Maneuverability

Instructions for implementation of the “ABS guide for vessel maneuverability”

V3.0

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Introduction
This guide is meant as the main instruction for the ship maneuverability part of the KTH course in marine dynamics on a master level. Most of the theoretical basis is however given in the ABS guide for vessel maneuverability. This guide will instruct the reader to take on the material and perform tutorial exercises which will lead to a final implementation of a fully functional computer code for vessel maneuverability. A set of homework assignments are also included and will be marked according to course program.

Lectures/workshops will be held on the content and will serve as further clarification to the tutorial exercises which are meant to be done in sequence in order to work with the material.

Good luck and have fun! 😊

Jakob Kuttenkeuler

Tutorial exercises
Here is a set of exercises to practice your skills in modeling and solving techniques for equations of motions, especially with focus on ship maneuverability. Work with them in sequence as they are designed to guide you through the material.

Exercise 1: Free falling ball with air resistance
You will here train your skills of mechanical modeling, handling equations of motions, numerical treatment of the equations by numerical integration through calculating the free fall of a table tennis ball. Now, read Appendix A and use Matlabs ODE45 function to solve the equation. I have done most of the work for you 😊. Generate the following function where you only need to fill in a few lines and swap the xxx for something nicer. Also, learn to understand the code.

```
function Xval_p = freefall_EOM(t,Xval,m,rho ,A,Cd, g)
% a few simple lines are missing_
z = Xval(1);            % [m] Position
zp = Xval(2);          % [m/s] velocity
zpp = xxx;            % [m/s^2] Acceleration
Xval_p = [zp ; zpp];  % New system state vector
```

Then unleash ODE45 and verify my results in Appendix A. Also check the terminal speed against analytical solution.

Hint: Tell Jakob you want to borrow a ball to experimentally measure the falling time from the top floor in Sing-Sing to compare with your calculations.
**Exercise 2: The ball "parabolic" path with air resistance**

In this exercise we go from a single state variable (z) in the previous exercise to two (z and x). The fine thing is that basically nothing changes, you simply extend the state variable to become \( X_{val} = [x, z, x_p, z_p] \).

The implementation of the EOM into the function `ballpath_EOM` is almost finished as shown below. You need only to swap the xxx for something more useful. No new lines are allowed to be added.

```matlab
function xval_p = ballpath_EOM(t, xval, m, r, rho, A, Cd)
    % The function calculates and returns the time derivative
    % of the state variable xval
    x = xval(1);            % [m] x-Position
    z = xval(2);            % [m] z-Position
    x_p = xval(3);          % [m/s] Horizontal velocity
    z_p = xval(4);          % [m/s] Vertical velocity
    Fd = xxx;               % [N] Air resistance
    alfa = xxx;             % [rad] Angle of resistance vector
    F = [xxx];              % [N] External forces
    M = [xxx xxx]
    Xpp = M*F;
    xval_p = [x; z; x_p; z_p];
end
```

a) Calculate the path of a ball with mass \( m = 0.1 \)kg being thrown at an elevation angle of 45° from a height of 1.5m and with an initial speed of 60m/s. **Neglect air resistance.** Compare with my results in Figure 1.

![Figure 1. Ball flight neglecting air resistance.](image1.png)

b) Now add air resistance. The diameter of the ball is 0.1m, drag coefficient \( C_d = 1.0 \) and the air density is \( \rho = 1.2 \)kg/m\(^3\). My calculations say that the ball is thrown \( \approx 15.7 \)m. Do you agree?
Exercise 3: Linear maneuver model
I want you to study the derivation of the linear maneuver model to understand the concepts.
- Read and learn to explain the derivation of the basic EOM for a surface ship as explained in Appendix B, leading to the standard formulation of the kinematics as in ABS eq. A3.1.
- Learn to explain the Taylor expansion of the external loads leading to the linear external forces as described in ABS appendix 3, sec 2.3 & 2.4, leading to eq. A3.9.
- Read ABS Appendix 3, sec. 2.5 on linear rudder modeling.

Exercise 4: Non-Linear maneuver model
a) Read and learn to understand the general concepts of the “derivation” as done in ABS Appendix 3, sec 4.1-4.4 leading to the
  - surge equation eq. A3.59,
  - sway equation A3.68 and
  - yaw equation A3.76.
  Note that these equations are re-organized into equations A1.1, A1.2 and A1.3 of which the right hand terms will serve as input to A1.10.
b) Examine EOM eq. A1.10 to realize that it is on the form \( \dot{\eta} = f(t, \eta) \) with state variables \( \eta = \begin{bmatrix} u & v & r \end{bmatrix}^T \) and control variables \( \delta_R \) and \( \mu_c \).
c) Implement eq A1.10 in a Matlab function with the following call

function Xval_dot = ABS_EOM(t,Xval, rho, uc);
where Xval is the same as \( \eta \). Use the following lines (from ABS Appendix 1, sec 3.1) for the VLCC:

