A Heuristic Downside Risk Approach to Real Estate Portfolio Structuring

- a Comparison Between Modern Portfolio Theory and Post Modern Portfolio Theory

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## Abstract
Portfolio diversification has been a subject frequently addressed since the publications of Markowitz in 1952 and 1959. However, the Modern Portfolio Theory and its mean variance framework have been criticized. The critiques refer to the assumptions that return distributions are normally distributed and the symmetric definition of risk. This paper elaborates on these shortcomings and applies a heuristic downside risk approach to avoid the pitfalls inherent in the mean variance framework. The result of the downside risk approach is compared and contrasted with the result of the mean variance framework. The return data refers to the real estate sector in Sweden and diversification is reached through property type and geographical location. The result reveals that diversification is reached differently between the two approaches. The downside risk measure applied here frequently diversifies successfully with use of fewer proxies. The efficient portfolios derived also reveals that the downside risk approach would have contributed to a historically higher average total return. This paper outlines a framework for portfolio diversification, the result is empirical and further research is needed in order to grasp the potential of the downside risk measures.
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Erik Hamrin
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1. INTRODUCTION

1.1 Background

The global financial market has in recent years experienced significant turmoil. The turbulence stemmed from a collapse in the value of mortgage-backed securities starting in the summer of 2007 (Blackburn, 2008). The defaults resulted in a credit crunch were financial institutions hoarded cash and required substantial increases in premiums before lending to one another. The global commercial real estate market was not unharmed. The transaction volumes contracted substantially in the first quarter of 2008 and amounted to less than half of the volume experienced one year earlier (CBRE, 2010). The liquidity was in 2010 still limited on a global level compared to the pre-crisis era. It was not only transaction volumes that contracted. Capital values on a global level for the prime office sector was down roughly 20% compared with the capital value level recorded in 2003. From the peak in the third quarter of 2007 to the through in the second quarter of 2009 capital values were down with roughly 40% reflecting the significant upward yield shift of the sector. The office rental levels declined rapidly and were in the third quarter of 2009 down approximately 30% since the peak in the third quarter of 2007 reflecting increased vacancy rates from tenants downsizing or even leaving their businesses. The commercial real estate sector in Sweden has not been unaffected by the imbalances in the financial market and witnessed declining capital values during 2008 and 2009 (IPD, 2010). The Swedish transaction volume decreased from strong levels amounting to 12% of the total European transaction volume in 2008 to less than 1% in 2009 (DTZ, 2010).

The severe impact of the financial crisis has triggered numerous debates regarding the structure of the financial market. A subject frequently addressed is the occurrence of high stakes in financial markets with uncertainty (Sen, 2008). It sounds reasonable that such presence could trigger financial turmoil. High financial stakes in uncertain markets often results in disproportionately high risk when set in contrast to the realized returns (Sen, 2008). This fact stresses the importance of sound practices of portfolio structuring in order to limit substantial losses in the presence of an economical down turn.

One of the most famous economists of our time, Harry Markowitz, was in 1990 awarded the Nobel Prize for his contributions regarding portfolio structuring (Rubinstein, 2002). Markowitz proved that an investor could receive diversification benefits by systematically and quantitatively structure a portfolio. The most important outcome of his work was that he showed that it was the variance of the total portfolio that should be regarded, thus not the individual variances of each individual asset. Markowitz framework is commonly referred to as Modern Portfolio Theory (MPT). The MPT has, despite its success, during the years received a lot of criticism from various researchers (Devaney et.al, 2006). Some of this critique refers to the assumption that asset returns are normally distributed. However, several studies show that asset returns not conform to the normality assumption. For instance real estate returns seem to exhibit other characteristics and do not conform to the characteristics of a normal distribution (Devaney et.al, 2006; Graff et.al, 1997; Graff & Young, 1995; Maurer et.al, 2004; Sivitanides, 1998). The mean variance framework also assumes that investors view deviations from the mean return in a symmetric manner. This assumption does not conform to investors’ risk preferences (Harlow, 1991). Harlow (1991) states that scientists have found that individuals view return dispersion in an asymmetric manner, which implies that losses weights more heavily against gains.
Alternative portfolio optimization techniques have been developed in order to avoid the pitfalls of the MPT and its suggested portfolio construction methodology. One of these alternative construction methods refer to the Downside Risk (DR) approach. Structuring a real estate portfolio according to the downside risk framework should be more appealing than using the traditional MPT framework (Sivitanides, 1998). The attractiveness of the DR framework comes from the framework’s superiority to create efficient portfolios without violating the asymmetric perception of risk that many investors have. The DR model is furthermore applicable on return distributions that do not have the characteristic of a normal distribution. Investors who care about downside risk should therefore preferably use the DR framework when constructing and evaluating different real estate portfolios.

The concept of downside risk is older than MPT, however the extensions are relatively recent (Harlow, 1991). Authors as Sivitanides (1998), Sing & Ong (2000) and Cheng & Wolverton (2001) have extended the DR measure and applied the concept on real estate portfolios and in different ways compared their results with the Mean Variance (MV) framework. All the authors suggest that further research is needed to grasp the potential of the DR measures, because of the relatively scarce empirical evidence that is available today. Furthermore, some of the research has been criticized for comparing the two optimization techniques in an inappropriate manner (Cheng & Wolverton, 2001). It is therefore of importance to pursue further studies, which also incorporates the relevant critiques. The previous research is clearly geographically defined and has, to the author’s knowledge, only been conducted utilizing data from the USA and Singapore.

The DR concept in this thesis relies on the heuristic methodology proposed by Estrada (2007) in order to derive an efficient set of real estate portfolios.

1.2 Objective

This thesis focuses on within real estate portfolio diversification considering allocation by property type and geographical location. The study sheds light on two different quantitative portfolio optimization approaches, namely the Modern Portfolio Theory (MPT) and its Mean Variance (MV) framework and secondly the Downside Risk (DR) approach and its semivariance framework. These two approaches aim to provide the investor with a framework that allows the investor to optimize the portfolio’s return and risk characteristics.

The objective of this study is to extend the previous research by applying the DR framework on a real estate only portfolio with asset allocation in Sweden. The thesis will in addition compare and contrast the DR results with the results of the MV approach considering asset allocation by property type and geographical location. The study will moreover give an indicative result regarding arithmetical average ex post total return performance derived from the suggested asset allocations of each of the two portfolio optimization approaches during the entire period of 1984-2009, as well as five year sub periods.

1.3 Thesis Structure

The thesis is structured in six chapters. The second chapter, following this section, refers to the methodology of the paper. The third chapter provides some basic concepts inherent in the portfolio optimization process as well as sections explaining, in detail, how the portfolios
mathematically are constructed and a short but concise literature review. The fourth part presents the data utilized. The fifth part guides the reader through the result. The final and sixth part of the paper consists of a conclusion and suggestions of further areas yet to be explored.

2. METHODOLOGICAL OVERVIEW

The method section will give a brief overview of the procedures applied in this paper, however for complete information I urge you to read the theory part of this paper. The theory part will explain and review the concepts in a more explicit manner than provided here. All formulas is stated in the theory part and thus not provided in this chapter.

2.1 Method

This thesis utilizes a quantitative approach in order to derive optimal real estate portfolios. The framework used in order to detect efficient real estate allocations relies among others to a large extent on the work of Markowitz (1959), Estrada (2007) and Geltner (1993).

The data has been provided by IPD and refers to real estate return data stemming from the real estate sector in Sweden between the years of 1984-2009. The total return data is divided between income return and capital growth. The data used in the analysis has been divided into proxies relating the returns to property sector and geographical region. A full data description is provided in chapter 4.

The input data for both the downside risk approach as well as for the mean variance approach is exactly the same. The property returns data has, prior to its use in the models, been desmoothed. This procedure has been executed in order to acknowledge the implication of valuation lag many times inherent in property return indices that are structured on individual property valuations. The desmoothening process relies on the work provided by Geltner (1993). I have assumed a valuation lag of 8 months in the desmoothening process. This assumption is what I find reasonable, however I have no empirical evidence for this being the actual case. Deriving such an exact estimate falls out of the scope and purpose of this thesis and available data would also put constraint on such analysis. I would furthermore argue that the relativities between the models, examined in this paper, would stay the same independent of the assumption regarding the extent of valuation lag. The reader should also be aware that the capital growth is the only return parameter that has been desmoothed, the income return is thus not affected by the desmoothening assumption.

The desmoothed property return data has been used as input in two different portfolio optimization processes briefly described in the introduction. These two approaches, the mean variance and the downside risk approach, will be thoroughly explained in the theory part of the paper, however it is probably good to describe some of their characteristics already in this section as the theory chapter could be experienced as relatively technical.

First, let us focus on the similarities and dissimilarities between the mean variance approach and the downside risk approach in order to get an intuitive feel with comfort of excluding the technicalities.
The aim of both approaches is to minimize the risk of the portfolio for each given return level. This is accomplished by diversification. The diversification, in this paper, is received by scattering the asset allocation, inherent in the portfolio, through different property types e.g. offices, retail, logistics, etc. but also through geographical regions. The overall aim is thus the same for both approaches – to minimize the portfolio risk in relation to the return characteristic of the portfolio.

The main difference between the approaches refers to the definition of risk. The mean variance approach uses the standard deviation to assess risk, while the downside risk concept relies on many different measures. However, the downside risk measures in this thesis refer to the semivariance. The standard deviation measures the deviation from the mean return that includes upside and downside deviations. The semistandard deviation excludes upside deviations and measures only the return deviation occurring below a certain return threshold level set by the perpetrator. This is the main difference between the concepts. For further information regarding the concept of standard deviation I propose you to read appendix I.

This paper has utilized three different threshold levels, or named differently target returns, for the downside risk concept. The first of these threshold levels of return refers to the mean of the total return distribution of 10.69%, the second and third return threshold level amounts to 8.5% and 12% respectively and have been arbitrarily determined.

I would argue that the downside risk concept is more complex both to grasp and to structure efficient portfolios with. The complexity descend, to a large extent, from the endogenous semicovariancematrix. The downside risk approach, in this thesis, is therefore structured according to Estrada’s (2007) proposed heuristic framework in order to avoid such an endogenity issue. Estrada’s (2007) framework allows for an exogenous semicovariance matrix, which implies that the portfolio optimization process is very similar to the mean variance approach. In short one could state that the only difference between the mean variance approach and the downside risk approach is that the variance is replaced with the semivariance and the covariance is replaced with the semicovariance within the portfolio optimization process in the downside risk setting.

The analysis part of this paper investigates among other things the allocation differences received from the two quantitative concepts. This is done by examining the property weights for each expected portfolio return, ultimately deriving the absolute differences between the allocations.

The efficient frontiers are also compared and contrasted. The comparison of the efficient frontiers have been utilized by deriving the downside risk that is inherent in the mean variance allocation and vice versa deriving the mean variance risk inherent in the proposed downside risk allocation.

The analysis chapter contains, in addition, a total return comparison acknowledging each model’s proposed asset allocation and its ex post total return performance. The arithmetical average total return per annum is calculated for the whole period as well as for five-year’s sub-periods.
3. THEORETICAL FRAMEWORK

This chapter provides an explicit review of the concepts utilized throughout this paper. Much of the contextual weight is put on the downside risk framework providing a brief history of the birth and development of the measure. However, the chapter is initiated by describing the implications of the real estate return distributions as well as smoothing issues.

