Elastic deformation mechanisms of softwoods in radial tension – Cell wall bending or stretching?

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Abstract

Radial softwood modulus $E_R$ is typically twice as high as the tangential modulus $E_T$. The reason for this is unclear, although cell geometry is likely to contribute. The established hexagonal honeycomb model for prediction of $E_R$ is based on a cell wall bending mechanism only. If cell wall stretching also takes place, the dependence of $E_R$ on relative density will be different. If experimental data for $E_R$ as a function of relative density show deviations from cell wall bending predictions, this may indicate the presence of cell wall stretching. A SilviScan apparatus is used to measure density distribution. A procedure by means of digital speckle photography is then developed for measurements of local $E_R$ within the annual rings of spruce. Comparison is made between experimental data and the two expected density dependencies from cell wall bending and from stretching. The hypothesis of cell wall stretching as a contributing mechanism is supported based on the observed linear dependence of $E_R$ over a wide density range.

Introduction

Wood is an anisotropic cellular solid and this must form the basis for interpretation of its mechanical behavior. For instance, in structural design, the anisotropy of mechanical properties should be considered to create efficient wood structures. The cellular nature of wood is also of critical importance, as exemplified by the strong correlation between mechanical properties and relative density. Transverse properties of wood tend to be neglected despite considerable practical importance. For instance, collapse of large wooden structures may initiate by transverse failure (Gustafsson 2003).

In the present study, the focus is on radial modulus of softwoods. Theoretical models based on micromechanics are helpful for our understanding of deformation mechanisms in composite materials. A micromechanical model for modulus of a cellular material, such as wood, starts with a geometrical idealization of its microstructure. Mechanics are then used to derive a theoretical model expressing the modulus as a function of cell wall properties and microstructural parameters, such as relative density.

One of the first micromechanical models of wood was presented by Barkas (1941). The model includes transverse modulus of spruce. This geometrical model is based on fibers with circular cross-sections and transversely isotropic properties. Such a material has the same modulus in the radial and the tangential direction. In his model, Barkas adds rays in the radial direction to explain the higher $E_R$ as compared with $E_T$.

Easterling et al. (1982) developed a micromechanical model for transverse moduli of balsa. Gibson and Ashby (1988) generalized this model to all types of wood. Wood is geometrically represented as a cell aggregate where each cell is a hexagonal unit cell. As transverse load is applied, the cell walls are assumed to deform by pure bending, according to classical beam theory. Equations for the transverse moduli are derived, where moduli are expressed as functions of relative density and cell wall modulus. Relative density is the relevant measure of density, as it represents the volume fraction of cell wall in a porous material. It is defined as the ratio between wood density and the density of the cell wall material, $\rho^*/\rho_c$.

Gibson and Ashby (1988) assumed a transversely isotropic material consisting of regular hexagons (hexagons with all edges of equal length and equal thickness and all angles equal). They compared model predictions with experimental data for radial modulus. However, careful scrutiny reveals that the agreement between data and predictions is not very convincing. This model of Gibson and Ashby needs critical evaluation, as its basis is that radial deformation in wood is completely controlled by cell wall bending.

Softwoods have a distinct annual ring structure with regions of low density (earlywood) and high density (late-wood). A micromechanical model should be able to describe the density modulus relationship at this scale. In contrast to previous studies on global density – modulus relationship, the present study focuses on local density – modulus relationships within the annual ring.

Persson (1997) and Harrington (2002) developed sophisticated models utilizing the finite element method. However, mechanisms of elastic deformation are not explicitly discussed, as the focus is on prediction of properties. Harrington (2002) does indeed mention cell wall bending and cell wall stretching as possible deformation mechanisms during radial loading and speculates that cell wall stretching may be important. Still, the subject of cell wall bending versus cell wall stretching during radial loading of softwoods remains open.
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The transverse anisotropy in softwoods is an important question, directly related to the topic of $E_R$. Barkas (1941) and Gibson and Ashby (1988) explain transverse anisotropy by the presence of rays. However, data in Boutelje (1962) show that softwood rays do not necessarily explain transverse anisotropy. Boutelje (1962) studied the swelling mechanisms in pine of different densities, comparing tissue with rays and tissue without rays. He concludes that the anisotropy of swelling depends on density, but not on whether the samples contain rays or not. This implies that rays do not significantly contribute to the radial stiffness and the approaches by Barkas (1941) and Gibson and Ashby (1988) to explain high $E_R$ may be fundamentally flawed in the case of softwoods.