% Nondimensional derivatives
a = [-0.001331 -0.000894 -0.000894 -0.001722]; % Coeffs from A3.55
b = [-0.001011 -0.000649 0.001016 -0.000619];
c = [ 0.001827 0.001543 0.0000004 -0.000813];
Xup = -0.00086;
Xvr = 0.01095;
Xv = 0.00287;
Xdd = -0.001;
Xrr = 0.0;
Xvnn = 0.000;
Xddmn = -0.00135;
Yvp = -0.0146;
Ys = 0.000063;
Yv = -0.011;
Yvv = -0.0398;
Yr = 0.00394;
Ydr = 0.0;
Yvr = -0.0152;
Yrp = -0.00025;
Yd = 0.00416;
Ydn = 0.00416;
Yrn = 0.00138;
Yvn = -0.00266;
Yvvn = 0.0;
Ysn = 0.000063;
Nrp = -0.000964;
Ns = -0.000033;
Nv = -0.00798;
Nvv = 0.00754;
Nr = -0.00294;
Nrr = 0.0;
Nrn = 0;
Nvr = -0.00495;
Nvp = -0.00005;
Nd = -0.00216;
Ndn = 0;
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\[ \begin{align*}
N_v & = 0.00138; \\
N_vv & = 0.0; \\
N_d & = -0.00072; \\
N_n & = -0.00033;
\end{align*} \]

Solve the equation for a period of 1000 seconds with \( u_c = 15 \) knots and \( \delta_R = 0 \). Plot your results (especially the path but also turning rate and velocities) and reflect on your results. Play around with simulations to learn to know your code and model. Does the model behave as you expected? If not, find out why etc...

**Exercise 5: Play with your model**

What is the penalty in resistance for 15° angle of sideslip caused by e.g. hard wind in the ship side? Try to turn some of the derivatives on/off to "examine" their influence.

**Exercise 6: Simulation of Circle Test**

When you have confidence in your code let us turn into hard-core ship maneuvering 😊. Implement and simulate a circle test with \( u_c = 15 \) knots and \( \delta_R = 35° \) executed after 60 seconds of simulation. Reflect on your results. You may compare to ABS Appendix 1, section 3.3 if you like... even if I do not agree with their results... Observe that the rudder deflection rate might affect the results significantly.

**Exercise 7: Simulation of Zig-Zag test**

Simulate a 10 degree zig-zag test at \( u_c = 15 \) knots and evaluate the results according to criteria. ABS does this in Appendix 1, section 3.4. Observe that the rudder deflection rate might affect the results significantly.
Homework assignments

**M1: Full scale maneuverability tests**
Perform evaluate and report ship maneuverability experiments according to ABS-standards. You will either perform a circle test or zig-zag test depending on ship availability, group sizes etc. Do not repeat ABS-instruction but focus on methodology, results and reflective comments. Please indicate in your report the amount of hours you spent on the assignment.

**M2: Equations of motions: modeling and solving**
Write a short summary (a “manual” that you can use later in life) on how to numerically solve equations of motions of the type that you encounter here. Include single degree of freedom problems as well as multiple DOF problems. Come up with a “new problem” to model and analyze as example (You can basically choose any mechanical system you find in your everyday life, at home etc.). You can also e.g. report your table tennis experiments vs analysis etc. Also describe with your own words pedagogically how to rewrite the second order ordinary differential equation as a system of first-order differential equations. I expect ≈2-4 pages. Please indicate in your report the amount of hours you spent on the assignment.

**M3: Numerical assessment of ship maneuverability**
Perform evaluate and report ship maneuverability experiments according to the exercises in this tutorial. I want your report to include a short executive summary of relevant theory (=2 pages) and results and discussion (=2 pages). Please indicate in your report the amount of hours you spent on the assignment.
Appendix A: Converting Second-Order ODE to a First-order System

Many of our engineering equations describing the motion and other dynamic behavior of systems are described in the form of differential equations and our task is to solve them which means that we need to predict the behavior of the system as function of time to come given a set of initial conditions. Consider the second order (second derivatives exist) ordinary (only one space variable) differential equation.

\[ \ddot{\eta} = f(t, \eta, \dot{\eta}) \]

with initial conditions

\[ \begin{cases} \eta(t_0) = \eta_0 \\ \dot{\eta}(t_0) = \dot{\eta}_0 \end{cases} \]  

(1)