3.1 Real Estate Return Distributions

Many quantitative models which determine the optimal asset allocation when considering risk and return is based on an assumption that returns are normally distributed (Sivitanides, 1998). Findings suggest that this assumption does not hold in a real estate context. Real estate return distributions are many times skewed to the left and thus not normally distributed. Devaney et.al (2006) found, for instance, that real estate return distributions in the UK, between the years 1981-2003, were not normally distributed. The authors concluded that real estate returns appear to behave differently than the case of equities and bonds. They further argued that using standard deviation measures proposed by the MV framework may generate misleading results. Additional studies on real estate return distributions have been made on data from the Russell-NCREIF data base during the period of 1980-1992. Graff & Young (1995) concluded that the individual annual property returns, represented in the index, were not normally distributed. They furthermore found that the skewness and asset specific risk changes year by year. A similar study was compiled using Australian property data during the period of 1985-1996, which conformed to the same conclusion, real estate returns are not normally distributed (Graff et.al, 1997). Maurer et.al (2004) found, in accordance with previous authors, that German real estate returns also were non-normally distributed.

3.2 Appraisal Smoothing

There are a number of articles written in the 1970s and 1980s that conclude that real estate exhibit a higher risk-adjusted return than other investment alternatives (Edelstein & Quan, 2006). Real estate would thus, according to these articles, be beneficial to include in asset portfolios, both with regards to risk and as an inflation hedge. However, these articles used appraisal based indices when concluding these relationships. More recent articles have shown, utilizing transaction based indices, that real estate as an asset class may not provide these benefits at least not to the extent previously argued. The difference, when using transaction based indices instead of appraisal based indices, refers to the implication of valuation smoothing which may be present in appraisal based indices. Valuation smoothing ascend from the valuation procedure where appraisers sometimes tend to look back on past transaction data in order to determining a value for a specific property. The problem with this approach is that commercial property transact infrequent and leave appraisers with little input when determining market values at specific times (Fisher et.al, 1994). Rational appraisers aim to filter out random transaction price noises, which for example may descend from the unique motivations behind a given transaction, when determining the market value for a specific property. In order to reduce the transaction price noise, appraisers tend to base their valuation on both transaction price observations of comparable sales, but also on previous appraisal based valuations. This methodology leads to smoothed property values over time on a disaggregate level. Appraisal based real estate indices are constructed by including individual property valuations and aggregate these valuations (Edelstein & Quan, 2006). The common
critique against this approach is that the aggregated real estate return figures are smoothed since the individual property valuations are smoothed. The smoothing implies that the variances of the return series are reduced and below a corresponding transaction based index. Smoothed return series results in risk underestimation, since a common way of measuring risk are through examining the deviations from the mean return. Smoothing issues can thus be viewed as errors stemming from appraisal estimates of individual property values.

Edelstein & Quan (2006) defines smoothing as the deviation of an index from one which is never observed. It is therefore natural that much of the literature on the subject is based on assumptions about the true return series, which is unobservable, the appraiser methodology as well as general practice. One way to overcome the problems inherent in appraisal based return indices is to use a method proposed by Geltner (1993), which reverse-engineer the returns of the aggregate property index. Geltner’s (1993) reverse-engineering method keeps the inherent characteristic of property returns, assumptions about an informational efficient real estate market can thus be relaxed. Geltner (1993) argues among others that the real estate market may not be informational efficient. This argument is reinforced by previous studies made on the U.S. market, which imply that return predictability has been possible. Let us now consider a model commonly referred to as “partial adjustment model” which provides a quantitative expression in explaining how appraisers determine market values for properties at a disaggregate level. Consider expression 1:

\[ V_t^* = \alpha V_t + \alpha \epsilon_t + (1 - \alpha) V_{t-1}^* \]  

(1)

Where:

- \( V_t^* \) = current appraisal value of the specific property
- \( V_{t-1}^* \) = the previous appraisal
- \( V_t \) = the contemporaneous transaction price evidence

\( \alpha \) is a number between 0 and 1 and represents the weight put on transaction price evidence by appraisers (Geltner, 1993). From this model it becomes clear that if appraisers were to put a large weight on comparable sales then this would yield a large error term in the estimated market value on a disaggregate level. A rational appraiser is therefore expected to put some weight on previous valuations in order to receive plausible market value estimates. However, the rational behavior of the appraisers becomes irrational when considering their valuations as input for an index. This can be shown with expression 2:

\[ V_t^* = \alpha V_t + (1 - \alpha) V_{t-1}^* \]

(2)

Expression 2 looks almost identical to expression 1 with one important difference, the error term is diversified away. The exclusion of the error term is based on the argument that in aggregate errors stemming from valuation estimates will largely diversify away between the different valuations included in the index (Geltner, 1993). Appraisal estimates of market values with the intention to use at an aggregate level would thus be most beneficial to use if appraisers put the entire weight, 1, on \( \alpha \) as the error term will be diversified away in aggregate. It should thus be noted that the most reliable appraisal method in disaggregate is not optimal on an aggregate level.
This study is based on the Swedish IPD index which consists of aggregated individual property valuations and should thus be desmoothed as many of the appraisals probably are affected by previous property valuations. The desmoothing method builds on a number of assumptions and it is sometimes argued that smoothed series does not exist, furthermore the extent of smoothening may vary across time (Cheng, 2001). However, this study will utilize the desmoothing method proposed by Geltner (1993).

In this thesis \( \alpha \) has been assumed to be 0.6 which implies that the average valuation lag is 8 months. This can be derived by utilizing expression 3 outlined by Geltner (1993):

\[
\bar{L} = \frac{1}{\alpha} - 1
\]  

Here \( \bar{L} \) depicts the average valuation lag. A \( \alpha \) of 0.6 implies that \( \bar{L} \) equals 0.667 years which corresponds to 8 months.

One could argue that \( \alpha \) should be larger or smaller than 0.6. However, it falls out of the scope of this thesis to derive an exact estimate of \( \alpha \). An exact estimate of \( \alpha \) would of course be beneficial to use, but such an estimate is not needed to fulfill the purpose of this paper. This is true since the allocation relativities between the models will stay equal no matter what chosen value of \( \alpha \).

### 3.3 Introduction to the Mean Variance Framework

The MV framework for structuring risk and return was first introduced by Markowitz in 1952 and was further extended by the author in 1959. Markowitz MV framework is commonly referred as Modern Portfolio Theory (MPT). MPT provides the tools to receive as modest risk as possible for any given return when constructing a portfolio of assets (Markowitz, 1959). The concept of minimizing risk for a certain return is utilized through diversification, which implies that the risk is decreasing by investing in more than one asset. The concept of diversification was well known before Markowitz presented his theories, however investors lacked the option of quantifying risk and return and were thus not able to construct optimal portfolios, at least not in a systematic manner. The penetrating power of the MPT has been enormous and the theory could be seen as a blueprint frequently utilized by various investors in order to find an efficient set of portfolios (King & Young, 1994). The risk, in the MV framework, is defined as the deviation from the mean of the return distribution (Markowitz, 1959). The deviation from the mean is defined as the standard deviation, which is the square root of the variance. The MV framework could both be utilized with ex ante returns as well as historical returns; the only difference is a small change in the necessary calculations. In short one could argue that the concept boils down to constructing an efficient frontier where no combinations of other assets can give a higher return without increasing the risk.

The following sections will introduce the main ideas inherent in the MPT. Basic statistical procedures are found in appendix I.
3.3.1 Portfolio Return & Portfolio Standard Deviation

This section will guide the reader through the necessary mathematical procedures in order to receive the volatility, measured as standard deviation, of a portfolio. The entire section builds on Markowitz proposed methodology as of 1959.

The return of a portfolio consisting of $N$ assets is calculated by taking the value-weighted average return across all the assets included in the portfolio. Consider expression 4:

$$ r_p = \sum_{i=1}^{N} w_i r_i, $$

where

$$ \sum_{i=1}^{N} w_i = 1 $$

$r_p$ is the return of the portfolio during period $t$, $w_i$ is the proportion of portfolio value in asset $i$ at the beginning of period $t$ and $r_i$ is asset $i$‘s return during that period. Expression 5 states that all the asset weights together must sum to one, which implies that all assets included in the portfolio must be equal to the total portfolio. These two expressions, 4 and 5, allow for short selling of assets, which implies that the investor can sell an asset that the investor does not own. This paper will not allow asset allocations with negative asset weights. Following constraint 6 will therefore apply:

$$ w_i \geq 0, \ for \ i = 1,2,...,N $$

The above constraint restricts the asset weights in the portfolio to only take positive or zero allocation figures, short selling is thus not allowed. For simplicity, the $t$ subsequent is dropped.

The volatility of a portfolio is computed by taking the square root of the portfolio variance. The formula to calculate the portfolio variance requires the perpetrator to calculate some of the presented time-series statistics in appendix I. Consider the expression 7, where $p$ denotes that it is a portfolio of assets, which defines the portfolio variance for $N$ assets:

$$ \sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2 \sum_{j=1}^{N} \sum_{j\neq i}^{N} w_i w_j \sigma_{ij} \quad (7) $$

where $\sigma_{ij}$ denotes the covariance between asset $i$ and asset $j$.

Then the portfolio standard deviation for a portfolio with $N$ assets can be expressed as:

$$ \sigma_p = \sqrt{\sigma_p^2} = \sqrt{\sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N} \sum_{j\neq i}^{N} w_i w_j \sigma_{ij}} \quad (8) $$
Expression 8 quantifies the risk of a portfolio when the investor view risk as standard deviation.

3.3.2 The Correlation Coefficient and Diversification

One of the most important aspects that Markowitz (1952; 1959) formalized was that he showed that it is not the individual asset’s own risk that is important to an investor, but rather the contribution of the single asset to the variance of the complete portfolio (Rubinstein, 2002). Consider figure 1, which graphically addresses the effect of diversification (Hishamuddin, 2006). The figure depicts that a fair share of the total risk can be diversified away by adding additional assets to the portfolio. Investing in one asset implies that the investor accepts to be exposed to the total risk inherent in the single asset. The investor can, however avoid some of the total risk by dividing his funds across more than one asset. In more detail the specific risk, the actual risk specific to the individual asset, can be totally diversified away and the investor is then only exposed to market related risk. The market risk refers to the movement of the general economy and is not diversifiable. Asset returns tend to react to changes in the money supply, interest rates, exchange rates, taxation, and government spending to name a few variables that are incorporated in the market risk. This implies that the investor in the MPT setting is restricted to receive compensation for market related risk exposure, thus not for asset specific risk that can be diversified away.

Figure 1

The contribution from a single asset in a portfolio is in the MPT recognized by the asset’s covariance with all the other assets within the portfolio. The covariance can be expressed as the correlation between asset i and asset j times the variance of each asset as stated in expression 9. This feature implies that expression 8 can be written as:

\[
\sigma_p = \sqrt{\sigma^2_p} = \sqrt{\sum_{i=1}^{N} w_i \sigma_i^2 + 2 \sum_{i=1}^{N} \sum_{j=i+1}^{N} w_i w_j \rho_{ij} \sigma_i^2 \sigma_j^2}
\]

(9)

Note that the covariance has been replaced by the correlation coefficient and the assets’ respective variance. Remember that the correlation coefficient can take any value ranging from -1 to 1. How does the correlation coefficient affect the standard deviation of the portfolio? For illustrative purpose two extreme examples, are presented below, where the correlation coefficient is zero and perfectly positive. When the correlation for the assets included in the portfolio equals zero then expression 9 simplifies to:

\[
\sigma_p = \sqrt{\sigma^2_p} = \sqrt{\sum_{i=1}^{N} w_i \sigma_i^2}
\]

(10)
Thus, when the correlation coefficients between the assets in the portfolio are equal to zero then the portfolio standard deviation equals the square root of the weighted sum of the variance. When the correlation coefficients within a portfolio are perfectly positively correlated then expression 9 can be written as:

\[ \sigma_p = \sqrt{\sigma_p^2} = \sqrt{\sum_{i=1}^{N} (w_i \sigma_i)^2} \]  \hspace{1cm} (11)

The result, from expression 11, is that perfectly positive correlation coefficients between all the assets in the portfolio do not reduce the overall volatility of the portfolio. This is true since the standard deviation simply is the weighted sum of all the asset variances.