Kahle and Woodhouse (1994) use a different approach to explain transverse anisotropy in spruce. The basis is the honeycomb model presented by Gibson and Ashby, although the hexagonal honeycomb shape is altered from the hexagonal case. Cell geometries are selected based on four recent micrographs of spruce containing hundreds of cells and the resulting elastic constants are calculated based on a numerical procedure. Tangential and radial moduli for four specimens from the literature (Carrington 1923) compare fairly well with their predictions. As their model is based on cell wall bending, this appears to support the assumption by Gibson and Ashby with respect to the basic deformation mechanism. However, the predictions by Kahle and Woodhouse are for specimens within a very limited density range. Predictions for a very limited density range are compared with four sets of data for $E_p$ and $E_t$. In addition, the cell structures chosen from spruce micrographs are not compared with data from these spruce specimens, but with data by Carrington (1923). The results are therefore not enough to exclude the possibility of cell wall stretching as a deformation mechanism in softwoods.

The objective of the present study is to test the hypothesis that elastic deformation in radial tension is controlled by cell wall bending. A competing hypothesis is that cell wall stretching is an additional deformation mode. Comparison between the cell wall bending model by Gibson and Ashby (1988) and data for different densities is not convincing, and although the model with modified cell geometries by Kahle and Woodhouse compare fairly well with predictions, the density range they tested was small. Models based on bending or stretching show different dependencies of $E_R$ as a function of relative density (see next section). A suitable set of radial modulus data for softwoods must therefore include a wide range of densities to test the different hypotheses. The present data are generated by loading spruce in radial tension and determination of the strain field at sub-annual ring scale. Due to the strong density gradient in spruce, modulus data may be collected from a wide range of densities.

Models

Softwoods are well suited for micromechanical modeling. The differences in cellular structure between species are not dramatic, as is often the case with different hardwoods. Softwood tracheids are characterized by a very high aspect ratio (length/diameter) so that the two-dimensional assumption inherent in honeycombs is motivated. In addition, tracheids are formed by cell division in the radial direction so that identical cells are laid down in a row. Models based on circular or square cell shapes are too simplistic compared with the real cell shape, whereas hexagonal cell geometries are closer to reality (see Figure 1a).

The mechanics of honeycomb structures is treated by Gibson and Ashby (1988). Consider an array of hexagons

![Figure 1](image-url)  (a) Micrograph of the cellular structure of spruce. (Micrograph by Dr. Stig Bardage, SLU, Uppsala.) (b) Regular honeycomb with cell shape angle $\theta = 30^\circ$. (c) Honeycomb with $\theta = 15^\circ$. 
To be 1500 kg/m
to describe the cell shape angle, \( \theta \). This angle is 30° for regular hexagons. The relative density, \( \rho^*/\rho_s \), is the ratio between the density of the wood material \( \rho^* \) and the cell wall density \( \rho_s \) (assumed to be 1500 kg/m"). This ratio is equivalent to the volume fraction of cell wall. For regular hexagons this is

\[
\frac{E_t}{\rho_s} = \frac{2}{3} \frac{t}{l} = \text{const.}
\]  (1)

If the cell shape angle is changed from 30°, the same basic formula holds, but the value of the constant will be changed as it contains an angular dependence. The relationship describes how the ratio between wall thickness and length is proportional to relative density. This equation is linearized, and a slight deviation, less than 5%, between the actual cell wall measures and relative density should be expected for the high-density regions. This equation is still used as it simplifies both calculations and, more importantly, the physical interpretation of the model.

