A two-phase annual ring model of transverse anisotropy in softwoods

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Abstract

Transverse anisotropy in softwoods is an important phenomenon of both scientific and industrial interest. Simple one-phase hexagonal honeycomb cell models for transverse moduli of softwoods are based on cell wall bending as the only deformation mechanism. In the present study, a two-phase annual ring model is developed and includes both cell wall bending and stretching as deformation mechanisms. The proportion of cell wall bending and stretching for different cases is analysed and the importance of stretching is confirmed. A two-phase annual ring model is presented based on fixed densities for earlywood and latewood. Such a model is motivated by the large difference in density between earlywood and latewood layers. Two-phase model predictions show much better agreement with experimental data than predictions from a one-phase model. Radial modulus is dominated by bending at low density and by stretching at high density. For tangential modulus, bending is more important at all densities.

1. Introduction

Wood shows transverse anisotropy so that the radial modulus \( E_R \) is significantly higher than the tangential modulus \( E_T \). This is an important phenomenon, despite the fact that it tends to be disregarded in structural design recommendations [1]. In addition, the phenomenon of surface checks due to shrinkage and swelling of wood products is influenced by transverse anisotropy. Due to the anisotropy of elastic properties, shrinkage in the tangential direction is much larger than in the radial direction. Micromechanics modelling is of interest in the context of transverse anisotropy, since global deformation is influenced by deformation at the scale of the cellular structure. Finite element analysis has been carried out taking cell scale deformation into account [2,3]. The models tend to be highly complex, and deformation mechanisms at the scale of cells are seldom discussed. Gibson and Ashby [4] presented an attractive hexagonal honeycomb model and compared scaling predictions for transverse moduli with experimental data as a function of relative density. Careful scrutiny reveals that the agreement is not very convincing. Kahle and Woodhouse [5] successfully applied Gibson and Ashby's cell wall bending model to predict wood properties. Four recent micrographs were used to calculate a distribution of cell geometries, and effective global moduli were the calculated. These moduli were compared with experimental values from the literature. Although agreement is good, the global density range of the experimental values is very limited. In a recent paper [6], we pointed out that Gibson and Ashby's wood predictions are based on a model with cell wall bending only, and this is likely to limit the predictive capability. We suggested that the deformation mechanism of cell wall stretching is important and needs to be accounted for. Both experimental and theoretical arguments were presented in support of this theory. Masters and Evans [7] published a general honeycomb model including cell wall stretching, and successfully predicted earlywood moduli for several species. The cell shape angle was adjusted for each species, based on experimental data. In contrast to the present work their model is a one-phase model and has only been applied to earlywood. In the present study, the objective is to develop a general model for predictions of transverse anisotropy in terms of moduli versus relative density for softwoods. Furthermore, we study the proportion of bending and stretching deformation associated with different cases. Important parameters include cell shape angle, relative density and cell wall modulus. We introduce a two-phase wood model based on a layered earlywood and latewood material. We then study the relative contribution towards anisotropy from earlywood and latewood phases, and in the combined two-phase wood material.

2. Model

In order to model the transverse elasticity of softwoods from basic properties, a cellular solids approach may be used. Gibson and Ashby [4] suggested a honeycomb geometry for the
description of wood. In the present study, two models will be dis-
cussed. First a one-phase honeycomb model is introduced, in-
cluding both bending and stretching of the cell walls. A two-phase
annual ring model is developed, describing the annual ring struc-
ture in wood.

2.1. One-phase model

Before the elasticity of honeycombs can be discussed, we need
to establish the geometry and make some assumptions. In Fig. 1,
the geometry of a honeycomb is defined. All six walls are assumed
to have the same length \( l \) and thickness \( t \). The angle \( \theta \), termed cell
shape angle, defines the angle between the radial direction and the
cell walls.