### 3.3.3 The Optimal Asset Allocation in the MPT Framework

The main focus so far has been associated with the statistical procedures and relating those procedures to portfolio construction. However, the solution to find the optimal combinations of asset weights has not yet been considered. The intention and result of structuring the asset weights optimally within the portfolio is an efficient frontier. The efficient frontier describes a frontline where no other portfolio compositions are more effective when bearing in mind the risk and return component of the portfolios. In order to uncover the asset weights of a two asset portfolio one can utilize expression 12:

\[ w_i = \frac{\sigma_j^2 - \sigma_{ij}}{\sigma_i^2 + \sigma_j^2 - 2\sigma_{ij}} \]  \hspace{1cm} (12)

The portfolio asset weights must sum, as stated previously, to 1. Expression 13 can therefore be applied:

\[ w_j = 1 - w_i \]  \hspace{1cm} (13)

To solve for the asset weights in a portfolio consisting of more than two assets becomes increasingly more difficult without computer power. The analysis in this paper solves for the asset weights through an iterative process in both Excel and the statistical software package SAS 9.2. These programs simply try different asset weights until the minimum variance portfolio is acquired.

### 3.3.4 The Efficient Frontier

An efficient portfolio is in the MPT framework defined as a portfolio where it is impossible to obtain a higher expected return without increasing the volatility of the portfolio (Markowitz, 1959). From this definition it follows that an inefficient portfolio is a portfolio where it is possible to obtain a higher expected return without increasing the volatility of the portfolio. The efficient frontier exhibit different asset compositions that on a portfolio level are efficient. The efficient frontier is thus consisting of many different efficient portfolios which each exhibit different expected returns and volatility characteristics. An efficient frontier is
graphically depicted in figure 2. The y-axis represents the expected return and the x-axis represents the volatility of the portfolios measured as standard deviation. The point represents the minimum variance portfolio, while the hyperbola represents possible portfolios that minimize the standard deviation for each expected return above the minimum variance portfolio.

**Figure 2**

It follows from the definition of an efficient portfolio that portfolios that are situated below the minimum variance portfolio are inefficient (Markowitz, 1959). This could clearly be observed in figure 2, where inefficient portfolios are represented by the continuation of the line below the minimum variance mark. The efficient frontier is derived by maximizing the return for all possible standard deviations or minimizing the standard deviation for all possible returns. The derivation of the efficient frontier can thus be obtained by utilizing the expression 14 and 15:

Max

\[ E(R_p) = \sum_{i=1}^{N} E(R_p)w_i \]  

(14)

Subject to

\[ \sigma_p = \sqrt{\sum_{i=1}^{N} \sum_{j=i+1}^{N} w_i w_j \sigma_{ij}} \]  

\[ \sum_{i=1}^{N} w_i = 1 \quad w_i \geq 0 \]  

(15)

In the above expression is the expected return maximized for each standard deviation. The optimization problem could alternatively be expressed as expression 16 and 17:

Min

\[ \sigma_p = \sqrt{\sum_{i=1}^{N} \sum_{j=i+1}^{N} w_i w_j \sigma_{ij}} \]  

(16)

Subject to

\[ E(R_p) = \sum_{i=1}^{N} E(R_p)w_i \]  

\[ \sum_{i=1}^{N} w_i = 1 \quad w_i \geq 0 \]  

(17)

The above expressions, 16 and 17, are basically the same as the previous ones, 14 and 15, and will yield the same result. The difference refers to how the optimization problem is defined. The last expression minimizes the standard deviation for the portfolio for each portfolio return. It is probably the task at hand that determines which of the optimization techniques
one prefers over the other. This implies that the two expressions are interchangeable with each other when deriving the efficient frontier.

### 3.3.5 Critique against the MV Framework

The MV concept has been widely used when determining asset allocation. However, the concept of the MV model has, during the years, received a lot of criticism from various researchers (Devaney et al., 2006). Some of this critique refers to the cause that the MV model is based on an assumption that returns are normally distributed (Sivitanides, 1998; Sing & Ong, 2000). Findings suggest that this assumption does not hold in a real estate context. A second critique relates to the assumption that investors are indifferent between meeting a certain Minimum Required Return (MRR) or simply fail to meet the MRR. It turns out that most investors do care about the MRR and that investors’ discernments are dominated of the concern of failing to meet such a MRR.

In order to avoid the problems related to the MV framework suggested by the MPT a number of researchers have devoted time to establish new risk measures; one of these risk measures is the semi-variance measure. This measure focuses on the deviations of return figures that occur below the MRR, such a risk measure is commonly referred to as a Downside-Risk (DR) (Sivitanides, 1998). This measure and portfolio optimization technique will be discussed in the following section.

### 3.4 Introduction to Downside Risk

Consider investment A in figure 3a and investment B in figure 3b. The figures show a hypothetical time series of returns for each investment respectively. Investment A has a standard deviation of 5 percent per annum and a mean return of 10 percent per annum (Blazer, 2001). Investment B has the same mean return as of A, but a higher standard deviation of 10 percent per annum. It is rather intuitive to graphically determine which of the investments that seems to be the more risky one.

**Figure 3a**

![Investment A graph](image)

**Figure 3b**

![Investment B graph](image)

*Managing downside risk in financial markets, 2001, p 105*

Investment B’s return is clearly deviating from its mean to a larger extent than investment A and most investors would argue that investment B is associated with the highest risk out of the two (Blazer, 2001). Most investors would furthermore relate the downside extremes of
investment B as some sort of risk. However, does the upside return deviations of investment B feel risky? Most investors, and probably yourself, would most likely not have any concerns with returns occurring above the mean return, thus the asymmetry of risk.

Let us now consider investment A and B again, however now by examining figure 4 instead. The shaded areas refer to the performance of the two investments where investment B performed worse than investment A. Many investors would identify the shaded areas as the excess risk of investment B compared with investment A (Blazer, 2001). Furthermore, the areas below the zero level of return, implying a loss of capital, would be of most concern.

**Figure 4**

![Graph showing the performance of investments A and B.](Managing_downside_risk_in_financial_markets, 2001, p 106)

From the above figure it becomes evidential that risk is related to relative performance and thus not absolute performance (Blazer, 2001). Risk is, more explicitly, related to doing worse than some alternative investment, in other words, a benchmark.

The appeals of using the downside risk measures are that they incorporate the above statements. Downside risk measures view and acknowledges the asymmetry of risk and makes it possible to utilize some sort of benchmark in the risk analysis (Blazer, 2001). Later sections will develop some of the existing downside risk measures and more technically show how the measures are composed and incorporated in to the portfolio optimization process. However, before moving on to the mathematical technicalities, it is probably useful to consider a graphical example which points out what one tries to optimize when utilizing and defining risk within a downside risk framework. Consider figure 5 which exhibit a normal distribution of returns in percentages from a hypothetical asset. The target rate of return is set to be slightly above 6 percent and the mean return of the asset is 8 percent.
The target rate of return can be any arbitrarily chosen return figure determined by the investor (Harlow, 1991). Blazer (2001) mentions many different benchmarks that can be used in order to determine the target return, which mainly depends on the investors’ preferences. Some of the benchmarks Blazer mentions refer to negative returns, real returns, risk-free rate of returns and sector index returns among others. The target rate of return is thus a return figure that the investor wants to exceed and in all circumstances avoid to fall below. The concept of downside risk is to allocate resources to assets which minimize the deviations occurring below the target return. This implies that return figures that fall below the target rate of return, in the figure, which is somewhat above 6 percent should be minimized. The returns that fall above the target return, in the example return figures that fall to the right of the target return, will be ignored in the calculation. The ignorance of the return figures falling above the target return comes from the risk asymmetry –in other words, returns that fall above the target return are not viewed as risk in a downside risk setting.

3.5 A Short Historical Recap Regarding Downside Risk Measures

Portfolio theory and the birth of downside risk measures was the result of two published research papers in 1952 (Nawrocki, 1999). The first article was a contribution from Markowitz, in which he outlined a quantitative framework for measuring portfolio risk and return. Markowitz (1952) utilized different quantitative measures such as variance, covariance and mean return in order to construct a portfolio that perceived minimum variance for each expected level of return.

Roy (1952) publicized a second paper with regards to portfolio theory in 1952. The aim of the paper was to determine the best risk-return tradeoff, as Roy believed that it would be impossible to mathematically fulfill the utility function of an investor (Nawrocki, 1999). Roy (1952) argued that investors prefer safety of the principal, the initial amount invested, and that investors set a minimum return that will safeguard the principal. The minimal acceptable return is referred to as the disaster level and the concept boils down, without going into details, to select assets that have the lowest probability of falling below the disaster level. Nawrocki (1999) states that Roy’s concept of minimizing the risk of losing the initial principal is a groundbreaking concept in the development of downside risk measures.

The importance of downside risk in portfolio construction was furthermore acknowledged by Markowitz (1959) in his famous monograph Portfolio Selection – Efficient Diversification of Investments. The downside risk measures developed in the article included two different measures referred to as below-target semivariance (SVt) and below-mean semivariance (SVm). Markowitz argued that downside risk is an important consideration when constructing
portfolios and that the semivariance measures would be beneficial to use when return distributions are not normally distributed. It turns out that both the ordinary variance measure and the semivariance measure provides the same allocation when returns are normally distributed, however if the distribution is non-normal semivariance measures provides a better asset allocation with regards to the risk return ratio (Nawrocki, 1999). The SVt and SVm utilize only the returns that fall below an arbitrarily chosen target return or the mean return respectively. The semivariance measure will be defined explicitly in later sections of the paper.

The semivariance measures were further investigated by various researchers. Some of this research argued and demonstrated that the semivariance measures were superior to the more ordinary variance measure (Quirk & Saposnik, 1962). Further studies on the subject were, among others, compiled by Mao (1970). Mao stated that semivariance is a better measure of risk than the variance measure when taking into consideration that investors want to avoid a loss of the principal invested. It has been shown, according to Mao (1970), that investors usually consider different hurdle rates, which an investment alternative should pass in order to be investable. However, passing a single hurdle rate alone for a specific investment is not satisfactory enough. Most often investors also evaluate the potential loss as a result of the investment. This statement is in line with argument proposed by Ang & Chua (1979) who argues that semivariance may be more consistent with investors’ natural perception of risk as the miscarriage to earn a specific target return.

The semivariance measure and the variance measure are both constrained measures when considering the utility function of the investor, since both measures are represented by a quadratic utility function. However, Bawa (1975) and Fishburn (1977) developed a new downside risk measure in which different utility functions could be incorporated. The measure is commonly referred to as Lower Partial Moment (LPM). The LPM measure will be defined in the next sections.

### 3.6 Lower Partial Moment

Harlow (1991) argues that risk measures of particular interest in finance are those measures that involve the left-tail of the return distribution. The returns that fall below a specific threshold level, or target rate of return, are referred to as Lower Partial Moments (LPMs) since only the left-tail of the return distribution is used in the scheming. LPMs are risk measures that were defined and developed by Bwa (1975) and Fishburn (1977). The LPM measures are structured in such a way that it allows the perpetrator to utilize different utility functions of the investor in the portfolio optimization process. The LPM risk measure is thus not constrained to a quadratic utility function, which is the case when using the mean-variance approach. The LPM risk measures represent many of the Von Neumann-Morgenstern utility functions and therefore reflect a vast amount of human behavior from risk loving behavior to risk neutral behavior to risk averse behavior (Fishburn, 1977). Harlow (1991) and Fishburn (1977) argue that the LPM measures are more coherent with most investors preferences since the measures only penalize deviations that occur below the threshold level of return. The view of risk as the deviation below the threshold level of return is thus, according to the authors, more consistent with investors’ preferences since investors’ weights losses more heavily against gains.
The following section will define the LPM measure and explain the measures intuitive appeal as well as its drawbacks when it comes to structuring an efficient set of portfolios. The following LPM sections will build up to the later sections, which will introduce the reader to a heuristic portfolio optimization approach. The heuristic approach is proposed in order to overcome the main drawback inherent in the LPM measures when structuring an efficient set of portfolios.