If the cell walls of the honeycombs are considered as beams, according to classical beam theory, the only deformation comes from bending of the cell walls. Classical beam theory may then be applied to the micromechanical scale of the hexagonal unit cell. The deformation of each individual wall, or beam, of the unit cell is added. Based on the global stress, the relative modulus \( E/E_s \) (\( E = \)modulus of honeycomb structure, \( E_s = \)cell wall modulus), can be calculated as a function of wall thickness, wall length and cell shape angle. Following the derivation in Gibson and Ashby (1988), this relationship becomes

\[
\frac{E_t}{E_s} = 2.3 \left( \frac{t}{l} \right)^3 = \text{const.} \left( \frac{\rho^*}{\rho_s} \right)^3
\]  (2)

for regular hexagons (\( \theta = 30° \)).

The constant 2.3 is a result of geometrical relationships and contains the cell shape angle, \( \theta \), but the \( t/l \) part of the relationship is independent of cell shape angle. As described earlier, \( t/l \) is directly proportional to relative density, \( \rho^*/\rho_s \), in this model.

Closer scrutiny of the cellular structure in softwoods, in this case spruce (see Figure 1a), reveals that the assumption of regular hexagons with a cell shape angle of \( \theta = 30° \) is not realistic, as the real cell has a much smaller cell shape angle. When the radial cell walls are more closely aligned in the radial direction (smaller \( \theta \)), the honeycombs are no longer isotropically elastic. Instead, the radial modulus becomes higher than the tangential as the cell walls become more oriented in the radial direction. For the case of cell walls parallel to the radial direction, a model that considers only bending becomes infinitely stiff. When there is no bending deformation, stretching must play a role. It should also be noted that Eq. (2) just like Eq. (1) uses a linearized term when it comes to the geometry.

Masters and Evans (1996) extended the ordinary honeycomb model to include stretching and hinging of cell walls. This model is of interest in the context of wood modeling, as it overcomes the limitation in Gibson and Ashby’s treatment (cell wall bending only). Masters and Evans present a purely theoretical argument, where they investigate honeycomb deformation. Stretching of cell walls and hinging of corners are allowed in addition to cell wall bending. The present paper will not derive the expressions, but they are available in Masters and Evans (1996).

To contrast with the cell wall bending mechanism, we may use the model by Masters and Evans to describe the relationship between radial modulus and relative density for the case of cell wall stretching only

\[
\frac{E_t}{E_s} = \text{const.} \left( \frac{\rho^*}{\rho_s} \right)^2
\]  (3)

Because radial modulus show different dependences on relative density in Eqs. (2) and (3), it becomes possible to test the two competing hypotheses of the cell wall bending or cell wall stretching mechanisms. Experimental data for radial modulus versus relative density is required. An experimental method by which we can measure the radial elastic modulus over a wide range of softwood densities is required. As softwoods do not show large differences in density between species (Wood Handbook 1999, Dinwoodie 2000), this is a challenging task.

Experimental

Materials

Spruce specimens from three different trees grown in the southern part of Sweden were cut to strips in the radial direction. The dimensions of the strips were 5 mm in the longitudinal direction and 2 mm in the tangential direction. In the radial direction, the strips were as long as the radius of the tree from pith to bark. The specimen was sawed using a double blade circular saw. The upper longitudinal surface was then sanded to give an optically good surface for the SilviScan measurements. Before testing, the specimens were conditioned in 50% relative humidity (RH) for more than a week to steady state condition.

Density measurements

The SilviScan-3 at STFI-Packforsk AB was applied to measure the density profiles of the strips. The measurement is based on a combination of microscopy of the cross-section and X-ray transmission in the tangential direction; the density profile of the wood strip is determined with a spatial resolution in the radial direction of 50 \( \mu \)m (Schimleck et al. 2002).

First, transverse cross-sections of the specimen are photographed in a light microscope. This is to determine cellular structure and exact specimen geometry for the X-ray transmission measurements. Another reason is to find the angle between the specimen direction and the annual rings if they are slanted. An image recognition technique determines the angle between rays and specimen direction, and the direction perpendicular to the rays is assumed to be the tangential direction.