The relative density \( \rho/\rho_s \) is defined as the density of the cellular
solid, in this case wood, divided by the cell wall density \( \rho_s \). In soft-
woods, the cell wall density \( \rho_s = 1500 \text{ kg/m}^3 \) [11]. The relative den-
sity is related to the geometry of honeycombs through the
relationship

\[
\frac{\rho}{\rho_s} = \frac{t}{l} \frac{3}{2} \cos \theta (1 + \sin \theta). \tag{1}
\]

The relative density is thus directly proportional to the aspect ratio
of the cell walls. For honeycombs, the relative modulus is defined as the
ratio between modulus of the cellular solid and the cell wall. Gibbons
and Ashby derived formulas for elastic deformation of hon-
yecombs where cell walls deform in pure bending. For honeycombs
with low relative density, i.e. very slender cell walls, bending of the
cell walls is obviously the most important deformation mechanism.

For the case of pure bending, the relative moduli will take the fol-
lowing forms:

\[
\frac{E_{\text{bb}}}{E_s} = \left( \frac{t}{l} \right)^3 \frac{\cos \theta}{(1 + \sin \theta) \sin^2 \theta}, \tag{2}
\]

\[
\frac{E_{\text{tb}}}{E_s} = \left( \frac{t}{l} \right)^3 \frac{1 + \sin \theta}{\cos^2 \theta}. \tag{3}
\]

The moduli for bending is proportional to the cube of \( t/l \), and thus also
to the cube of the relative density. The dependence of radial
bending modulus \( E_{\text{bb}} \) on cell shape angle is also interesting. As
the angle \( \theta \to 0 \), the modulus \( E_{\text{bb}} \to \infty \). Hence, the model provides
unrealistic predictions for small angles.

For the case of pure cell wall stretching, the relationships for relative
modulus will take the following forms [7]:

\[
\frac{E_{\text{ts}}}{E_s} = \frac{1}{1 + \sin \theta} \cos \theta, \tag{4}
\]

\[
\frac{E_{\text{ts}}}{E_s} = \frac{1 + \sin \theta}{1 + \sin \theta} \cos \theta \tag{5}
\]

In the pure cell wall stretching case, the relative honeycomb mod-
ulus will be directly proportional to relative density and \( E_{ts} \) will att-
in reasonable values when the cell shape angle \( \theta = 0 \).

A model with combined bending and stretching, based on the
work by Masters and Evans [7], is created through the combination
of Eqs. (2) and (4) for the radial direction and (3) and (5) for the
tangential direction.

\[
\frac{E_{\text{bb}}}{E_s} = \left( \frac{t}{l} \right)^3 \frac{\cos \theta}{(1 + \sin \theta) \sin^2 \theta} \frac{1 + \sin \theta}{1 + \sin \theta} \cos \theta \tag{6}
\]

\[
\frac{E_{\text{tb}}}{E_s} = \left( \frac{t}{l} \right)^3 \frac{1 + \sin \theta}{\cos^2 \theta} \frac{1 + \sin \theta}{1 + \sin \theta} \cos \theta \tag{7}
\]

2.2. Two-phase annual ring model

A simple two-phase model is developed in order to describe
contributions from earlywood and latewood. There is a large differ-
ence in density between earlywood and latewood. If densities of
earlywood and latewood are given, Eqs. (6) and (7) may be used to
predict their properties.

Composite mechanics theories are then used to calculate the
global modulus from the earlywood and latewood moduli. In the
tangential direction, earlywood and latewood layers will have
equal strain, so a Kelvin model is used. This gives the relationship

\[
E_{\text{t}} = E_{\text{early}} V_{\text{early}} + E_{\text{late}} (1 - V_{\text{early}}). \tag{8}
\]

where \( V_{\text{early}} \) is the volume fraction of the earlywood, \( E_{\text{early}} \) and \( E_{\text{late}} \)
are the tangential moduli for earlywood and latewood calculated
using the one-phase model. In the radial direction, strains in the
earlywood and latewood will be different. Equilibrium requires
equal stress in layers of earlywood an latewood. A Voigt model is
therefore used, giving the relationship

\[
E_{\text{r}} = \left( \frac{V_{\text{early}} E_{\text{early}} + 1 - V_{\text{early}} E_{\text{late}}}{E_{\text{early}}} \right)^{-1}. \tag{9}
\]