3.6.1 Definition of the Lower Partial Moment

Fishburn (1977) was first to extend the LPM measure into the n-order LPM measure which incorporates different utility functions of the investor in the expression. Consider the expression 18:

\[ LPM_n(\tau, R_i) = \int_{-\infty}^{\tau} [(\tau - R_i)]^{n} df(R_i) \]  

(18)

\( \tau \) is the target return or threshold return, \( R_i \) is the return of asset \( i \), \( df(R_i) \) is the probability density function of the return on asset \( i \) and \( n \) denotes the order of moment. The order of moment describes the investors’ preferences regarding the return deviation occurring below the target return. One can divide the LPM measure in different classes by changing the number of \( n \) (Sing & Ong, 2000). Regularly used classes of the LPM measure are the probability of a loss \( n=0 \), the target shortfall \( n=1 \), the target semi-variance or mean target semivariance \( n=2 \) and finally the target skewness \( n=3 \). However, one can also interpret the power of \( n \) as a measure of risk aversion. One can therefore interpret \( n=0 \) as an investor that is a risk lover, interpret \( n=1 \) as an investor who is risk neutral and interpret \( n=2 \) as a risk averse investor (Nawarocki, 1999). The n-order LPM measure can also be described as a discrete distribution and can therefore be depicted as expression 19:

\[ LPM_n(\tau, R_i) = \left( \frac{1}{T} \right) \sum_{t=1}^{T} [\text{Min}(R_{it} - \tau, 0)]^{n} \]  

(19)

where \( T \) denotes the number of return observations for asset \( i \) and the \( \text{Min} \) implies that the smallest of the two values \( 0 \) or \( (R_{it} - \tau) \) will be raised by the power of \( n \).

3.6.2 Co-Lower Partial Moment

In order to use the semivariance measure, \( n=2 \) in the LPM expression, in the Capital Asset Pricing Model (CAPM) Hogan and Warren (1974) extended the measure into a co-semivariance concept. The co-semivariance is an asymmetric risk measure, which defines the relative risk between a risky asset and an efficient market portfolio (Sing & Ong, 2000). The co-semivariance measure was further extended to the n-order LPM construction, which is commonly referred as generalized or asymmetric co-LPM (GCLPM). The GCLPM can be depicted as expression 20:

\[ GCLPM_n(\tau, R_i, R_j) = \int_{-\infty}^{\tau} \int_{-\infty}^{R_j} [(\tau - R_j)]^{n-1} (\tau - R_i) df(R_i, R_j), \]  

(20)

\[ GCLPM_n(\tau, R_i, R_j) \neq GCLPM_n(\tau, R_j, R_i), \]
\[ GCLPM_n(\tau, R_i, R_j) = LPM_n(\tau, R_i), \quad \text{when} \quad R_i = R_j \]

d\text{F} \text{ refers to the joint probability density function of the returns of asset } i \text{ and } j. \text{ It should be noted that when the return of asset } i \text{ and } j \text{ is equal then the asymmetric co-LPM can be written as the more simple } LPM_n \text{ expression defined above. However, it is most common that } R_i \neq R_j \text{ (Estrada, 2007). The discrete form of the GCLPM can be defined as 21:}

\[ GCLPM_n(\tau, R_i, R_j) = \left( \frac{1}{T} \right) \sum_{t=1}^{T} \left[ \text{Min}(R_{it} - \tau, 0) \right]^2 \frac{1}{(\tau - R_{it})} \] (21)

The first definition of the CLPM did not incorporate the target return, \( \tau \). The target return was instead equal to the risk-free interest rate. However, Natell and Price (1979) and Harlow and Rao (1989) proposed GCLPM as defined above and the expression is in its form unrestricted when considering the target return. In other words, the perpetrator can arbitrary chose which ever target return that make sense for the purpose.

The next section will shed light on the semivariance measure which is one of the risk measures incorporated in the LPM structure.

### 3.7 Semivariance

The semivariance was proposed as a downside risk measure by Markowitz (1959). The semivariance measure the downside variance by incorporating return figures that fall below either a certain arbitrarily chosen target return (SVt) or the return figures that fall below the mean return (SVM). It should be noted that the semivariance is one of the measures incorporated in the n-order LPM structure when \( n=2 \). This section will follow the framework proposed by Estrada (2008, 2007, 2004) and will concentrate on the semivariance measure. The semivariance of asset \( i \)'s return with respect to a specifically chosen target return is defined by expression 22:

\[ \sum_{i \tau}^2 = E\left[ \text{Min}(R_i - \tau, 0)^2 \right] = \left( \frac{1}{T} \right) \sum_{t=1}^{T} \left[ \text{Min}(R_{it} - \tau, 0) \right]^2 \] (22)

Where \( R_{it} \) is the return of asset \( i \) during the time period \( t \), \( T \) is the number of observations, \( \tau \) is the threshold rate or target rate of return. The notation of \( \text{Min} \) in the formula implies that the smallest of the two values \( 0 \) or \( R_{it} - \tau \) will be squared. It could clearly be seen, in the above formula that return figures falling above the chosen target return are ignored in the calculation. The semideviation, which measures the volatility below the target rate of return, \( \tau \), could be calculated by taking the square root from the semivariance. The semideviation from the below-target semivariance can be defined as expression 23:

\[ \sum_{i \tau} = \sqrt{\sum_{i \tau}^2} = \sqrt{\left( \frac{1}{T} \right) \sum_{t=1}^{T} \text{Min}(R_{it} - \tau, 0)^2} \] (23)
The formulas above are rather intuitive; however it is harder to solve for the semivariance of portfolio returns. Consider the following optimization problem, expression 24 and 25, in which the investor wants to minimize the semivariance of the portfolio for each given target return:

\[
\min_{x_1, x_2, \ldots, x_n} \sum_{p=1}^{T} \frac{1}{T} \sum_{i=1}^{T} \left[ \min (R_{p,t} - \tau, 0) \right]^2
\]  

(24)

Subject to:

\[
\sum_{i=1}^{n} x_i E_i = E^T, \quad \sum_{i=1}^{n} x_i = 1, \quad \text{and} \quad x_i \geq 0
\]  

(25)

The \( R_{p,t} \) express the returns of the portfolio, \( \sum_{p=1}^{T} \) denotes the semivariance of the portfolio, \( E^T \) is the target return, \( E_i \) denotes the expected return for asset \( i \) and \( x_i \) is the weight of the portfolio invested in asset \( i \). In order to find the minimum semivariance portfolio one has to acknowledge that the semicovariance matrix is endogenous (Estrada, 2008). The endogeneity of the semicovariance matrix implies that a change in asset weights affects the periods in which the portfolio underperforms the chosen target return. This consecutively affects the semicovariance matrix. The endogeneity issue inherent in the semicovariance matrix can be solved by using different black-box numerical algorithms, however those procedures are clearly out of the scope for this paper.

The following section will explain the limitation of the endogeneity issue regarding the semicovariance matrix. Later sections will instead of using optimal algorithms propose a heuristic framework for estimating the semivariance in the portfolio optimization process.

### 3.7.1 The Implication of Endogeneity of the Semicovariance Matrix

The semivariance as a measure of risk was defined by Markowitz (1959). Markowitz proposed that the semivariance for a portfolio could be estimated by expressions 26 and 27:

\[
\sum_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sum_{ij} \quad (26)
\]

Where \( \sum_{ij} \) could be expressed as:

\[
\sum_{ij} = \left( \frac{1}{T} \sum_{i=1}^{K} (R_{i,t} - \tau)(R_{j,t} - \tau) \right)
\]  

(27)

\( K \) is the periods in which the target return exceeds the portfolio return. The advantage of the above expressions is that they yield an exact estimation of the portfolio semivariance (Estrada, 2007). However, the exact estimation of the portfolio semivariance leads to an endogenous semicovariance matrix. The endogeneity of the semicovariance matrix implies that a change in asset weights affects the periods in which the portfolio underperforms the chosen target return. Estrada (2007) provides an illustrative example regarding the
endogeneity issue when constructing a portfolio where the risk is quantified and defined as 
semideviation, which the interested reader might find useful. However, in short, one could 
say that the endogeneity of semicovariance matrix forces the perpetrator to a vast number of 
calculations to be able select the portfolio with the lowest semideviation. In order to find the 
most optimal portfolio one has to calculate all the returns for each portfolio, then from the 
received returns calculate the semideviation of each portfolio and from all the portfolios select 
the portfolio with the lowest semideviation. These steps might not seem to troublesome or 
tedious, however as the number of assets included increases and, as a result, the number of 
feasible portfolios increases the method of minimizing the semideviation of a portfolio for a 
given target return becomes obdurate. It is thus no problem to calculate the semideviation for 
one portfolio. The problem arises when one wants to find portfolios with the lowest 
semideviation for each arbitrarily chosen target return -in other words finding the efficient 
frontier.

Many studies propose a heuristic approach in order to estimate the semideviation of a 
portfolio, the next section will briefly present some of these approaches and the analysis, in 
this paper, will utilize a heuristic approach in order to find an efficient set of portfolios.

3.8 The Heuristic Approach in Measuring Downside Risk

Many researchers have proposed different solutions in order to overcome the endogeneity of 
the semicovariance matrix and the complexity it brings to the necessary calculations (Estrada, 
2007).

Hogan and Warren (1972) developed an algorithm which, according to them, solves the 
endogeneity issue of the semicovariance matrix. Ang (1975) provides an alternative portfolio 
selection model, where the essential difference is that a linear framework is utilized in the 
portfolio optimization process instead of the quadratic programming proposed by Markowitz. 
Nawrocki (1983) propose a heuristic approach, which builds on the framework provided by 
Elton et.al (1976). Nawrocki and Staples (1989) extend the heuristic approach developed by 
Nawrocki (1983) by incorporating the LPM risk measure. Harlow (1991) provides a mean-
semivariance efficient frontier in his analysis, however it is unclear how the frontiers have 
been achieved. Markowitz et.al (1993) converted the mean-semivariance problem into a 
quadratic problem by utilizing fictitious securities and then applying the critical line algorithm 
proposed by Markowitz (1959). More recent studies on the subject refer to the studies of 
Athayde (2001) and Ballestero (2005). However, this section will focus on the heuristic 
approach proposed by Estrada (2007) to tackle the endogeniety issue of the semicovariance 
matrix when constructing an efficient frontier of portfolios with regards to minimizing 
semideviation.

Estrada (2007) argues that his heuristic approach solves the endogeniety issue of the 
semicovariance matrix. Furthermore, the heuristics proposed allows all mean-semivariance 
problems to be solved with the same framework many times utilized in the mean-variance 
problems. The portfolio semivariance can, according to Estrada (2007) be estimated with the 
expression 28:

\[ \sum_{\tau} x_{\tau} x_{j} \sum_{y_{\tau}} \]  

(28)
Where \( \sum_{ij\tau} \) is Estrada’s (2002, 2007, 2008) definition of the semicovariance between asset \( i \) and \( j \). The semicovariance is defined as follows:

\[
\sum_{ij\tau} = E\left\{ \text{Min}(R_i - \tau, 0)\text{Min}(R_j - \tau, 0) \right\} = \\
\left( \frac{1}{T} \right) \sum_{j=1}^{T} \text{Min}(R_{it} - \tau, 0)\text{Min}(R_{jt} - \tau, 0)
\]

(29)

The above semicovariance definition, 29, creates a symmetric semicovariance matrix, which implies that \( \sum_{ij\tau} = \sum_{ji\tau} \) which is not the case when utilizing Markowitz (1959) definition of the portfolio semivariance. With the intention to create an efficient set of portfolios with regards to the expression proposed by Markowitz (1959) one has to know if the portfolio underperforms the target return, which results in the endogeniety issue described previously. However, if one applies the heuristic proposed in this paper one only need to know whether the assets included in the portfolio underperforms the target return not the entire portfolio (Estrada, 2007). Estrada (2007) tests for the accuracy of the model and concludes that it gives a very close approximation of the portfolio semivariance. Moreover, when the heuristic approach differs from the optimal approach it does so by slightly overestimating the semivariance.

