Optimal Design of a 6-DoF Haptic device

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Abstract—The work presented in this paper is motivated by the use of haptics in applications of medical simulation, particularly simulation of surgical procedures in hard tissue such as bone structures. In such a scenario haptic device characteristics such as stiffness, motions, suitable workspace and device footprint are key design factors. This paper presents a procedure for optimal design of a parallel kinematic structure for a 6-Dof haptic device. For optimization, performance indices such as workspace volume, kinematic isotropy and static actuator force requirements are defined. A specific Jacobian matrix normalization is introduced for defining the kinematic isotropy and actuator force requirement indices. For defining the optimization problem, a novel multi-criteria objective function is introduced. Based on this objective function, a genetic algorithm is used to solve the multi-objective and non-linear optimization problem. Also, sensitivity analysis of the performance indices against each design parameter is presented as a basis for selecting a final set of design parameters for prototype development. Finally, using these results, a prototype was implemented.

Keywords—Design and optimization, Haptic devices, Parallel mechanism, Genetic algorithm.

I. INTRODUCTION

A haptic device is a robot-like mechanism that provides an extra sense of touch; force/torque feedback capability to the human operator based on what he/she discovers and interacts with in a virtual or remote environment. Application of these devices is emerging in various fields such as medicine, telerobotics, engineering design, and entertainment [1]. The work presented in this paper is related to the use of haptics in applications of medical simulation. This research work on the haptic device is one important component, towards achieving manipulation capabilities and force/torque feedback in six degrees of freedom (6-DoF) during simulation of surgical procedures in hard tissue such as bone structures [2]. Such procedures involve removing bone by drilling or milling, including the processing of channels and cavities, hence requiring 5-6 DoF and capability of reflecting stiff contacts.

A. Literature review

Many 6-DoF haptic devices have been developed and some of these have been commercialized since the late 1940s [3]. The Phantom by Massie [3, 4], HAPTION Virtouse 6D35-45 [3], and Freedom 6S [3] are haptic devices based on serial mechanisms with suitable workspace for the intended application but with insufficient stiffness and force/torque capacity. Many researchers have proposed haptic devices using parallel mechanisms. S. Lee proposed a new 6-DoF haptic desktop device [5], based on non-floating actuators which has a relatively large orientation workspace. He used a multi-criteria based design index to obtain optimal design parameters. The modified Delta device and Haptic master were developed by Tsumaki et al [6], based on an optimal gimbal mechanism and three floating actuators. Gosselin and Martins [7] also proposed an optimal parallel haptic device for desktop applications. However all these devices have either too limited workspace, too high inertia or too limited stiffness to be used with stiff contacts in our scenario. On the other hand, the 6-DoF Haptic Cobot [8] has sufficient stiffness, but the size (footprint) and weight of this device is not suitable for the medical applications in question. Nor is this device optimized to achieve the best performance.

To find an optimal solution for the proposed parallel kinematic structure we have to handle a multi-criteria design optimization problem which is a constrained nonlinear optimization problem with no explicit analytical expression. The gradients and Hessians algorithms that generally converge to a local minimum are hence not suitable for solving this problem. An interval analysis based approach was recently applied by F. Hao [9] to solve a multi-criteria design problem for parallel manipulators. This method determines design parameter spaces that satisfy all design constraints, but requires explicit analytic expressions of all constraints.

Genetic algorithms, on the other hand, seem to be a good candidate for these multi-criteria problems due to their good convergence property and robustness. S.H. Lee [5], J.H. Lee [10], D. Stan [11], and Yoon Kwon Hwang [12] used a genetic algorithm approach for multi-criteria optimization of parallel kinematics machines and parallel haptic devices respectively.

B. Motivation

The application context, surgery in bone structures, leads to two main haptic device requirements that are not simultaneously met by any commercially available device that we have found. These main requirements are [1]

- Haptic feedback in six degrees of freedom to allow both force and torque feedback from a virtual tool operating in a (narrow) channel or cavity.
- Device stiffness and force/torque performance that allow realistic simulation of stiff tool-to-bone contacts.