To calculate the fraction of earlywood from global density data
in the literature the formula

\[
V_{\text{early}} = \frac{\rho - \rho_{\text{late}}}{\rho_{\text{early}} - \rho_{\text{late}}} \tag{10}
\]

is used, where \( \rho \) is the density of the clear wood. According to Koll-
mann [12], typical earlywood in softwood has a density of about
\( \rho_{\text{early}} = 250 \text{ kg/m}^3 \), corresponding to a relative density \( \rho_{\text{early}}/\rho_s = 0.17 \). Latewood has a density of \( \rho_{\text{late}} = 750 \text{ kg/m}^3 \), correspond-
ing to a relative density \( \rho_{\text{late}}/\rho_s = 0.5 \).

2.3. Determination of cell wall modulus and cell shape angle

Instead of assuming values for cell shape angle and cell wall
modulus and calculating the radial and tangential moduli, the in-
verse can be done. In the case of a one-phase material, the following
relationships hold for a specific density (see Eqs. (6) and (7))

\[
E_{\text{r}} = f_1(E_s, \theta), \tag{11}
\]

\[
E_{\text{t}} = f_1(E_s, \theta). \tag{12}
\]
Experimental data for relative density, $E_R$ and $E_T$ are used as starting point. From Eqs. (1), (6) and (7), it is apparent that at a given relative density, the anisotropy ratio between $E_R$ and $E_T$ is controlled by $\theta$, whereas $E_s$ controls the magnitude of $E_R$ and $E_T$. Since there are two equations and two unknowns, there is a unique solution for $\theta$ and $E_s$.

In the case of the two-phase model, the procedure to determine $\theta$ and $E_s$ is slightly more cumbersome. Experimental data for $E_R$ and $E_T$ at a given density are again used as input. Because of the need to use Eqs. (6) and (7) in combination with Eqs. (8) and (9), we obtain the following relationships:

$$E_R = \left[ \frac{V_{\text{early}}}{E_{\text{mod}}(E_s, \theta)} \right]^{-1} \left( \frac{1}{E_{\text{late}}(E_s, \theta)} \right),$$

$$E_T = V_{\text{early}}E_{\text{late}}(E_s, \theta) + (1 - V_{\text{early}})E_{\text{late}}(E_s, \theta).$$

In the two-phase model, there are four different variables: $\theta_{\text{early}}$, $\theta_{\text{late}}$, $E_{\text{early}}$, and $E_{\text{late}}$. In order to reduce the number of unknowns, it is assumed that earlywood and latewood have the same $E_s$ and $\theta$.

3. Results and discussion

3.1. One-phase model – parametric study

To better understand the implications of the model, some parametric studies are performed. In Figs. 2–5, the solid lines are the moduli of the stretching–bending model (SB-model). The dotted lines are data from the original honeycomb model suggested by Gibson and Ashby, where the deformation mechanism is bending of the cell walls (B-model). The dashed lines are the moduli for a honeycomb model where only stretching is allowed (S-model).

Fig. 2 presents the modulus dependence on relative density if the cell shape angle is 30°. Honeycombs with this cell shape angle are termed regular honeycombs. Due to the symmetries that arise from this particular geometry, a regular honeycomb will be isotropic, i.e. have the same modulus in all directions. This holds regardless of the assumed deformation mechanism, thus the following three relations hold for any regular honeycomb.

$$E_R = E_T, \quad E_{\text{RB}} = E_{\text{Tb}}, \quad E_{\text{Rs}} = E_{\text{Ts}}.$$  

It is clear from Fig. 2 that the proposed SB-model, with a combination of stretching and bending of the cell walls, will be less stiff than any of the other two models. The reason for this is that the strain in the SB-model receives contributions from both deformation mechanisms (stretching and bending), whereas the other models only include strains from one of the mechanisms.