### 3.8.1 The Heuristic Semistandard Deviation of a Portfolio

When the semicovariances between the assets have been calculated expression 28 is applied in order to derive the portfolio semistandard deviation according to the framework proposed by Estrada (2007). Expression 28 is very similar to expression 16 used in the mean variance approach, the difference is the use of the semivariance of each asset instead of the variance and the semicovariance instead of the normal covariance.

The portfolio optimization procedure is from this stage on exactly the same as in the mean variance approach. Semistandard deviation is minimized for each expected return following the exact same reasoning and constraints as the mean variance approach. Similar short selling constraint as described in the mean variance approach has been applied in the downside optimization procedure as well. For short selling constraint see expression 25. Negative asset weights are thus not allowed.

The optimization procedure has been solved by utilizing the add-in application solver provided in the Excel package.

### 3.9 Real Estate and Quantitative Portfolio Structuring

It is important to note that the quantitative approach to real estate portfolio structuring has some draw backs, at least in a real estate only portfolio context (Coleman & Mansour, 2005). This is due to the characteristics of the real estate sector, which in many ways is unequal to the stock and bond market where quantitative approaches to portfolio structuring frequently is
used. The arguments used to criticize the quantitative structuring of real estate allocations refer to the assumptions about normality, as discussed in previous sections, the liquidity of the market and the asset heterogeneity. The real estate market is illiquid, while the stock and bond market is liquid, the bond and stock market is homogeneous while the real estate market is heterogeneous. These implications is reflected in Bajtelsmit and Worzala (1997) study where they find that only 23.9% of the U.S pension funds use the MPT approach in order to derive optimal real estate portfolios. The most common method is instead, according to the same study, general experience/intuitive diversification with 53.7% of the respondents followed by simple correlation analysis with 37.3% of the respondents.

3.10 Earlier Research

The research that has investigated the applicability of downside risk concepts within real estate portfolios is limited at least to the author’s knowledge. The number of journal articles grows in numbers if other asset types such as stocks and bonds are allowed in the investment scope of the portfolio.

This section will give a brief review over some of the research that has been compiled regarding downside risk, either in a mixed asset portfolio including real estate or in a real estate portfolio only context. The section will thus present some of the empirical results already achieved.

Sivitanides (1998) applied the downside risk concept on the real estate sector, which he divided into four property types’ office, retail, R&D and warehouse. Sivitanides (1998) was able to show that the downside standard deviation increased with an increase in the target return. Sivitanides found, furthermore, that the proxy allocation differed as a result of the chosen target return. More specifically, as the target return was increased so was the weight on warehouses, while the retail weight in the portfolio declined. The paper also concludes that the downside risk efficient frontier always exhibit lower volatility figures than the standard deviation figures achieved by the mean variance approach. Sivitanides (1998) argues that the mean variance approach overestimates the risk inherent in real estate only portfolios. However, this argument is provocative and has been criticized. Authors as Cheng and Wolverton (2001) state that Sivitanides (1998) argument is invalid since the two approaches are incomparable when considering the risk dimension. The last finding of Svitaniades (1998) study was the similarity between the DR approach and the MV approach for target returns that ranged between 0% to the average return of the NCREIF index. For these target returns both approaches allocated similar weights on each of the proxies, however as the target return was increased above the mean return so was the divergence of the proxy allocation between the two approaches.

Cheng (2001) compared the downside risk concept with the mean variance concept using bootstrap simulation. Bootstrap simulation refers to a statistical procedure allowing the perpetrator to extend the number of, in this case, return observation. The portfolios were constructed by utilizing four different asset types namely, stocks, bonds, T-bills and real estate. Cheng (2001) concludes that the downside risk approach is an appealing alternative to utilize when structuring a portfolio. The result reveals furthermore that the downside risk approach creates more realistic portfolios in the sense that they complies better too institutional investors’ actual real estate allocation. Cheng (2001) empirically shows that the
downside risk approach improves the portfolio performance by consistently creating higher median returns than the mean variance approach.

Sing and Ong (2000) applied the downside risk concept on a three asset portfolio consisting of real estate, stocks and bonds. They found that the downside risk approach is efficient to use for risk averse investors. This stem from the superiority of the downside risk measure to create portfolios which exhibit a significant safety margin before falling below the investors’ threshold level of return for the same level of expected return compared to the mean variance approach. Sing and Ong (2000) concluded, in accordance with Cheng (2001), that the downside risk approach allocates less portfolio weight towards the real estate sector in comparison with the mean variance approach. Estrada (2007) applied his heuristic downside risk framework on a multi asset portfolio consisting of stocks, bonds and real estate. Estrada (2007) found contrary to Sing and Ong (2000) and Cheng (2001) that the downside risk approach allocate a higher portfolio weight towards the real estate sector than the mean variance approach. However, Estrada (2007) found the same relationship as, Sing and Ong (2000) and Cheng (2001), regarding increased portfolio weight on stocks in the downside risk approach compared with the mean variance approach.

4. DATA

This chapter will briefly explain the methodology behind some of the IPD measures and provide some descriptive statistics of the data used in the analysis of this paper.

4.1 The Real Estate Return Data

The data in this study is provided by the IPD, which is an organization that tracks the real estate return performance in a large number of countries around the world. IPD publishes indices in 15 European markets and seven additional indices outside the European boarder (IPD, 2011). IPD argues that their indices, built on fairly long time-series, are suitable for risk analysis, forecasting and financial derivatives. IPD provide standardized performance measures such as total return, income return, capital growth, rental value growth to name a few. The first three of these measures are of interest in this paper.

The input in the form of return figures is compiled by utilizing appraised valuations of real buildings. IPD (2011) argues that all the valuations included in the indices have been produced for investment purpose by real estate professionals. IPD claims that appraisal based indices for the moment provides the most robust, convincing and transparent records of the movement of the market. Moreover, according to IPD, transaction based indices tend to underestimate market downturns as a result of the illiquidity of the real estate market, which in some way is avoided when utilizing valuation based indices.

The IPD index that tracks the Swedish real estate performance includes standing investments (IPD, 2011). This implies that completed and leasable properties are included, while properties that are purchased sold or in the course of development during the measurement period are excluded. The reason behind this standpoint is to avoid bias in the valuation influenced by abnormal profits or losses which may be generated through active management. Moreover, the use of standing investments also provides consistency with indices for other types of investment assets. The Swedish IPD index is unfrozen, which implies that published
returns can be restated. This may cause retroactive restatements if additional real estate assets would provide a more comprehensive and accurate report of past market performance.

4.2 IPD’s Relative Sources of Return

The total return in IPD’s indices can be divided into two subgroups namely capital growth and income return (IPD, 2011). The capital growth can in turn be divided between the yield impact and the rental value growth. Figure 6 graphically describes the inherent parameters that all together constitute the total return. The total return is calculated as the change in capital value, less any capital expenditure incurred, plus net income, expressed as a percentage of capital employed over the actual period (IPD, 2011). Capital growth is the change in capital value, less capital expenditure, expressed as a percentage of capital employed over the actual period. The income return is a percentage figure representing the net income relative the capital employed over the actual period. The rental value growth represents the change in market rental value stated as a percentage figure relative to the rental value at the start of the period. The yield impact measures the relative impact of yield movements on capital values expressed as a percentage figure. More specifically a positive percentage figure implies that yields have compressed adding to the capital value, a negative percentage figure has the reversed impact. All the formulas to calculate the total return and its relative source of return can be found in the appendix II.

4.3 Index Time-weighting

All the IPD indices are time-weighted with a return calculation period of one calendar month (IPD, 2011). Returns for longer periods are calculated by chain-linking individual monthly returns. The total return is the sum of the capital and income parameters and is calculated for each monthly measurement period. The Swedish IPD index lacks the convenience of monthly valuation data as input. The valuation that is used as input is instead interpolated in order to estimate the monthly valuation values. If the valuation figures for the different properties inherent in the index are occurring on different dates then IPD apply either one of the two methodologies described next. The first method holds the values flat from one valuation date to the next, while the second method uses an interpolating technique which results in restatements of previously published returns. The computation technicalities can be found in the appendix II.
4.4 Data Description

The Swedish IPD index tracks officially back to 1997, however there exist longer time-series with the drawback of a less comprehensive market coverage. The analysis in this paper utilizes the longest time-series there is for the Swedish real estate market dating back to 1984. Using IPD’s return figures pre 1997 implies that the market coverage is less convincing, however for this paper it provides the best solution since it would be too few data points if only considering return figures from 1997 and onwards. The Swedish IPD index was in 2009 estimated to cover 26% of the professional real estate investment market. Exhibit 1 provides a description of the data. The increase in market coverage becomes evident when considering that the number of properties included in the index for 1984 only was approximately half of the number compared with the number of properties included in 2009. From the exhibit it becomes clear that the proxies are divided into property type e.g. Retail Shopping Centres, Office and so forth as well as between geographical regions. The analysis in this paper utilizes both property type and geographical region as proxies for the portfolio structuring. This is inline with the findings of Lee (2001) who argues that the most important proxies to consider when constructing a real estate portfolio are property type and geographical region.

Offices in Malmö, Residentials in Malmö and Residentials in the Rest of Sweden have been excluded in the analysis. The exclusion of these proxies is the result of their relatively short return-series starting in 1997. If these proxies were included it would result in an allocation overweight towards these assets, since the short return-series would imply a lower standard deviation compared with the other proxies, which exhibit longer return-series.

Exhibit 1

<table>
<thead>
<tr>
<th>IPD Proxies &amp; Number of Properties</th>
<th>1984</th>
<th>1990</th>
<th>1997</th>
<th>2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retails Shopping Centres</td>
<td>8</td>
<td>22</td>
<td>44</td>
<td>37</td>
</tr>
<tr>
<td>Retails Other Retail</td>
<td>59</td>
<td>95</td>
<td>187</td>
<td>223</td>
</tr>
<tr>
<td>Offices Stockholm CBD</td>
<td>19</td>
<td>36</td>
<td>80</td>
<td>81</td>
</tr>
<tr>
<td>Offices Stockholm Central Area</td>
<td>31</td>
<td>49</td>
<td>142</td>
<td>118</td>
</tr>
<tr>
<td>Offices Rest of Greater Stockholm</td>
<td>25</td>
<td>44</td>
<td>184</td>
<td>84</td>
</tr>
<tr>
<td>Offices Göteborg</td>
<td>31</td>
<td>45</td>
<td>125</td>
<td>77</td>
</tr>
<tr>
<td>Offices Malmö</td>
<td>-</td>
<td>-</td>
<td>70</td>
<td>48</td>
</tr>
<tr>
<td>Offices Other Major Cities</td>
<td>25</td>
<td>44</td>
<td>184</td>
<td>31</td>
</tr>
<tr>
<td>Offices Rest of Sweden</td>
<td>-</td>
<td>-</td>
<td>61</td>
<td>8</td>
</tr>
<tr>
<td>Industrial</td>
<td>25</td>
<td>58</td>
<td>213</td>
<td>43</td>
</tr>
<tr>
<td>Hotel &amp; Other Commercial</td>
<td>18</td>
<td>43</td>
<td>104</td>
<td>62</td>
</tr>
<tr>
<td>Residentials Stockholm Central Area</td>
<td>91</td>
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<td>196</td>
<td>26</td>
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<tr>
<td>Residentials Rest of Greater Stockholm</td>
<td>51</td>
<td>49</td>
<td>177</td>
<td>63</td>
</tr>
<tr>
<td>Residentials Göteborg</td>
<td>44</td>
<td>44</td>
<td>88</td>
<td>27</td>
</tr>
<tr>
<td>Residentials Malmö</td>
<td>-</td>
<td>-</td>
<td>51</td>
<td>18</td>
</tr>
<tr>
<td>Residentials Other Major Cities</td>
<td>20</td>
<td>24</td>
<td>92</td>
<td>61</td>
</tr>
<tr>
<td>Residentials Rest of Sweden</td>
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<td>-</td>
<td>29</td>
<td>1</td>
</tr>
<tr>
<td>Other</td>
<td>8</td>
<td>18</td>
<td>153</td>
<td>22</td>
</tr>
<tr>
<td>All Property</td>
<td>502</td>
<td>727</td>
<td>2180</td>
<td>1030</td>
</tr>
</tbody>
</table>

Exhibit 2 provides descriptive statistics for each property type aggregated on country level. The results show that Residential units have been the most attractive investment generating
the highest average yearly return of 13.8%. The least attractive proxy when considering average yearly return figures has been Other property with a return of 6.2%. Moreover Other property exhibit a wide return range with a minimum return of -51% and a maximum return of 57.4%. The standard deviation figures are the lowest for Residential implying that return figures for this property type historically has deviated less from the mean return compared with the other proxies included. It actually already now becomes rather clear that the characteristics of the residential proxy will result in a portfolio allocation with much weight on residential units at least from a mean variance perspective.