Mechanical structures that are currently used in similar devices include serial as well as parallel configurations. However, parallel kinematic structures have several significant advantages compared to serial ones, e.g. high...
stiffness, high accuracy and low inertia with the actuators located on the fixed base, thus enabling high transparency. However, the performance of parallel kinematic structures highly depends on their geometry and dimensions. Thus, considering a set of structural design parameters is of vital significance to achieve desired/optimal performance.

II. DESIGN METHODOLOGY AND KINEMATIC STRUCTURE

A design methodology for a more systematic design and optimization procedure for haptic devices was developed and presented by the authors in [1]. In the first step a list of specifications was obtained in dialogue with a tentative user. Then, three different conceptual models were developed and analysed in the MBS software Adams View®. On the basis of these analyses one mechanism was selected for further development. In the next step a kinematic model of the selected mechanism was developed as a basis for an optimization to achieve the desired performance. The optimization process of the design methodology is the main focus of this paper.

A. Specification and requirements

The list of preliminary specifications for the design of the device is given below:

- Six actuated degrees of freedom (6-DoF input/output)
- The whole device [footprint+height] should fit within the space of 250x250x300 [mm].
- The translational workspace should be a minimum of 50x50x50 [mm] and the rotational workspace should be ±40 degrees in all directions with no singularities within that space.
- The TCP force and torque peak performance should be at least 50 [N] and 1 [Nm] respectively.
- The stiffness of the device should be a minimum of 50 [kN/m].

B. Structure and kinematics of the selected mechanism

In the conceptual design phase, three different mechanisms were studied and simulated [1] to select the best candidate structure for the task at hand. As a result a modified JP Merlet parallel kinematic structure [9] was selected due to its relatively large workspace, low inertia and its ability to provide enough stiffness. This mechanism consists of a fixed base, a moving platform, and six identical legs connecting the platform to the base as shown in Fig 1.

![Fig 1. Geometric description of the 6-DoF haptic device to be optimized.](image)

Each leg consists of an active linear actuator fixed to the base, a spherical joint, a proximal link, and a universal joint. For structural design optimization, six design parameters were considered: motion range of actuator ($l_{min}, l_{max}$), length of proximal link $c_i$, radius of base $r_b$, radius of platform $r_p$, angle between the base pair of joints $2\beta$ and platform pair of joints $2\alpha$. The attachment point pairs are symmetrically separated 120° and lie on a circle, both on the base and the platform. The platform attachment points are rotated 60° clockwise from the base attachment points.

To define the performance indices for optimization, a kinematic model of the selected mechanism is developed. To derive the inverse kinematics, a closed-loop vector equation based on the kinematic diagram and notation in fig 2 is derived as

$$\theta_{B_l} = \theta_{D_l} + \theta_{R_p} \cdot \theta_{P_l}, \quad i = 1...6 \quad (1)$$

Where $\theta_{B_l}$ is the base joint coordinates of leg $i$ in the base frame $\{B\}$, $\theta_{D_l}$ is the platform joint coordinates of leg $i$ in the platform frame $\{P\}$. Here $\theta_{R_p}$ is a rotation matrix, representing roll-pitch-yaw rotations in terms of Euler angles.

For simplicity, only one of the six joint-coordinate loop equations is shown, since this equation holds for all six loops. The inverse kinematics determine the change in coordinates of the active joints (actuator lengths $l_{iz}$) corresponding to a given change in pose $\{x,y,z, \theta, \phi, \psi\}$ of the platform. So solving equation (1) for each actuator length ($l_{iz}$) using the known fixed length of the proximal link $c_i$ gives the solution as

$$l_{iz} = \bar{p}_l - \bar{q}_l - \bar{q}_l \cdot (\bar{p}_l - \bar{h}_l)^2 - (\bar{p}_l - \bar{h}_l)^2, \quad i = 1...6 \quad (2)$$

Equation (2) is equivalent to $l_1 = f_1(x,y,z, \phi, \theta, \psi)$. Two solutions exist for each actuator but in practical application due to collision between the legs and the moving platform, only one of the solutions for each leg is reachable. Furthermore the actuation ($l_{iz}$) of each linear actuator is limited by a fixed range ($l_{iz} \leq l_{iz} \leq l_{iz}$), $i = 1...6$.