To simplify solving the equation we now want to rewrite the second order ordinary differential equation as a system of 2 first-order differential equations by introducing the new state variables \( \eta_1 \) and \( \eta_2 \). Rearrange (1) into

\[ \begin{cases} \eta_1 = \eta \\ \eta_2 = \dot{\eta} \end{cases} \]

which gives that (1) can now be rewritten

\[ \begin{cases} \dot{\eta}_1 = \eta_2 \\ \dot{\eta}_2 = f(t, \eta_1, \eta_2) \end{cases} \]

with initial conditions

\[ \begin{cases} \eta_1(t_0) = \eta_0 \\ \eta_2(t_0) = \dot{\eta}_0 \end{cases} \]  

(2)

To further explain the simple strategy – here is an example:

The free fall (in the negative z-direction) of an object in air from 10 meters height can be written as

\[ \ddot{z} = -g + \frac{\rho AC_d}{2m} |\dot{z}| \dot{z} \]

with initial condition \( z_0 = 10 \)

(3)

\( \dot{z}_0 = 0 \)

Which is by the introduction of Introducing \( z_1 = z \) and \( z_2 = \dot{z} \). rewritten as the system of first order equations on the form

\[ \begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -g + \frac{\rho AC_d}{2m} |\dot{z}| \dot{z} \end{cases} \]

(4)

\( \dot{z}_1(0) = z_0 \)

\( \dot{z}_2(0) = \dot{z}_0 \)
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with initial condition \( z_0 = 10 \)

\( \dot{z}_0 = 0 \)

which can now be integrated by e.g. the Matlab solver ODE45. When use (5) and ODE45 to calculate the fall of a table tennis ball from a height of 10 meters I get the results visualized in Figure 2.

![Graph showing z-position, velocity, and acceleration over time for a table tennis ball falling from a height of 10 meters.](image)

**Figure 2.** The results of integration of EOM for free falling table tennis ball.

![Graph showing drag coefficients as function of Reynolds number for various smooth, axially-symmetric bodies.](image)

**Figure 3.** Drag coefficients as function of Reynolds number for sphere etc.
Appendix B: EOM for surface ships

This section will derive the ship-fixed equation of motion stated in ABS eq. A3.1 with the simplification of \( x_{cg} = 0 \), i.e. that the ship center of gravity is at mid ship. The starting point is the definitions and notation shown in ABS Fig. A6 and the insight that Newton’s second law of motion applies.

At time \( t_0 = 0 \) the ship origin is located at the origin of the global frame of reference. Consider the ship as a rigid body with constant mass \( m \) (which might be including added mass). Newton’s second law of motion in the three degrees of freedom gives

\[
\begin{align*}
F_{x0} &= m\ddot{x}_0 \\
F_{y0} &= m\ddot{y}_0 \quad \text{(in global coordinate system)}, \\
M_{z0} &= I_{zz}\ddot{\psi}
\end{align*}
\]

Where the use of subfix "0" indicates that the equation is in reference to the global coordinate system.

Next step is to express equation (6) in the ship fixed coordinate system. From ABS Fig. A6 it is seen that the velocities are transformed between the ship referenced and the earth fixed system by the relations

\[
\begin{align*}
\dot{x}_0 &= u\cos\psi - v\sin\psi \\
\dot{y}_0 &= u\sin\psi + v\cos\psi
\end{align*}
\]

where the following substitution is done \( u = \dot{x} \) and \( v = \dot{y} \). After differentiation with respect to time (7) becomes

\[
\begin{align*}
\ddot{x}_0 &= \dot{u}\cos\psi - \dot{v}\sin\psi + (-u\sin\psi - v\cos\psi)r \\
\ddot{y}_0 &= \dot{u}\sin\psi + \dot{v}\cos\psi + (u\cos\psi - v\sin\psi)r
\end{align*}
\]

The forces can in a similar manner be transformed from the global system to the ship-fixed coordinate system as

\[
\begin{align*}
X &= F_{x0}\cos\psi + F_{y0}\sin\psi \\
Y &= -F_{x0}\sin\psi + F_{y0}\cos\psi
\end{align*}
\]

where the external loads in the ship fixed reference system is expressed as \( X \), \( Y \) and \( N \). The relation from (6) can now be expressed in the local ship fixed coordinate system by inserting (8) into (6) and then substituting into expressions in (9) which gives Newton’s second law of motion (often referred to as the equation of motion EOM) in the ship fixed coordinate system as

\[
\begin{align*}
X &= m(\ddot{u} - vr) \quad \text{(surge)} \\
Y &= m(\ddot{v} + ur) \quad \text{(sway)} \quad (= \text{ABS eq. A3.1 in local coords).} \\
N &= I_{zz}\ddot{\psi} \quad \text{(yaw)}
\end{align*}
\]

The equation of motion is now derived and ready to be used to simulate the ship behavior in the water as soon as the external loads \( X \), \( Y \) and \( N \) on the left hand side is available ☺.