For low densities, a consequence of Eqs. (6) and (7) is that

$$\lim_{\theta \to 0} E_s = E_{\text{RB}} \quad \text{and} \quad \lim_{\theta \to 0} E_T = E_{\text{Tb}}.$$  

hence bending is the dominating mechanism for low density materials. For higher relative densities, greater than approximately 0.3, the relative modulus shows an almost linear relationship with relative density. This indicates that the higher density region is dominated by cell wall stretching.

In softwoods the real cell shape angle is not 30°, but smaller, i.e. the radial walls are more parallel to each other. Measurements of cell shape angle by Watanabe et al. [9] suggests values ranging from 7° to 19° for different species of softwoods. If the cell shape angle $\theta$ is changed to 15°, the material will no longer be isotropic, as is observed in Fig. 3. Here, the solid lines are the moduli calculated using the SB-model. The radial modulus increases, and the tangential modulus decreases, as compared with the case when $\theta = 30°$.

The reason for the anisotropy can be found by comparing predictions for $E_{\text{RB}}$ and $E_{\text{Tb}}$, the dotted lines in Fig. 3, representing the B-model. The anisotropy is very strong, especially at high density. As the cell shape angle decreases the radial modulus will increase strongly. This angular dependence will be described in detail later. In the S-model (dashed in Fig. 3) the anisotropy is much less pronounced. For the SB-model, the transition from bending dominated deformation to stretching dominated deformation occurs at a lower relative density in the radial direction, as compared with the isotropic honeycomb.

In the tangential direction, on the other hand, deformation will become more dominated by bending as the angle decreases. This is because the modulus for the B-model (dotted) will be much lower than the modulus associated with the S-model, for all densities. If, instead of fixing the cell shape angle, the relative density is fixed, it is possible to plot the dependence of modulus on cell shape angle. In Fig. 4 the relative density is set to 0.17. This density corresponds to the relative density of an earlywood layer in softwood [12].

From the solid lines representing the SB-model in Fig. 4(a) and (b), it is clear that there is a substantial anisotropy for normal cell shape angles $\psi$, in the region from about 10° to 20°. This observation of anisotropy is in good agreement with transverse earlywood moduli measurements performed by Farruggia and Perre [13] and...
The deformation in tangential loading. Watanabe et al. [10] also gives experimental modulus data as a function of the density for earlywood. Though the proposed model only tells relative modulus values, there it shows a reasonable agreement of how much stiffer the highest density measurements is to the lowest density measurements. The tangential modulus $E_T$ becomes very low, since it is dominated by cell wall bending, $E_T \approx E_{T_b}$. The modulus calculated using the pure cell wall stretch model is much higher, $E_{T_b} \gg E_T$.

As previously mentioned the radial modulus for pure bending $E_{R_b} \to \infty$ as $\theta \to 0$. The reason is that no bending is induced in the radial walls since they are parallel to the applied force. In the S-model the radial modulus, $E_{R_b}$, is less sensitive to the angle, but increases slightly when the cell shape angle becomes smaller. Pure stretching would result in lower modulus than pure bending when $\theta < 8^\circ$, see Fig. 4. Kollmann [12] suggests that the relative densities for typical latwood in softwoods are about 0.5. Fig. 5 shows the predicted modulus dependence on cell shape angle for latwood.

The dependence of radial modulus $E_R$ on cell shape angle is much less pronounced in latwood as compared to earlywood. At cell shape angles $\theta < 23^\circ$, the radial modulus associated with the stretching model, $E_{R_b}$, is less than that of the bending model, $E_{R_b}$. Since $\theta < 15^\circ$ in many softwoods, the stretching model is very important for the description of radial latwood modulus.