When considering the third moment of expected returns, the skewness, it becomes evident that the proxies do not exhibit any significant skewness. A distribution should have skewness figures above 1 or less than -1 in order to reveal the true characteristic of a skewed distribution (Cheng, 2001).

The kurtosis figures in exhibit 2 are excess kurtosis values and a positive value indicates a platykurtic distribution. All the proxies except Hotel & Other Commercial exhibit kurtosis statistics that exceeds the kurtosis of a normal distribution they are thus platykurtic. This implies that the tails of the distributions are longer and heavier and the distribution is, moreover, flatter around the mean of the distribution than in the case of a normal distribution. The proxies in this analysis with the largest excess kurtosis values are Other, Residential and Retail. Historical total return figures have behaved differently for the proxy Hotel & Other Commercial as the excess kurtosis is negative. A negative excess kurtosis value implies a leptokurtic distribution. The characteristic of this distribution is a more peaked curve than the normal distribution with shorter and thinner tails as well.

**Exhibit 2**

<table>
<thead>
<tr>
<th></th>
<th>Retail</th>
<th>Office</th>
<th>Industrial</th>
<th>Residential</th>
<th>Hotel &amp; Other Com.</th>
<th>Other</th>
<th>All Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.107</td>
<td>0.103</td>
<td>0.096</td>
<td>0.138</td>
<td>0.106</td>
<td>0.062</td>
<td>0.107</td>
</tr>
<tr>
<td>Median</td>
<td>0.088</td>
<td>0.095</td>
<td>0.085</td>
<td>0.123</td>
<td>0.118</td>
<td>0.038</td>
<td>0.094</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.113</td>
<td>0.150</td>
<td>0.135</td>
<td>0.107</td>
<td>0.127</td>
<td>0.212</td>
<td>0.136</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.351</td>
<td>0.104</td>
<td>0.212</td>
<td>(0.106)</td>
<td>(0.323)</td>
<td>0.246</td>
<td>(0.174)</td>
</tr>
<tr>
<td>Kutosis</td>
<td>1.113</td>
<td>0.581</td>
<td>0.865</td>
<td>1.391</td>
<td>(0.323)</td>
<td>2.694</td>
<td>0.869</td>
</tr>
<tr>
<td>Min.</td>
<td>(0.193)</td>
<td>(0.255)</td>
<td>(0.200)</td>
<td>(0.130)</td>
<td>(0.233)</td>
<td>(0.510)</td>
<td>(0.231)</td>
</tr>
<tr>
<td>Max.</td>
<td>0.325</td>
<td>0.431</td>
<td>0.427</td>
<td>0.358</td>
<td>0.402</td>
<td>0.574</td>
<td>0.397</td>
</tr>
</tbody>
</table>

Note: N = 26 for 1984-2009

Four different tests of normality, found in exhibit 3, were conducted in order to strengthen the conclusion drawn from exhibit 2. The results of the normality test strengthen the conclusion drawn above. The null hypothesis can be rejected for the proxies Residential and Other property implying that these proxies do not share the same characteristics as a normal distribution. These results are encouraging for the downside risk concept as the framework is unrestricted to the normality assumption. The null hypothesis can however not be rejected for the remaining proxies on a 95% confidence level.
### Exhibit 3

**Tests for Normality**

<table>
<thead>
<tr>
<th>Test</th>
<th>Statistic</th>
<th>p Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retail</td>
<td>Shapiro-Wilk</td>
<td>W</td>
</tr>
<tr>
<td></td>
<td>Kolmogorov-Smirnov</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>Cramer-von Mises</td>
<td>W-Sq</td>
</tr>
<tr>
<td></td>
<td>Anderson-Darling</td>
<td>A-Sq</td>
</tr>
<tr>
<td>Office</td>
<td>Shapiro-Wilk</td>
<td>W</td>
</tr>
<tr>
<td></td>
<td>Kolmogorov-Smirnov</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>Cramer-von Mises</td>
<td>W-Sq</td>
</tr>
<tr>
<td></td>
<td>Anderson-Darling</td>
<td>A-Sq</td>
</tr>
<tr>
<td>Industrial</td>
<td>Shapiro-Wilk</td>
<td>W</td>
</tr>
<tr>
<td></td>
<td>Kolmogorov-Smirnov</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>Cramer-von Mises</td>
<td>W-Sq</td>
</tr>
<tr>
<td></td>
<td>Anderson-Darling</td>
<td>A-Sq</td>
</tr>
<tr>
<td>Residential</td>
<td>Shapiro-Wilk</td>
<td>W</td>
</tr>
<tr>
<td></td>
<td>Kolmogorov-Smirnov</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>Cramer-von Mises</td>
<td>W-Sq</td>
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<td></td>
<td>Anderson-Darling</td>
<td>A-Sq</td>
</tr>
<tr>
<td>Hotel</td>
<td>Shapiro-Wilk</td>
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<td></td>
<td>Kolmogorov-Smirnov</td>
<td>D</td>
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<tr>
<td></td>
<td>Cramer-von Mises</td>
<td>W-Sq</td>
</tr>
<tr>
<td></td>
<td>Anderson-Darling</td>
<td>A-Sq</td>
</tr>
<tr>
<td>Other</td>
<td>Shapiro-Wilk</td>
<td>W</td>
</tr>
<tr>
<td></td>
<td>Kolmogorov-Smirnov</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>Cramer-von Mises</td>
<td>W-Sq</td>
</tr>
<tr>
<td></td>
<td>Anderson-Darling</td>
<td>A-Sq</td>
</tr>
</tbody>
</table>

Note that a p value below 0.05 implies that the null hypothesis can be rejected and the distrib. is thus not normally distributed.

### 5. RESULT

The proceeding result sections will reveal the result of the MV approach and the DR approach and compare and contrast these findings. The subsequent section will graphically present the efficient frontiers of each of the models and thereafter present their respective property allocation. The section following will focus on the result difference between the models. The result section will be finished off with a historical return comparison presenting some possible portfolio scenarios and thus give an indication of which of the models that have outperformed the other in an historical total return perspective.

#### 5.1 Frontiers

This section presents four different efficient and inefficient frontiers. One of the frontiers refers to the MV approach proposed by Markowitz (1959) and the following three efficient frontiers refer to the DR approach proposed by Estrada (2007). It is important to note that the
frontiers define risk in dissimilar ways and that all the frontiers are efficient when considering each frontiers risk definition.

Figure 7 depicts the MV frontier with risk defined as standard deviation on the x-axis and expected return on the y-axis. The solid line represents the efficient frontier, which implies that no other portfolio composition is efficient when considering the expected return and standard deviation. The dotted line represents the inefficient frontier which depicts portfolio compositions that have a higher risk and less expected return than the portfolios situated on the efficient frontier. The minimum variance portfolio is found where the efficient and inefficient frontiers foregather. The MV minimum variance portfolio yields an expected return of 10.78% per annum and a standard deviation of 12.36%.

\[ \text{Figure 7} \]

The following three efficient frontiers, figure 8-10 depicts the DR approach with three different target returns. Figure 8 depicts the efficient and inefficient frontier with a target rate of return of 8.5%. The minimum semivariance portfolio for the case of 8.50% target return yields an expected return of 11.09% and a downside standard deviation of 6.04%.

\[ \text{Figure 8} \]

An increase of the target return, in the DR setting, results in a right shift of the minimum semivariance frontier. This becomes clear by comparing the graphics of figure 8 and figure 9.
Figure 9

One can clearly notice that the minimum semivariance portfolio with a target return of 10.69%, figure 9, exhibit higher downside volatility than the portfolio with a target return of 8.50%, figure 8. This characteristic is natural since portfolios with an increasingly higher target return broadens the risk span as relatively higher return figures would be viewed as risk in that setting. The portfolios optimized with a target return of 10.69% exhibit a minimum semivariance portfolio yielding an expected return of 12.03% per annum and with a downside standard deviation of 7.06%. Now consider the next efficient frontier in figure 10 with a target rate of return of 12.00%. The minimum semivariance portfolio has yet again shifted to the right, however to a much lesser extent than the comparison between the previous two charts. The minimum semivariance portfolio exhibits an expected return of 12.25% per annum and a downside standard deviation of 7.12%.

Figure 10

The above findings are in line with the papers of Harlow (1991), Sivitanides (1998), Sing and Ong (2000), Coleman and Mansour (2005) which all have investigated the implication of different target rate of returns in a DR setting. They argue and empirically conclude that when the target rate of return is increased so is the downside volatility of the minimum semivariance portfolio.

5.2 Allocations

Let us proceed and leave the efficient frontier analysis, for a while, and instead consider the resulting property allocations stemming from the different optimization techniques. Exhibit 4, on the next page, provides a presentation of the different property allocations. Note that the
DR allocations are divided, as previously, in three different sections in accordance to their specific target rate of return. The first complete horizontal row states the expected portfolio return for all the portfolios included in the exhibit and all the figures are in percentages. Before going into any depth in comparison between the approaches it becomes evident that all the models allocate the same percentage weight on the same proxies as the expected return is below 8.00%. The portfolio allocation corresponds to 65% of Offices in Rest of Greater Stockholm and 35% of the portfolio allocation refers to Other properties when the expected return is set to 7.50%. When the target return is increased with 50 basis points to 8.00% allocation discrepancies becomes evident between the MV and DR approach. The MV model minimizes the standard deviation by allocating less portfolio weight, -18 percentage units, on Other property, increase the allocation, +10 percentage units, towards Offices in Rest of Greater Stockholm and incorporating a new proxy Retail Shopping Centres, +9 percentage units, compared to the weight received when the expected return amounted to 7.50%. All the DR approaches, regardless of chosen target return, allocate the portfolio weight equal when the expected return is set to 8.00%. The DR allocation uses fewer proxies, when the expected return is equal to 8.00%, than the case of the MV approach. The portfolio weight is divided between Offices Rest of Greater Stockholm, 90%, and Other Properties, 10%. The DR approach with a target rate of return amounting to 8.50% uses consistently fewer proxies in the portfolio optimization process than the case of the other DR scheming and the MV approach as the target return increases. Put differently, when the target rate of return is increased in the downside risk setting so is the number of proxies used in the portfolio optimization process. The DR portfolio allocations are rather different from each other until the expected return reaches 15.00%. The explanation for this occurrence is that there does not exist any other proxy that yields such a high expected return characteristic. All the DR models and the MV model are therefore forced to allocate the entire portfolio allocation towards this proxy. The reasoning and logic is the same when the expected return is set to 6.20%. There are no other proxies that are expected to have such a low return and the models are therefore forced to allocate the entire portfolio weight towards this single proxy.