![Fig 2. Kinematic diagram of 6-DoF haptic device of leg $i$.](image)

The forward kinematics determine the pose of the platform $\{P\}$ with respect to the base frame $\{B\}$, given the actuators length ($l_{iz}$). In general for a 6-DoF parallel haptic device with this configuration, a closed loop analytical solution is not available for the forward kinematics. In fact, multiple solutions are possible due to the six unknown and six non-linear equations, without considering potential collisions, actuator range and limits on joint motion. An iterative approach, the Newton Raphson method, described in...
equations (3-4) is used in this work to compute the task space coordinates as a function of actuator coordinates.

\[ X_n = X_a + [J(X)] \delta a \]  
\[ J(X) = \frac{f(X)}{\partial X} \quad i,j = 1...6 \]  

Where, \( X_n \) is the current root-value, \( J \) is the Jacobian matrix, \( f(X) \) is the function derived in equation (2), and \( X_{n+1} \) represents the next \( X \)-value that is to be determined. The Jacobian matrix in (4) is derived, by taking partial derivatives of \( f(X) \) with respect to all six dependent variables as 

\[ J = J(X)X \]  

\[ \mathbf{J} = \begin{bmatrix} J_{11} & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & J_{66} \end{bmatrix} \]

\[ \mathbf{s} = \begin{bmatrix} J_{11} \\ \vdots \\ J_{66} \end{bmatrix} \]

\[ \mathbf{J} = \mathbf{s} \mathbf{s}^{-1} \]

\[ \mathbf{s} = \begin{bmatrix} [J_{11}] & \cdots & [J_{66}] \end{bmatrix} \]

where \( \mathbf{s} \) is a scaling matrix.

A. Workspace

Workspace is the working space that the haptic device can operate within. We use a Constant–Orientation Workspace (COW) to describe and analyse the workspace of the proposed 6-DoF mechanism. The COW is defined as the three dimensional space that can be reached by TCP when the platform is kept at constant orientation. The boundaries of this space was determined using inverse kinematics while taking into account the constraints such as actuator stroke length, rotation of universal and spherical joints and collision between links. A Cartesian workspace within a range of \( \pm 75 \) mm along all three axes was scanned using an evenly spaced grid. Finally, the volume of the workspace can be calculated as

\[ V = \int dv \]

Where \( dv \) is the volume of a grid element. The optimization criterion is to maximize the workspace volume while keeping the footprint (size) of the device as a constraint.

B. Kinematic Isotropy Index

The kinematic isotropy index \( (II) \) indicates how evenly the device produces motions (velocities) in all directions in the workspace. A haptic device is called “isotropic” if at least in one point of the workspace some of its kinematic properties are homogenous with respect to all directions. The isotropy index is defined as the ratio of minimum to maximum singular values of the normalized Jacobian matrix, according to

\[ II = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \]

where \( \sigma_{\text{min}} \) and \( \sigma_{\text{max}} \) represent the minimum and maximum singular values of the normalized Jacobian matrix. If, at a certain point, the isotropy index approaches zero, it indicates operation close to singular points in the workspace. To represent the average of the device isotropy index over the whole workspace, a global isotropy index is defined as

\[ GII = \frac{1}{V} \int II dv \]

A higher value of \( GII \) represents a mechanism with a better isotropy characteristic within its workspace, and thus the criterion is to maximize this index.

