When Near Becomes Super Far: From Rayleigh to **Optimal Near / Far-Field Boundaries**

Sajad Daei

May 17, 2025

Joint work with Gabor Fodor ⋆ † and Mikael Skoglund †
† KTH Royal Institute of Technology
⋆ Ericsson Research, Stockholm

Workshop on Near-field Communications and Sensing

Roadmap

1. History: Rayleigh rule
2. Motivation: Why Rayleigh is no longer enough
3. Mismatch metrics & optimal boundaries
4. Key numerical insights
5. Take-aways for Hardware, DSP, and network layers

History: Rayleigh Distance

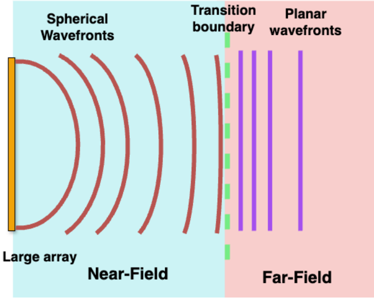
Definition

- Distance from an antenna/aperture beyond which wavefronts can be approximated as planar (Far-Field).
- Marks transition from Near-Field (spherical) to Far-Field (planar) behavior.

Formula:

$$R_{\mathrm{Ray}} \triangleq \frac{2D^2}{\lambda}$$

- D: Aperture diameter or maximum dimension
- λ: Wavelength



Visualization of near and far fields

Why It Matters

Understanding transition distance is crucial for accurate sensing, communications and beamforming design, as it defines the region of validity for planar-wave assumptions.

Why *Rayleigh* Is No Longer Enough

Rayleigh's 1903 Rule

$$R_{\text{Ray}} = \frac{2D^2}{\lambda}$$

- Assumes **max** phase error $\frac{\pi}{8}$ (center-edge).
- **6G mmWave/sub-THz**: $\lambda \approx 1\,\text{mm}$, $D \approx 1\,\text{m}$

$\Rightarrow R_{\text{Ray}} \approx 2\,\text{m}$ (indoor link!)

If you rely on R_{Ray}

- **Array-gain drop:** 3 to 6 dB
- **Localization bias error:** 5–20 cm bias
- **Pilot overhead:** doubled for UL channel estimation

Engineering Insight

Rayleigh distance is an outdated **geometric** criterion. Modern 6G systems need metric-driven boundaries for array gain, spectral efficiency, and localization accuracy.

Near-Field **Opportunities**

3-D beam focusing

~1 cm

indoor positioning–centimetre-level without UWB.

Depth-resolved MU-MIMO: users on the *same* AoA but different ranges become separable $\Rightarrow$ extra spatial DoF.

Zero-overhead sensing: narrow-band TX already encodes range/Doppler in spherical curvature $\Longrightarrow$ free radar snapshot.

Channel Model: Near vs. Far Field

Uplink Scenario (LoS, ULA)

Radiative Near-Field (Spherical Wavefront):

$$\mathbf{h}_{\mathrm{NF}}[n] = \sqrt{g_T g_R[n]}\frac{\lambda}{4\pi R_n} e^{-jk_\lambda R_n}$$

Far-Field (Planar Wavefront):

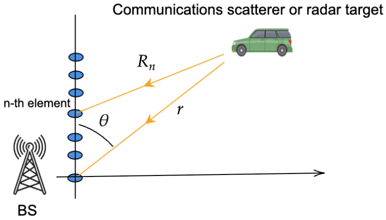
$$\mathbf{h}_{\mathrm{FF}}[n] = \sqrt{g_T g_R}\frac{\lambda}{4\pi r} e^{-jk_\lambda (r - nd\cos\theta)}$$

Range definition:

$$R_n = \sqrt{r^2 + (nd)^2 - 2rnd\cos\theta}, \ \ n = 0, \dots, N_r - 1$$

Wave number: $k_\lambda \triangleq \frac{2\pi}{\lambda}$

Assumptions: Isotropic elements, free space channel model



System Setup Illustration

4 Application-Driven "Mismatch Metrics"

Worst-case element-wise Mismatch

$$E_{\ell_\infty}(r) \triangleq \max_{n,\theta} \left| \frac{1}{R_n} e^{-jk_\lambda (R_n - r)} - \frac{1}{r} e^{jk_\lambda nd\cos\theta} \right|$$

Normalized ℓ_2 Mismatch

$$E_{\ell_2}(r) \triangleq \max_\theta \frac{\|\mathbf{h}_{\mathrm{NF}} - \mathbf{h}_{\mathrm{FF}}\|_2}{\|\mathbf{h}_{\mathrm{NF}}\|_2}$$