Now when we have familiarized our selves with the outcomes of the MV and DR approach it is time to compare the two approaches with each other.
### Exhibit 4

#### Ex Post Portfolio Compositions

<table>
<thead>
<tr>
<th>MV Portfolios</th>
<th>6.2</th>
<th>7.0</th>
<th>7.5</th>
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<th>8.5</th>
<th>9.0</th>
<th>9.5</th>
<th>10.0</th>
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<th>14.5</th>
<th>15.0</th>
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</thead>
<tbody>
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<td></td>
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<tr>
<td>Retails SC</td>
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<td>9</td>
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</tr>
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<td>Industrial</td>
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<tr>
<td>Other</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

| DR Portfolios (TR=8.5%) | | | | | | | | | | | | | | | | | | | |
|-------------------------|-----|-----|-----|-----|-----|-----|-----|------|------|------|------|------|------|------|------|------|------|------|
| Retails SC | 0 | 0 | 0 | 20 | 52 | 84 | 93 | 80 | 67 | 54 | 41 | 28 | 14 | 1 | 0 | 0 | 0 | 0 |
| Offices RG Sthlm | 0 | 40 | 65 | 90 | 80 | 48 | 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Offices Gbg | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Industrial | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Residential Sthlm CA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 31 | 66 | 100 |
| Residential RG Sthlm | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 20 | 33 | 46 | 59 | 72 | 86 | 99 | 69 | 34 | 0 |
| Residential Gbg | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Residential OM Cities | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Other | 100 | 60 | 35 | 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| DR Portfolios (TR=10.69%) | | | | | | | | | | | | | | | | | | | |
|---------------------------|-----|-----|-----|-----|-----|-----|-----|------|------|------|------|------|------|------|------|------|------|------|
| Retails SC | 0 | 0 | 0 | 20 | 52 | 71 | 70 | 65 | 59 | 52 | 43 | 33 | 22 | 11 | 0 | 0 | 0 | 0 |
| Offices RG Sthlm | 0 | 40 | 65 | 90 | 80 | 48 | 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Offices Gbg | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Industrial | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Residential Sthlm CA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 10 | 14 | 17 | 21 | 27 | 65 | 100 |
| Residential RG Sthlm | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 12 | 23 | 34 | 42 | 48 | 54 | 60 | 63 | 32 | 0 |
| Residential Gbg | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 7 | 9 | 10 | 13 | 15 | 17 | 21 | 23 | 25 | 0 |
| Residential OM Cities | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 11 | 12 | 13 | 12 | 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| Other | 100 | 60 | 35 | 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| DR Portfolios (TR=12%) | | | | | | | | | | | | | | | | | | | |
|------------------------|-----|-----|-----|-----|-----|-----|-----|------|------|------|------|------|------|------|------|------|------|------|
| Retails SC | 0 | 0 | 0 | 20 | 52 | 67 | 62 | 57 | 51 | 47 | 42 | 30 | 18 | 6 | 0 | 0 | 0 | 0 |
| Offices RG Sthlm | 0 | 40 | 65 | 90 | 80 | 48 | 6 | 3 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Offices Gbg | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 5 | 3 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Industrial | 0 | 0 | 0 | 0 | 0 | 0 | 23 | 20 | 15 | 12 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Residential Sthlm CA | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Residential RG Sthlm | 0 | 0 | 0 | 0 | 0 | 0 | 11 | 23 | 35 | 44 | 50 | 56 | 63 | 69 | 58 | 26 | 0 |
| Residential Gbg | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 7 | 11 | 14 | 18 | 18 | 14 | 0 | 0 |
| Residential OM Cities | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Other | 100 | 60 | 35 | 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**Note:** Asset weights displayed in the table are in percentage numbers. The difference in allocation between the MV approach and DR approach starts to be notable for expected portfolio returns above 8%. Proxies with zero allocation in all the models have been excluded.
5.3 Comparison Between MV Approach and DR Approach

This section will highlight the difference between the DR outcome and the MV outcome regarding the efficient frontier result, as well as the allocation result.

5.3.1 Frontier Comparison

The efficient frontier analysis relies on four different efficient frontier figures. The first three figures 11, 12, 13 compare the DR efficient frontiers with the MV efficient frontier from a DR perspective. This basically boils down to an investigation determining how the MV efficient frontier performs when evaluated within the DR setting. The fourth and last figure, 14, has a MV perspective and evaluates how the DR efficient frontiers perform in a MV context. Consider figure 11 depicting the DR efficient frontier with a target rate of return of 8.50% and the MV efficient frontier. The MV approach clearly exhibit larger downside standard deviation than the DR efficient frontier for each chosen target return.

Figure 11

![DR Efficient Frontier TR=8.5% & MV Efficient Frontier](image)

This result is reflected in figure 12 and figure 13 as well. It is interesting to note that figure 12 depicts the two frontiers with the least difference in downside volatility when considering the minimum variance portfolio of the MV approach and the minimum semivariance portfolio of the DR approach. This can to some part be explained by the chosen target rate of return of 10.69% since it refers to the mean of the historical proxy return. Remember, from previous chapters that MV approach minimizes deviations from the mean return, both on the upside as well as on the downside. The only difference in the optimization approach, in this setting, between the DR and MV is thus that the DR approach only minimizes deviations below the mean return. When the target return in the DR setting is increased to 12%, figure 13, so is the gap between the MV efficient frontier and the DR efficient frontier, which is intuitive as the DR model no longer optimizes from the mean of the return distribution.

Figure 14 depicts the MV efficient frontier and the three DR frontiers with the target returns of 8.5%, 10.69% and 12% viewed from a MV perspective. The result is rather logical the MV approach is superior to the DR approach when viewed from a MV perspective. We can yet again observe that the DR frontier with a target return of 10.69% ends up the closest to the MV efficient frontier following the same reasoning as previously. These outcomes are natural, since the two approaches optimize the proxy allocation in accordance to each of the models.
risk definitions. The DR efficient frontiers are optimized by minimizing downside volatility, while the MV efficient frontier minimizes standard deviation.

Figure 12

DR Efficient Frontier TR=10.69% & MV Efficient Frontier

Figure 13

DR Efficient Frontier TR=12.00% & MV Efficient Frontier

Figure 14

MV Efficient Frontier & Various DR Efficient Frontiers
5.3.2 Allocation Comparison

Exhibit 5 states the percentage unit differences between the MV and DR approach considering the property allocation. It is not until the expected return reaches 8% that allocation differences start to occur between the two approaches. The most notable difference in the range of 8.0-9.0% expected return refer to the office sector in Rest of Greater Stockholm where the DR approach allocate a significantly higher percentage share. The MV approach allocates instead a larger percentage share towards the proxy Retail Shopping Centres within the same expected return range. The next major allocation difference between the two approaches occurs in the expected return range of 10.0-14.5%. The MV optimizes the allocation by allocating a significant share of the portfolio weight towards the residential sector in Other Major Cities, while the DR approach optimizes the allocation by concentrating the allocation towards residential in Rest of Greater Stockholm.

The difference in allocations, between the models, reflect the different correlation scheming’s, but also the difference in the definition of risk. The DR (MV) approach successfully chose combination of property types which each contribute to minimize the semivariance (variance) of the portfolio. The interesting part to note is that the allocations differs, implying that a DR optimal allocation is unequal to a MV optimal allocation.

**Exhibit 5**

<table>
<thead>
<tr>
<th>Return portfolio</th>
<th>DR Portfolios (TR=8.5%)</th>
<th>DR Portfolios (TR=10.69%)</th>
<th>DR Portfolios (TR=12%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retails SC</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Offices RG Sthlm</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Offices Gbg</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Industrial</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Residential Sthlm CA</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Residential Sthlm</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Residential Gbg</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Residential OM Cities</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Other</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: Allocation difference between the MV approach and the DR approach.
The difference displayed in the table are in percentage units.
5.4 The Total Return Dimension

One way of comparing the performance of the two portfolio optimization techniques is to investigate ex post arithmetic average return, Exhibit 6. This has been done by using the minimum variance allocation and minimum semivariance allocation previously presented. The arithmetic average total return is then calculated for the entire data period from 1984 to 2009 keeping asset weight within the portfolios static. The total return calculation has in addition also been computed with a five year holding period starting in 1985 and ending in 2009. This exercise is somewhat hypothetical since the asset weights determined both by the MV approach and DR approach is based on data stretching from 1984-2009, an investor active in 1984 would thus not be able to receive these allocation suggestions since the data series would not exist.

Exhibit 6

<table>
<thead>
<tr>
<th>Ex post return based on efficient frontier proxy allocation</th>
<th>Retail SC</th>
<th>Residentials SHLM CA</th>
<th>Residentials RG SHLM</th>
<th>Residentials Gbg</th>
<th>Residentials OMC</th>
<th>Arithmetic avg. return</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DR</strong></td>
<td><strong>TR=8.5%</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Holding p 1984-2009</td>
<td>65%</td>
<td>35%</td>
<td></td>
<td></td>
<td></td>
<td>11.68%</td>
</tr>
<tr>
<td>Holding p 1985-1989</td>
<td>65%</td>
<td>35%</td>
<td></td>
<td></td>
<td></td>
<td>23.44%</td>
</tr>
<tr>
<td>Holding p 1990-1994</td>
<td>65%</td>
<td>35%</td>
<td></td>
<td></td>
<td></td>
<td>2.48%</td>
</tr>
<tr>
<td>Holding p 1995-1999</td>
<td>65%</td>
<td>35%</td>
<td></td>
<td></td>
<td></td>
<td>10.83%</td>
</tr>
<tr>
<td>Holding p 2000-2004</td>
<td>65%</td>
<td>35%</td>
<td></td>
<td></td>
<td></td>
<td>10.28%</td>
</tr>
<tr>
<td>Holding p 2005-2009</td>
<td>65%</td>
<td>35%</td>
<td></td>
<td></td>
<td></td>
<td>11.09%</td>
</tr>
<tr>
<td><strong>DR</strong></td>
<td><strong>TR=10.69%</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Holding p 1984-2009</td>
<td>39%</td>
<td>12%</td>
<td>45%</td>
<td>4%</td>
<td></td>
<td>12.18%</td>
</tr>
<tr>
<td>Holding p 1985-1989</td>
<td>39%</td>
<td>12%</td>
<td>45%</td>
<td>4%</td>
<td></td>
<td>24.88%</td>
</tr>
<tr>
<td>Holding p 1990-1994</td>
<td>39%</td>
<td>12%</td>
<td>45%</td>
<td>4%</td>
<td></td>
<td>3.57%</td>
</tr>
<tr>
<td>Holding p 1995-1999</td>
<td>39%</td>
<td>12%</td>
<td>45%</td>
<td>4%</td>
<td></td>
<td>11.41%</td>
</tr>
<tr>
<td>Holding p 2000-2004</td>
<td>39%</td>
<td>12%</td>
<td>45%</td>
<td>4%</td>
<td></td>
<td>10.66%</td>
</tr>
<tr>
<td>Holding p 2005-2009</td>
<td>39%</td>
<td>12%</td>
<td>45%</td>
<td>4%</td>
<td></td>
<td>10.45%</td>
</tr>
<tr>
<td><strong>DR</strong></td>
<td><strong>TR=12%</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Holding p 1984-2009</td>
<td>36%</td>
<td>2%</td>
<td>53%</td>
<td>9%</td>
<td></td>
<td>12.11%</td>
</tr>
<tr>
<td>Holding p 1985-1989</td>
<td>36%</td>
<td>2%</td>
<td>53%</td>
<td>9%</td>
<td></td>
<td>24.38%</td>
</tr>
<tr>
<td>Holding p 1990-1994</td>
<td>36%</td>
<td>2%</td>
<td>53%</td>
<td>9%</td>
<td></td>
<td>4.02%</td>
</tr>
<tr>
<td>Holding p 1995-1999</td>
<td>36%</td>
<td>2%</td>
<td>53%</td>
<td>9%</td>
<td></td>
<td>11.35%</td>
</tr>
<tr>
<td>Holding p 2000-2004</td>
<td>36%</td>
<td>2%</td>
<td>53%</td>
<td>9%</td>
<td></td>
<td>10.64%</td>
</tr>
<tr>
<td>Holding p 2005-2009</td>
<td>36%</td>
<td>2%</td>
<td>53%</td>
<td>9%</td>
<td></td>
<td>10.34%</td>
</tr>
<tr>
<td><strong>MV</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Holding p 1984-2009</td>
<td>70%</td>
<td>13%</td>
<td>17%</td>
<td>9%</td>
<td></td>
<td>10.95%</td>
</tr>
<tr>
<td>Holding p 1985-1989</td>
<td>70%</td>
<td>13%</td>
<td>17%</td>
<td>9%</td>
<td></td>
<td>21.49%</td>
</tr>
<tr>
<td>Holding p 1990-1994</td>
<td>70%</td>
<td>13%</td>
<td>17%</td>
<td>9%</td>
<td></td>
<td>2.32%</td>
</tr>
<tr>
<td>Holding p 1995-1999</td>
<td>70%</td>
<td>13%</td>
<td>17%</td>
<td>9%</td>
<td></td>
<td>9.91%</td>
</tr>
<tr>
<td>Holding p 2000-2004</td>
<td>70%</td>
<td>13%</td>
<td>17%</td>
<td>9%</td>
<td></td>
<td>9.71%</td>
</tr>
<tr>
<td>Holding p 2005-2009</td>
<td>70%</td>
<td>13%</td>
<td>17%</td>
<td>9%</td>
<td></td>
<td>10.87%</td>
</tr>
</tbody>
</table>

