1. Exercise, Lecture 8, October 23

Let $X \subseteq \mathbb{P}^2_k$ be the cuspidal cubic given by the equation $y^2 z = x^3$. The base ring k we assume to be algebraically closed.

Exercise I. Let $\mathscr{O}_X(1)$ denote the restriction of $\mathscr{O}(1)$ (on the projective plane) to the cusp X. Let s be a global section of $\mathscr{O}_X(1)$ that has support on the regular (non-singular) set of X. The degree of $\mathscr{O}_X(1)$ is the degree of the divisor we get from such a section s. What is the degree of $\mathscr{O}_X(1)$?

Exercise II. Let $\pi \colon Y \longrightarrow X$ denote the normalization of X, and let $\mathscr{L} = \pi^* \mathscr{O}_X(1)$.

(1) Determine the degree of $\mathscr{L}$, and determine $H^i(Y, \mathscr{L})$ for all i.
(2) Determine the linear system V that corresponds to the composite mapping $Y \longrightarrow \mathbb{P}^2$.
(3) Is V base point free?
(4) Does the linear system V separate points?
(5) Does the linear system V separate tangent vectors?
(6) Show that the complete linear system $\Gamma(Y, \mathscr{L})$ gives an embedding of Y into a projective space.