In latwood, the tangential modulus $E_T$ is dominated by bending since the bending modulus $E_{T_b}$ is smaller than the tangential stretching modulus $E_{T_s}$, regardless of the cell shape angle. However, stretching of the cell wall plays a larger role than in earlywood, since the difference between $E_{T_{sb}}$ and $E_{R_{sb}}$ is larger than in earlywood (Fig. 4).

### 3.2. Predictions of $E_s$ and $\theta$

Density and modulus data listed in the literature for a number of softwoods are presented in Table 2. A one-phase model was developed for different woods by fitting cell shape angle and cell wall modulus in the SB-model to experimental data for density, $E_R$ and $E_T$. A two-phase model was also developed using the procedure described in the modelling section. In both procedures it is possible to solve for the cell wall modulus and cell shape angle in the model, so that experimental data are matched perfectly. The purpose is to investigate if the resulting cell wall modulus and cell shape angle data compare well with experimental data. From this procedure it becomes apparent that anisotropy is controlled by cell shape angle in the one-phase model. The resulting values for the different wood species are found in Table 2.

The angle for the one-phase material in Table 2 is larger ($23–26^\circ$) than typical angles in real wood ($6–21^\circ$) [9], whereas the two-phase material shows more realistic values (11–14°). In the one-phase model there is a large scatter in calculated cell wall modulus $E_w$, whereas calculated $E_s$ shows much less scatter in the two-phase model. The cell wall moduli in Table 2 may be compared with the 10 GPa suggested by Cave [14]; the 15 GPa suggested by Watanabe et al. [9] and the 4 GPa suggested by Bergander and Salmén [15]. Thus, the present values for $E_s$ are too high, and the geometrical model is less stiff than a real softwood cell structure. There are two main reasons why the model is softer. First, the chosen idealised geometry maximises the extent of bending during tangential loading. The second reason is that the
latewood imposes some constraint on radial earlywood deformation, which is not taken into account in the present model. The stiff latewood layer is limiting earlywood strains in a region close to the earlywood/latewood interface. This observation has been confirmed in several studies. [6,16].

It is also interesting to look at the intermediate results for the two-phase model in Table 1. The table shows the radial and tangential moduli for earlywood and latewood, and the respective anisotropy ratios between these two. It can be noted that the anisotropy of both earlywood and latewood is much larger than the anisotropy in clear wood (combined earlywood and latewood). This is in good agreement with experimental modulus measurements, see Table 3. The data are obtained at sub-annual ring scale, from different species and different experimental set-ups. None of the listed references measured radial and tangential modulus for both earlywood and latewood. It is clear that there is a significant anisotropy in earlywood. For latewood there is less data available, but it is likely that the anisotropy is large there as well.

### 3.3. Two-phase annual ring model – parametric study

Predictions from the two-phase model can be plotted as a function of density, since we chose a fixed value for cell shape angle based on Table 2 ($\theta = 12.75^\circ$). We can then predict $E_R/E_t$ and $E_t/E_s$ for the density range between earlywood and latetwood densities (softwood density is assumed to be controlled by the proportions of earlywood and latewood) using the composite models described in the modeling section. The resulting functions are presented in Fig. 6. Tangential modulus, $E_t$, is a linear function of density, as expected for a model where two layers are loaded parallel to their length direction. The radial modulus, $E_R$, is a non-linear function of density. This is again a consequence of the composites model, where the layers in this case are in series. The $E_R/E_t$ anisotropy ratio is largest as we approach compositions close to only earlywood (low density) or only latewood (high density). The anisotropy is thus lower in clear wood (combined earlywood and latewood) as compared with in the earlywood or latewood layers separately.