C. Force requirement Index

The force requirement index \( (FI) \) is defined as the maximum magnitude of an actuator force required for a unit applied load on the tool center point (TCP). As the applied load on the TCP is related by the Jacobian matrix to the forces required on the actuators, the force requirements index is defined as the maximum singular value of the normalized Jacobian matrix as

\[ FI = \sigma_{\text{max}} \]

A smaller value of force requirement index indicates a better device and implies that the device can provide more force/torque with the same actuators. A global force requirement index which represents the average of the force requirement over the selected workspace is defined as

\[ GFI = \frac{1}{V} \int (FI) dv \]

A smaller value of the force requirement index implies that less capacity of the actuators is required i.e. this index should be minimized.

D. Multi-criteria objective optimization

For the optimization process, a multi-criteria objective function is needed in order to combine the three design indices.
transformed to a dimensionless domain as

\[
V_{\text{d}} = \frac{V - V_{\text{min}}}{V_{\text{max}} - V_{\text{min}}},
\]

(12)

\[
GI_{\text{d}} = \frac{GI_{\text{max}} - GI}{GI_{\text{max}} - GI_{\text{min}}},
\]

(13)

\[
GFI_{\text{d}} = -\frac{GFI}{GFI_{\text{max}}} - GFI_{\text{min}},
\]

(14)

Where the max and min indices are based on the user requirements, the selected min-max range of the kinematic parameters to be optimized, and simulation experiments [1, 3].

In the above equations the optimization target is to maximize volume \((V)\) and global isotropy \((GI)\) indices, and to minimize the global force requirement index, hence \((GFI)\) is negated in equation (14). Secondly, normalization is used such that all indices contribute equally in the optimization process. In this normalization each index is divided by a fixed index value calculated based on the mid values of the given design parameters space according to equation (15) and Table I. Finally, a multi-criteria design objective function is defined based on these indices as

\[
GDI = \min \left[ \frac{GI}{GI_{\text{max}}}, \frac{GFI}{GFI_{\text{max}}}, \frac{VI}{VI_{\text{max}}} \right],
\]

(15)

where subscript \(m\) indicates mid values of the parameter space. The main advantage of this new approach as compared to the traditional objective function presented in [5, 10, 11], is to assure that all design indices are equally active in the optimization process.

Before starting the optimization process, we have to define the allowed range of the design parameters as per the specification of the device, see table I. The bounds of design parameters were selected on the basis of footprint (size) of the device, to avoid collision of the links, and to satisfy initial design requirements. A genetic algorithm (GA) [15] with parameter specification given in table II was used to solve the multi-criteria nonlinear optimization problem.

IV. RESULTS AND DISCUSSIONS

A. Optimum solution

The optimal design parameters given by the optimization process are given in table I. This is a global optimum solution for the given objective function as the genetic algorithm explores the whole parameter space within bounds.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Min</th>
<th>Max</th>
<th>Optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>(l) [mm]</td>
<td>120</td>
<td>150</td>
<td>129.4159</td>
</tr>
<tr>
<td>(c) [mm]</td>
<td>120</td>
<td>150</td>
<td>125.4555</td>
</tr>
<tr>
<td>(r_b) [mm]</td>
<td>100</td>
<td>125</td>
<td>118.1799</td>
</tr>
<tr>
<td>(r_p) [mm]</td>
<td>40</td>
<td>60</td>
<td>54.9920</td>
</tr>
<tr>
<td>(\beta) [deg]</td>
<td>10</td>
<td>30</td>
<td>18.1519</td>
</tr>
<tr>
<td>(\alpha) [deg]</td>
<td>10</td>
<td>30</td>
<td>10.5485</td>
</tr>
<tr>
<td>Volume index, VI</td>
<td>0.8</td>
<td>1</td>
<td>0.9790</td>
</tr>
<tr>
<td>Global isotropy index, GI</td>
<td>0.156</td>
<td>0.24</td>
<td>0.1885</td>
</tr>
<tr>
<td>Global force index GFI</td>
<td>0.076</td>
<td>0.64</td>
<td>0.8522</td>
</tr>
</tbody>
</table>

B. Performance variation and sensitivity analysis

The set of optimal design parameter values obtained from the genetic algorithm was used to evaluate the performance of the device. Figure 3 shows the reachable workspace volume of the device with optimal design parameters. The colored surface shows the outer boundary of the workspace that could be reached. The required workspace [50x50x50] mm can easily be enclosed in this reachable workspace.