The second step is to use X-ray transmission from the tangential direction, as described above. If necessary, the specimen is rotated by the SilviScan to ensure that the transmission measurements are perpendicular to the annual rings. The transmitted X-ray intensity is used for determination of the density as a function of radial coordinate.
Tensile testing, strain distribution measurement and modulus estimation

The SilviScan specimens were cut in the radial direction and again conditioned at 50% RH. Tensile testing in the radial direction was performed by an Instron universal testing machine with pneumatic grips and a 500 N load cell. The testing was performed in a conditioned room (50% RH). The gauge length of the specimen was 5 mm, and each specimen contains between one and three annual rings. The crosshead movement during testing was 0.5 mm/min.

Strains were monitored by means of a GOM Aramis 1.3 m Digital Speckle Photography (DSP) system. During DSP measurements, the transverse cross-sectional specimen surface is photographed. After testing, images are correlated, and deformation and strains are calculated. To use the DSP, a speckle pattern must be applied to the specimen surface. It is important that this pattern does not alter the mechanical properties of the specimen. Here, ordinary laser printer toner was applied. Because the toner does not bind to the surface, no effect on mechanical properties could be noted. The DSP camera has a resolution of 1280 x 1024 pixels corresponding to a resolution of approximately 5 μm/pixel on the specimen. Displacements and strains are calculated using facets consisting of 15 x 15 pixels. This gives a spatial strain resolution of approximately 50 μm. Algorithms used in the DSP technique are explained in detail by Sjödahl (1995).

The DSP system compares the first image, called reference image, each subsequent image to calculate strains. Due to imperfections in the image correlation, there is a small random error in the strain fields. As all images are correlated to the first image, the amplitude of the error is constant for each image. The amplitude of the error is dependent on the quality of the speckle pattern. A method to determine the accuracy of the strain field is to perform a rigid body translation of specimen. During this translation, strains are zero in the specimen. However, due to the previously mentioned imperfections in the image correlation processes, a non-zero strain field will be measured. The amplitude or these strains is a measure of the error in strain measurement and is approximately 4%. It should be noted that the absolute error is nearly constant during the testing, and thus the relative strain measurement error decreases when strains are larger.

From the strain distribution, an apparent modulus is calculated. This is carried out by division of the nominal stress of the specimen by the local strains as calculated from DSP. The local modulus can then be expressed as a function of radial coordinate.

Results and discussion

In Figure 2, the density distribution from pith to bark is presented for one spruce specimen. The large range of densities within just one annual ring is apparent. The relative density, \( r' / r_s \), ranges from approximately 0.1 to 0.7. Throughout each annual ring, there is a gradual increase in density, and it is thus hard to define a specific fraction as earlywood. From the high-density areas in the latewood, there is an instant drop in density over the annual ring border to the first wood formed early in the spring. Despite the large density variation within one ring, the average ring densities do not vary much between individual rings. There is only a small increase in average ring density, as the tree grows older. Jernkvist and Thuvander (2001) pointed out the functional gradient characteristics of density distributions in the spruce annual ring. Gradually changing density distributions have a mechanical function by reduction of interface strain concentrations as compared with the case of abrupt change in density. The large difference between earlywood and latewood densities indicates that the procedure by Gibson and Ashby (1988) is problematic relying on a honeycomb structure of given density to predict global modulus. The contributions from earlywood and latewood regions to global modulus will not be equal, although this is the assumption in the case of the average density concept.

Figure 2  Relative density \( r' / r_s \) as a function of radial distance coordinate for spruce. The pith center has a 0-mm coordinate. Crosses represent average density for one annual ring.
Large strains are presented as lighter areas and the low strain latewood regions are dark. In this particular specimen, the annual rings are somewhat slanted, as compared to the loading direction. However, comparison with specimens with rings perpendicular to the loading direction show that this does not significantly change the stiffness. Figure 3b shows the radial strain from the radial section in Figure 3a. The strong localization of strains is apparent. The smallest strains in the latewood are 0.32%, whereas the largest strains are more than a factor of two, larger, up to almost 0.8%. This clearly illustrates the different contributions to modulus from earlywood and latewood.