Table 4 lists the relative importance of the bending mechanisms for earlywood and latewood in the two directions. In the radial direction, bending is important in the earlywood, while stretching dominates in the latewood. In the tangential direction, bending is more important in both earlywood and latewood. These observations are also illustrated in Figs. 4 and 5 for the calculated cell shape angle $\theta = 12.75^\circ$.

### 3.4. Predictions of transverse moduli

Based on the average of the $E_s$-values in Table 2 for the two-phase model, we selected a fixed value for effective $E_t$ in all softwoods ($E_s = 20.5$ GPa for the two-phase model and $E_s = 25$ GPa for the one-phase model). The cell shape angle is also fixed at $\theta = 12.75^\circ$ and $\theta = 25^\circ$, respectively. Based on density as the only input, we can then predict $E_R$ and $E_t$ for all softwoods. In Fig. 6(a), $E_R$ predicted from the two-phase model is plotted versus relative density. Fig. 6(a) also contains experimental data for softwoods [11,17]. The agreement for the two-phase model is reasonably good, considering the nature of the data (many different sources) and the generality of the model. The agreement with the one-phase model predictions is not good. At low densities, earlywood dominates and bending is relatively important, see Table 4. At higher densities, latewood dominates and there is very little bending (stretching dominates). Despite the linear nature of the stretching

### Table 2

Cell wall modulus and cell shape angles for softwoods calculated using a one-phase model (Eqs. (6)-(9)) and the two-phase model

<table>
<thead>
<tr>
<th>Softwood</th>
<th>One-phase model</th>
<th>Two-phase model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cell wall modulus, $E_s$ (GPa)</td>
<td>Cell wall modulus, $E_s$ (GPa)</td>
</tr>
<tr>
<td></td>
<td>Cell shape angle (°)</td>
<td>Cell shape angle (°)</td>
</tr>
<tr>
<td>Norway Spruce</td>
<td>26</td>
<td>21</td>
</tr>
<tr>
<td>Sitka Spruce</td>
<td>31</td>
<td>25</td>
</tr>
<tr>
<td>Scots Pine</td>
<td>14</td>
<td>16</td>
</tr>
<tr>
<td>Douglas Fir</td>
<td>17</td>
<td>20</td>
</tr>
</tbody>
</table>

### Table 1

Input data and intermediate results from simulations of two-phase honeycomb material

<table>
<thead>
<tr>
<th>Softwood</th>
<th>Density (kg/m²)</th>
<th>$E_s$ (MPa)</th>
<th>$E_t$ (MPa)</th>
<th>$E_s/E_t$</th>
<th>$E_R$ (MPa)</th>
<th>$E_t$ (MPa)</th>
<th>$E_s/E_t$</th>
<th>$E_R/E_t$</th>
<th>$E_t$ (MPa)</th>
<th>$E_s/E_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norway Spruce</td>
<td>390</td>
<td>710</td>
<td>430</td>
<td>1.65</td>
<td>532</td>
<td>65</td>
<td>8.08</td>
<td>5103</td>
<td>1366</td>
<td>3.73</td>
</tr>
<tr>
<td>Sitka Spruce</td>
<td>390</td>
<td>900</td>
<td>500</td>
<td>1.80</td>
<td>675</td>
<td>76</td>
<td>8.86</td>
<td>6230</td>
<td>1589</td>
<td>3.92</td>
</tr>
<tr>
<td>Scots Pine</td>
<td>550</td>
<td>1100</td>
<td>570</td>
<td>1.93</td>
<td>522</td>
<td>44</td>
<td>12</td>
<td>4205</td>
<td>920</td>
<td>4.57</td>
</tr>
<tr>
<td>Douglas Fir</td>
<td>590</td>
<td>1300</td>
<td>900</td>
<td>1.44</td>
<td>508</td>
<td>62</td>
<td>8.16</td>
<td>4858</td>
<td>1294</td>
<td>3.75</td>
</tr>
</tbody>
</table>
Table 3
Experimental moduli data of softwoods at the sub-anual ring scale