In order to visualize the variation of isotropy and force requirements indices in the optimized workspace, the TCP is moved in a circular path in the x-y plane with small increments in radius and when the radius reaches the maximum limit the TCP is shifted to the next x-y plane with a small increment in the z-direction. At each small grid isotropy and force requirements indices are measured. Figure 4a shows that the device has good “isotropic” behaviour around the central position of the workspace while the isotropy reduces as the TCP moves away from center. Similarly the force requirements are small around the center of workspace while it increases as the TCP moves away from central point as shown in fig 4b. This characteristic is also quite obvious from the isotropy definition of the device. From the index values corresponding to the optimal parameter set and by analysis made in MBS Adams, it is concluded that workspace and isotropy requirements as represented in section II are fulfilled.

Fig 3: A 3D illustration of workspace with the optimal design parameters.

Fig 4: a) Variation of isotropy index within optimized workspace
C. Sensitivity plots and analysis

In the design process of the mechanical structure it is important to know how sensitive the device performance is to the changes in design parameters. Thus we made a sensitivity analysis of the performance indices based on the GA results, for a variation of design parameter values within the selected bounds (Table I). Figure 5a) shows the sensitivity of isotropy index to variations in actuator and proximal link lengths. The 3D surface clearly indicates that keeping similar and not too long actuator and proximal link lengths result in better isotropy. The optimization result obtained from the genetic algorithm (table I) agrees with the sensitivity analysis (optimal point within the red surface in plot 5a). Similarly plot b) represents the sensitivity of the global force requirement index to variations in proximal and actuator link lengths. The plot shows again that similar and not too long proximal and actuator link lengths result in a better structure, (blue region in plot 5b).

One important point to note from fig 5 is the relation between the isotropy index and the force requirements index. Here, the sensitivity plots show that a device with better “isotropic configuration” also has lower actuator force requirements.

The effects of the base plate and platform radius on isotropy and force requirements indices are presented in fig 6. In both cases, increasing the radius, results in better performance. The red region in plot 6a) and blue region in 6b) are corresponding to the optimum results obtained from the genetic algorithm (table I). It is clear that both the base and platform radius are important in the design of the device and that the performance is quite sensitive to these parameters.

It is concluded from the sensitivity analysis that the selected design parameters play important roles in the device performance, thus should be considered in design optimization. If, for some reason, the choice of parameter values deviates from the optimization result, the sensitivity analysis provides guidance as to how the values maybe selected without deteriorating performance too much.
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Fig 7: a) The sensitivity of the GII (upper) and b) GFI (lower) to joints angle of base and platform dimensions respectively.

To further understand the results obtained from the genetic algorithm, an additional analysis was made. In this analysis one design parameter at a time was changed around the optimal value while the other five parameters were kept constant at the optimal value in optimization process. This was repeated for all six design parameters. The trends of performance indices obtained with this approach were same as for the genetic algorithm (fig 5-7), except for the base radius where the result is slightly deviated.

D. Prototype development

A prototype of the optimally designed 6-DoF haptic device has been built, shown in fig 8. The final set of design parameters was selected on the basis of results obtained from GA optimization and sensitivity plots.

V. CONCLUSION

This paper presents design optimization and sensitivity analysis for a 6-DoF haptic device based on a parallel kinematic structure. For optimization, the performance indices workspace volume, kinematic isotropy and actuator force requirements are defined; the latter two indices are based on a normalized Jacobian matrix. The three indices are combined in a multi-criteria objective function and used together with a genetic optimization algorithm. The optimization solution obtained is further investigated by sensitivity analyses against each individual design parameter. For the finally selected mechanism design, simulation shows that the optimal parameter values provide good isotropic properties within the required workspace and that the variation of isotropy and force requirements is small within workspace. A prototype of this design has been built and is currently under evaluation.

REFERENCES