Figures 2 and 3b both have the radial coordinate as the x-axis and the information can be combined in a new graph (see Figure 4). Based on the strains in Figure 3b, the local modulus $E_r$ is calculated, as described previously, at a nominal stress of 5 MPa. The strain fields were monitored during loading to detect local non-linear deformation or failure events in the specimen.

Figure 4 presents density and local modulus from a single annual ring. The authors are not aware of any data in the literature presented at this fine scale. The functional gradient structure of the spruce annual ring is apparent in terms of the smooth variation in $E_r$. In addition, the strong correlation between $E_r$ and $\rho^r/\rho_s$ is apparent. The radial modulus $E_r$ varies within one ring from approximately 600 MPa to well over 1200 MPa. It should be noted that these values for modulus are average modulus over 50 μm, i.e., the strain resolution of the DSP. In the transition region between earlywood and latewood, the strong correlation between $\rho^r/\rho_s$ and $E_r$ is illustrated clearly. However, there is an interesting deviation between the distribution in $\rho^r/\rho_s$ and $E_r$ at the sharp transition interface between latewood and earlywood. However, radial modulus changes more slowly than density. This is due to the biaxial stress state in this region, as discussed by Jernkvist and Thuvander (2001). The softer earlywood is constrained by the stiff latewood. In particular, the lateral Poisson contraction is prevented by the...
high latewood stiffness in the tangential direction. As a consequence, the effective radial modulus is increased and measured modulus decreases slower than expected. Figure 5 presents data for $\rho^*/\rho_s$ and $E_R$ for all the spruce specimens examined. Due to the high resolution of strain data, $E_R$ is obtained over a large range of relative densities. The data is better suited for comparison with micromechanical model predictions for $E_R$ than data for different softwood species. Because of the wide density distribution in spruce, the range is large, and in addition, the cellular structure does not vary to the same extent as between different species. The plot contains more data in the lower density range, as low-density earlywood dominates the annual ring. The modulus values are in the same range as for the radial mechanical tests by Farrugia and Perré (2000). Our technique allows data extraction at a finer resolution and from numerous annual rings in the same specimen. For this reason, the extent of data is considerable, which is advantageous for comparison with theoretical predictions.

The plot in Figure 5 also includes two functions for two different types of micromechanical models [see Eqs. (2) and (3)]. The model by Gibson and Ashby [Eq. (2)] predicts $E_R$ to depend on relative density to the power of three. This is a consequence of the cell wall bending assumption in the model. The non-linear line is a least square fit to Eq. (2), where the constant const is adjusted to data and modulus must be zero at $\rho^*/\rho_s=0$. The second model is based on cell wall stretching. It is expressed in Eq. (3) and provides the basis for the linear regression line in Figure 5.

Figure 5 demonstrates that the power law model correlates poorly with experimental data for spruce. In contrast, the linear stretching model shows better agreement with experimental data. The good correlation is in support of the hypothesis that cell wall stretching is present as a deformation mechanism in radial tension loading. Although Figure 5 indicates that stretching is quite important, a theoretical analysis is required to be able to discuss properly the relative importance of stretching and bending.

Conclusions

The present study indicates that elastic deformation in the radial direction of softwoods contains a significant contribution from cell wall stretching. In contrast, the Gibson and Ashby (1988) model applied to wood contains only bending. The presented cell wall stretching model predicts a linear relationship between local radial modulus and relative density. Local data show strong correlation with this linear model over a wide density range, whereas correlation is poor with the power law dependence expected for cell wall bending.

To obtain data for $E_R$ over a wide range of softwood densities, a new experimental approach is applied to spruce. DSP is used to determine local $E_R$ at several locations within each annual ring. The density variation is much larger than that obtainable from measurements on macroscopic specimens from various softwood species. This wide range makes the data particularly interesting and suitable for comparison with micromechanical models. With the help of DSP, $E_R$ can be estimated with high resolution within a single annual ring. The data for local $E_R$ vary between 600 and 3000 MPa, which is in the range expected from literature data.

The large variation in local modulus due to the density variation in the annual ring structure is neglected in many modeling efforts. How this local variation translates into the average modulus measured globally is not clear. An
improved micromechanics model of softwood would probably be needed to take this variation into account.

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