NMSE of model-based estimators (e.g., compressed sensing)

$$\mathrm{NMSE} \triangleq \max_\theta \mathbb{E}_{\mathbf{w}}\left[\frac{\|\mathbf{h}_{\mathrm{NF}} - \widehat{\mathbf{h}}\|_2^2}{\|\mathbf{h}_{\mathrm{NF}}\|_2^2}\right] = \underbrace{(1-\eta)}_{\triangleq \mathrm{NMSE}_{\mathrm{bias}}} + \frac{1}{L\rho_{\mathrm{p}}\|\mathbf{h}_{\mathrm{NF}}\|_2^2}$$

Spectral-eff. loss

$$E_{\mathrm{SE}}(r) \triangleq \max_\theta \log_2\left(\frac{1+\rho_{\mathrm{d}} G}{1+\frac{\eta^2 G^2 \rho_{\mathrm{d}}}{\frac{G\eta\rho_{\mathrm{d}}}{L\rho_{\mathrm{p}}}+\eta G+\frac{1}{L\rho_{\mathrm{p}}}}}\right)$$

Notations (quick view)

- $G \triangleq \|\mathbf{h}_{\mathrm{NF}}\|_2^2$
- $\eta = \frac{|\mathbf{h}_{\mathrm{NF}}^H \mathbf{h}_{\mathrm{FF}}|^2}{\|\mathbf{h}_{\mathrm{NF}}\|_2^2 \|\mathbf{h}_{\mathrm{FF}}\|_2^2}$ is the array gain efficiency
- ρ_{d} and ρ_{p} are the data and pilot SNR. L is the number of pilots.

"Pick the metric that aligns with your layer – and stay outside its red zone!"

Metric-Aligned "Traffic Light" for Practical Design

Hardware

- Calibration budgets
- PA/LNA linearity
- ADC dynamic range

Baseband DSP

- NMSE floor (–13 dB)
- Fisher-info positioning
- Pilot reuse & grouping

Network Layer

- SE margin / MCS tables
- Cell-edge robustness
- ISAC resource split

Near-field is a *resource*–leverage depth, curvature, and power rather than masking them.

Optimal Transition Distances

Optimal transition distance

$R_{\mathrm{OPT},\ell_\infty},\ R_{\mathrm{OPT},\ell_2},\ R_{\mathrm{OPT,SE}} = \min\{r:\ E(r') \leq \text{tolerance}\,\forall r' \geq r\}$

- Closed-form boundaries for Worst-case Mismatch:

$$R_{\mathrm{EPF}} \triangleq \inf\left\{r \geq 0:\ \sup_{r' \geq r}\left[\frac{D^2}{2r'^3} + \frac{2}{r'}\left|\sin\left(\frac{k_\lambda D^2}{4r'}\right)\right|\right] \leq \delta_\infty\right\}. \tag{1}$$

$$\boxed{R_{\mathrm{SPF}} \triangleq 2\sqrt{\frac{k_\lambda D^2}{6\delta_\infty}}\cos\left(\frac{1}{3}\arccos\left(\frac{3}{2k_\lambda}\sqrt{\frac{6\delta_\infty}{k_\lambda D^2}}\right)\right)} \tag{2}$$

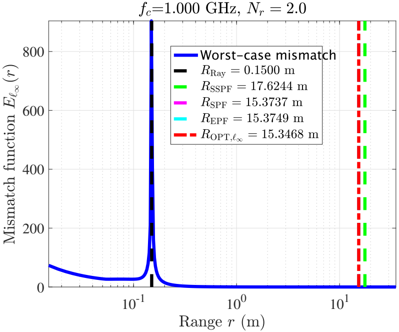
$$\boxed{R_{\mathrm{SSPF}} \triangleq \sqrt{\frac{k_\lambda D^2 + D}{2\delta_\infty}}}. \tag{3}$$

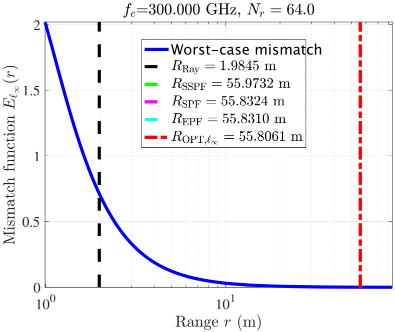
Our metrics provide precise, **application-specific** boundaries!