<table>
<thead>
<tr>
<th></th>
<th>Earlywood</th>
<th>Latewood</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Radial direction (MPa)</td>
<td>Radial direction (MPa)</td>
</tr>
<tr>
<td>Modén</td>
<td>600–1500</td>
<td>1000–3000</td>
</tr>
<tr>
<td>Watanabe</td>
<td>200–1200</td>
<td>50–400</td>
</tr>
<tr>
<td>Farruggia</td>
<td>740–800</td>
<td>160–230</td>
</tr>
</tbody>
</table>

Table 4
Factors for the relative importance of the bending mechanism for earlywood $\rho/\rho_s = 0.17$ and latewood $\rho/\rho_s = 0.5$

<table>
<thead>
<tr>
<th></th>
<th>Radial</th>
<th>Tangential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earlywood</td>
<td>0.71</td>
<td>0.96</td>
</tr>
<tr>
<td>Latewood</td>
<td>0.05</td>
<td>0.71</td>
</tr>
</tbody>
</table>

A value of 1 means that there is only bending; 0 is only stretching of the cell walls. The factors are calculated as $e^{-4}\mu_0/\mu_s$ and $e^{-4}\mu_0/\mu_s$.

function for a one-phase hexagonal honeycomb, the composite nature of a two-phase model causes a fairly non-linear prediction, as already pointed out in connection with the parametric study.

In Fig. 6(b), $E_T$ predictions for two-phase and one-phase models are presented with experimental data. The predictions are in agreement with softwood data in the literature, although the scatter in data is considerable. At lower densities, the discrepancy becomes larger. However, compared with the corresponding figure in Gibson and Ashby [4], based on a regular one-phase hexagonal model, the predictions are closer to data. Based on Table 4, we are able to conclude that cell wall bending is the dominating mechanism at all densities during tangential loading. Due to the composite nature of the earlywood/latewood structure, $E_T$ still shows a linear dependence on relative density. This is despite the fact that the modulus of a one-phase honeycomb deforming under bending only, depends on the relative density to the power of three.

The two-phase SB-model with fixed relative densities for earlywood (0.17) and latewood (0.5), as well as fixed cell shape angle ($15^\circ$) and cell wall modulus (20.5 MPa) provides predictions in good agreement with many softwoods, although density is the only input to the model.

4. Conclusions

A two-phase annual ring model based on hexagonal honeycomb cells is presented. It is used to model transverse anisotropy in softwoods. The model takes both deformation from bending and stretching of the cell walls into account. During tangential loading, bending is the dominating mechanism in the earlywood. During radial loading, stretching is the main mechanism in the latewood. A combination of bending and stretching deformation takes place during radial loading of earlywood and tangential loading of latewood.

Predictions from a one-phase model is compared to experimental data, but the agreement is not satisfactory. Instead a two-phase model is developed, based on two regions of constant densities for earlywood and latewood. Each of the two regions are honeycombs, and the free parameters of the two honeycombs (cell wall modulus and cell shape angle) are determined using a fitting procedure to global experimental data for wood. The resulting average cell wall modulus and cell shape angle are then compared with previously reported data. The predicted cell shape angle is $12.5^\circ$, which is in good agreement with experimental observations. The resulting average cell wall modulus is 20.5 GPa. This value may be too high, although Watanabe et al. [9] used 15 GPa in their predictions. The value should be viewed as an effective cell wall modulus. It incorporates limitations connected with the simplicity of this unit cell model and that only one parameter, wood density, is required as input in order to predict $E_R$ and $E_T$. The earlywood and latewood calculations from the two-phase model results in a radial modulus that is more than eight times higher than the tangential in earlywood, and four times higher in the latewood. Thus, both earlywood and latewood are highly anisotropic, and more anisotropic than clear wood. The reason is that the soft earlywood layer strongly reduces radial modulus, whereas its influence on tangential modulus is weaker.

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