Note: Asset weights held static for the entire period and all sub periods. The DR approach outperforms the MV approach in the return dimension both for the entire period as well as most sub periods.

However, it is still interesting to note that in an ex post perspective the DR approach seems appealing. The DR approach has clearly outperformed the MV approach in the total return dimension for the entire calculation period of 1984-2009. The best performing portfolio was the DR portfolio with a target return of 10.69%, when examining the entire period, with an arithmetic average total return of 12.18%. The same portfolio also yielded the single highest sub period arithmetical average total return of 24.88%. The DR portfolio with a target return
of 8.5% outperformed the MV portfolio for all periods presented here. The DR portfolio with a target return of 10.69 outperformed the MV portfolio for all periods except between the years of 2005-2009, when the MV portfolio yielded 42 basis points higher return. The case is similar to the DR portfolio with a target return set to 12.00% it outperforms the MV approach in all sub periods except the last period where the MV approach yields 53 basis points higher return. These results are not satisfactory enough to determine which of the models that is most likely to outperform the other, however it gives an indication which clearly is in favor for the DR portfolio concept. Additional analysis is needed in order to determine which, the DR or the MV, approach that is most beneficial to use when optimizing portfolios and considering the return performance.

6. CONCLUSION

This thesis shows that it is achievable to construct and optimize a real estate portfolio by utilizing quantitative approaches. The paper has furthermore outlined and showed that it is possible to apply a heuristic downside risk approach based on the methodology of Estrada (2007). However, the quantitative approaches examined here are in the real world probably most beneficial when evaluated together with more qualitative measures. This is true since the real estate sector exhibit many characteristics, e.g. liquidity constraints and heterogeneity issues, that is hard to quantify into a general model based on pure return series. With that stated, I would argue that there is no question regarding the intuitiveness and appeal of using the downside risk measures presented in this paper, which clearly exhibits characteristics which investors ought to appreciate. The appeal descends from the downside risk measures applicability on return distributions that deviate from the normal distribution and the asymmetry of investors risk preferences. The downside risk measures also allow for greater flexibility and the investor is able to elaborate on the measure depending on the investment requirements and the degree of risk aversion. The mean variance approach clearly lack these characteristics and the implication of considering upside potential as risk makes the measure in some sense unrealistic.

The portfolio semivariance and the heuristic downside risk approach, as defined in this paper, clearly exhibits other characteristics than the more common variance measure and its mean variance framework. This becomes evident when scrutinizing the portfolio allocations between the models and the efficient frontiers. The two portfolio optimization approaches do not provide the investor with the same portfolio allocations. The mean variance approach allocates e.g. a larger share of the portfolio weight towards the residential sector in Other Major Cities, while the downside risk approach instead allocates a larger share of the portfolio weight towards the residential sector in the Rest of Greater Stockholm area. The number of proxies used in the optimization process does also deviate between the two approaches. The downside risk approach reaches, in general, efficient diversification with less number of proxies compared to the mean variance approach.

When investigating the efficient frontiers between the approaches it becomes clear that these deviate. The downside risk efficient frontier is superior to the mean variance frontier from a downside risk perspective. This in turn implies that the mean variance frontier is superior to the downside risk approach when evaluated from a mean variance setting. This is very logical since the models optimize the allocation in accordance with their respective objective. The mean variance approach minimizes the standard deviation for each given expected return and
the downside risk approach minimizes the semivariance for each given expected return. What matters is how the risk is defined.

At time of writing this thesis the scientific community lacks a method or measure to determine which of the models that is the most superior when considering the risk parameters. Instead one has to focus on the return dimension, which is fully comparable between the downside risk approach and the mean variance framework. This paper has showed that if history where to repeat itself a portfolio structured in accordance with the downside risk framework would benefit the most. The mean variance framework beats the downside risk approach, marginally and only ones, for the sub period of 2005-2009.

This thesis is a result of an empirical approach and is in this sense not general. However, the scope is new and hopefully an inspiration to peruse further research on the subject. These could both be empirical and more theoretically geared. It would be of great importance to extend the outcome of this thesis by applying an optimal downside risk approach instead of the heuristics outlined here and elaborate on the differences. The return dimension could also be investigated with statistical methods such as bootstrapping in order to provide a more robust result. An area of interest, today, refers to time varying correlation. It would be of great importance to extend the downside risk approach by incorporating such measures and investigate its effect on diversification.

I am sure we will experience new measures which incorporates the downside risk in a better way than Markowitz mean variance framework. The downside risk methods and measures ought to grow in popularity in the aftermath of the financial crisis. Who wants to loose the initial principal?
REFERENCES


APPENDIX I

Time-Series Statistics

The following section stem from Geltner & Miller (2001)

Return statistics measure the central tendencies and dispersions in the returns to single assets or portfolios. Time-series statistics implies that the statistics are taken across multiple periods of time. These statistics can be compiled on a single assets basis or single portfolio basis as well as on multiple assets and multiple portfolio bases. In order to be able to construct efficient portfolios when considering the risk and return defined as in the MPT framework on has to consider the following time-series statistics: the mean return, the variance, the standard deviation, and the covariance.

The mean return is the average value of returns across time. The mean return can either be interpreted as the expected performance (ex ante), based on the view that the history provides a good estimate of the future performance, or as the historical performance (ex post). The ex post arithmetical average return can be written in the following manner:

\[ \bar{r} = \left( \frac{1}{T} \right) \sum_{t=1}^{T} r_t \]

Where \( \bar{r} \) is the ex post mean return, \( T \) is the number of return periods and \( r_t \) is the return at period \( t \). It should be noted that it for real estate returns can be more proficient to estimate the ex ante returns with other procedures than the mean, because of the autocorrelation often inherent in historical real estate return figures (Geltner & Miller, 2001). However, as this paper will have an historical perspective those procedures are overlooked and the above definition of the mean is utilized throughout the paper.

The variance is the most common risk measure investors utilize when quantifying risk (Geltner & Miller, 2001). The variance measure the return dispersion about the mean return. The following definition of the variance measure estimates the variance from an historical sample of periodic returns:

\[ \sigma^2 = \frac{\sum_{t=1}^{T} (r_t - \bar{r})^2}{T - 1} \]

The variance is rather unintuitive to interpret as the variance is measured in units of squared returns. A more intuitive measure of the return dispersion about the mean return is the standard deviation which is measured in units of returns. The standard deviation is many times referred to as the volatility (Geltner & Miller, 2001). The standard deviation is simply the square root of the variance and is, from a historical series of periodic returns, computed in the following manner:

\[ \sigma = \sqrt{\frac{\sum_{t=1}^{T} (r_t - \bar{r})^2}{T - 1}} \]

It should be noted that the standard deviation measures both idiosyncratic risk as well as the systematic risk (Geltner & Miller, 2001). The idiosyncratic risk relates to the asset specific risk which can be diversified away by optimizing the asset allocation. The systematic risk
inherent in a specific asset can unfortunately not be diversified away by adding additional assets or change the asset weights within a portfolio. The systematic risk will always be present no matter of portfolio composition, more on that in later sections.

The above expressions are enough to calculate the standard deviation of a single asset, however the covariance must be incorporated in order to calculate the standard deviation of a portfolio. The covariance measure the pair wise comovement between two assets. The covariance estimate, more specifically, how the periodic returns from the two assets move together. The covariance can be defined in the following manner when utilizing historical returns:

\[
\sigma_{ij} = \frac{\sum_{t=1}^{T} [(r_{it} - \bar{r}_i)(r_{jt} - \bar{r}_j)]}{T-1}
\]

Where \(r_{it}\) is the return for asset \(i\) in the time period \(t\) and \(r_{jt}\) is the return for asset \(i\) at the time period \(t\). The covariance needs to be included in the portfolio structuring since the measure describes how much an additional asset in a portfolio contributes to diversification. In other words what effect is realized when adding an additional asset to the portfolio when examining the portfolio variance or the portfolio standard deviation. It may be easier to interpret the importance of the covariance and its implication for portfolio structuring if one examines the cross-correlation coefficient often simply referred to as the correlation coefficient. The correlation between two assets ranges from -1 to 1. A correlation of -1 is interpreted as perfect negative correlation, a correlation of 0 implies that the assets have no correlation and a correlation of 1 is interpreted as perfect positive correlation. The correlation measure is, in accordance with the covariance, the comovement between returns of two assets, however the difference lies in the unit of measure. The covariance is expressed in unit of squared return, while the correlation as stated previously only takes either a negative or positive value ranging from -1 to 1. Two assets which have negative correlation, when considering their return figures, imply that the returns from the two assets are moving in opposite direction. Accordingly, assets with a positive correlation move in the same direction. Asset compositions with a negative correlation contribute to larger diversification benefits than asset compositions which exhibit positive correlation. The correlation coefficient between assets \(i\) and \(j\) is defined as:

\[
\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}
\]

From the above formula it becomes clear that the covariance between to assets also can be written as:

\[
\sigma_{ij} = \sigma_i^2 \sigma_j^2 \rho_{ij}
\]

The adjacent section will elaborate on the above time-series statistics in order to construct an efficient portfolio in accordance with the MPT framework.
APPENDIX II

All formulas stem from IPD’s index guide as of January 2011

**Total return**

\[
TR_t = (CV_t - CV_{(t-1)} - CExp_t + CRpt_t + NI_t) * 100
\]

**Income return**

\[
INCR_t = \frac{(NI_t)}{(CV_{(t-1)} + CExp_t)} * 100
\]

**Capital growth**

\[
CVG_t = \frac{(CV_t - CV_{(t-1)} - CExp_t + CRpt_t)}{CV_{(t-1)} + CExp_t} * 100
\]

Where:

\(TR_t\) is the total return in month \(t\)

\(CV_t\) is the capital value at the end of month \(t\)

\(CExp_t\) is the capital expenditure (includes purchase and developments) in month \(t\)

\(CRpt_t\) is the capital receipts (includes sales) in month \(t\)

\(NI_t\) is the day-dated rent receivable during month \(t\), net of property management costs, ground rent and other irrecoverable expenditure