efficiently establish robust flutter boundaries. Using subcritical experimental data for model validation, the estimated robust boundaries were found to be valid even at supercritical (unstable) conditions.

7 REFERENCES