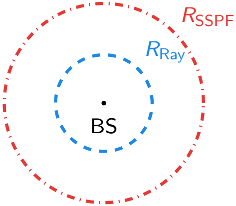
Numerical Insights and Comparisons

Worst-case element-wise Mismatch

The tolerance error is $\delta_\infty = 0.001$ 1/m





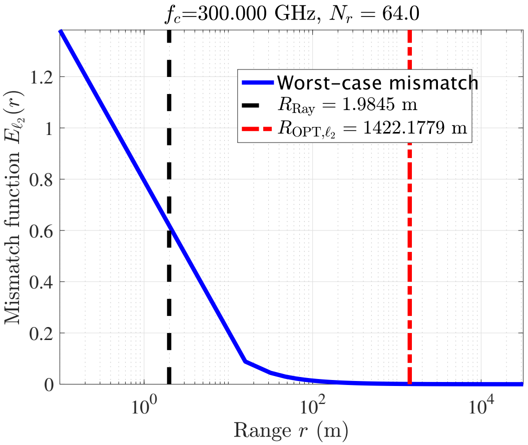


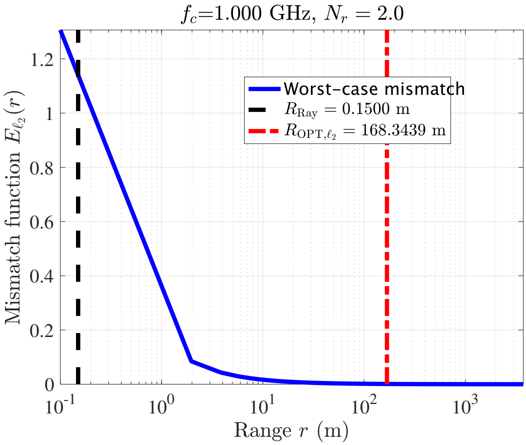
- Rayleigh boundary severely underestimates NF region with element-wise metric.
- Optimal boundary up to 10-100×Rayleigh

Numerical Insights and Comparisons

Worst-case ℓ_2 Normalized Mismatch

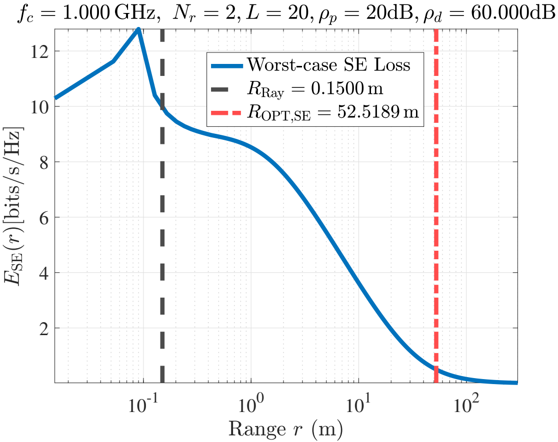
The tolerance error is $\delta_{\ell_2} = 0.001$

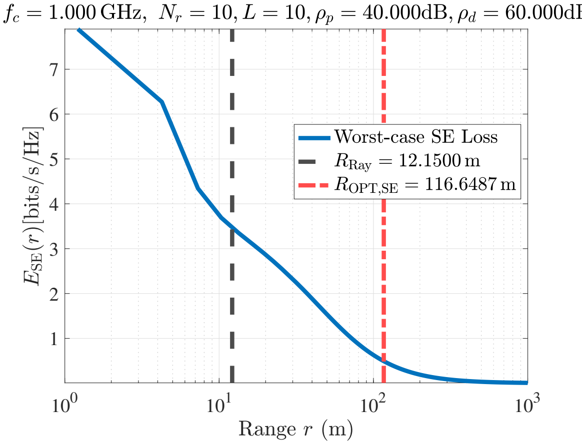




SE Mismatch

The tolerance error is $\delta_{\text{SE}} = 0.5$ bits/s/Hz





Take-Home Messages

- **Near–field = an opportunity, not a limitation**
  Depth, curvature, and power are intrinsic resources.
- Plane-wave Rayleigh yardstick $\longrightarrow$ **10–100** $\times$ **too optimistic** at mmWave/sub-THz.
- **Three metric-driven boundaries** $\{E_{\ell_\infty}, E_{\ell_2}, E_{\mathrm{SE}}\}$
  - hardware safety
  - signal-processing fidelity
  - network throughput
- $\approx$ **1 cm** indoor positioning, **depth-resolved MU-MIMO**, and **zero-overhead sensing** are already inside every UL packet.

3 dB — bias-free array gain margin

−13 dB — NMSE floor (LS/MMSE/CS/Super-res)

0.5 bits — SE loss

Rethink your link budget: embrace *depth-aware* communications!

Thank you!

Questions?

Contact: `sajado@kth.se`